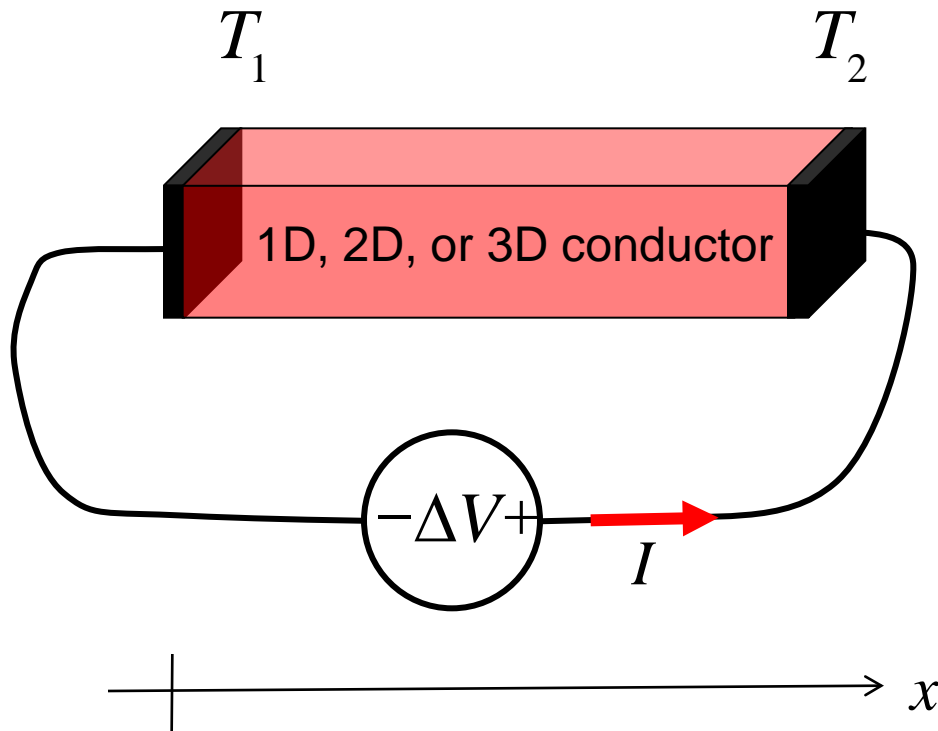# ECE-656: Fall 2009

Lecture 9:
Coupled Current Equations

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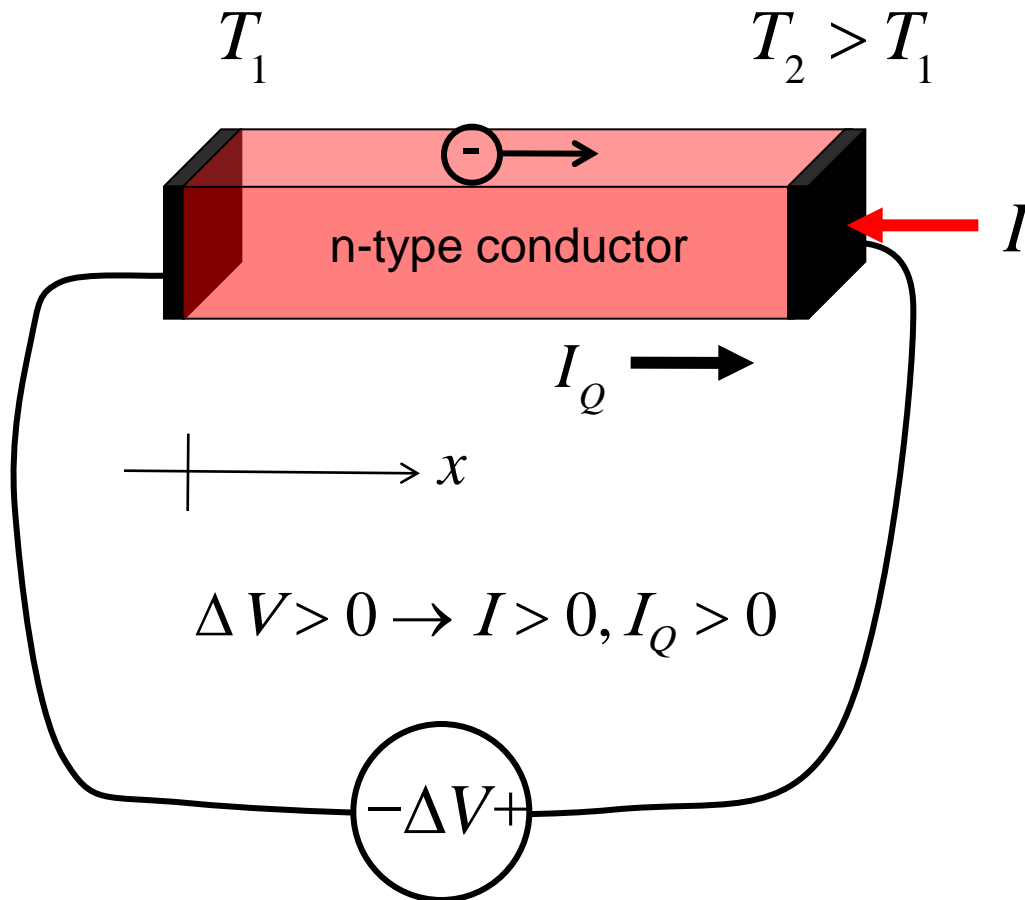
charge and heat currents

$$I = G\Delta V - [SG]\Delta T$$

$$I_Q = T[SG]\Delta V - K_0 \Delta T$$

driving "forces:"

1) differences in voltage
   (electrochemical potential)

2) differences in <u>temperature</u>

signs of the coefficient

T_1 $T_2 > T_1$

n-type conductor

I

I_Q

x

$\Delta V > 0 \rightarrow I > 0, I_Q > 0$

$-\Delta V+$
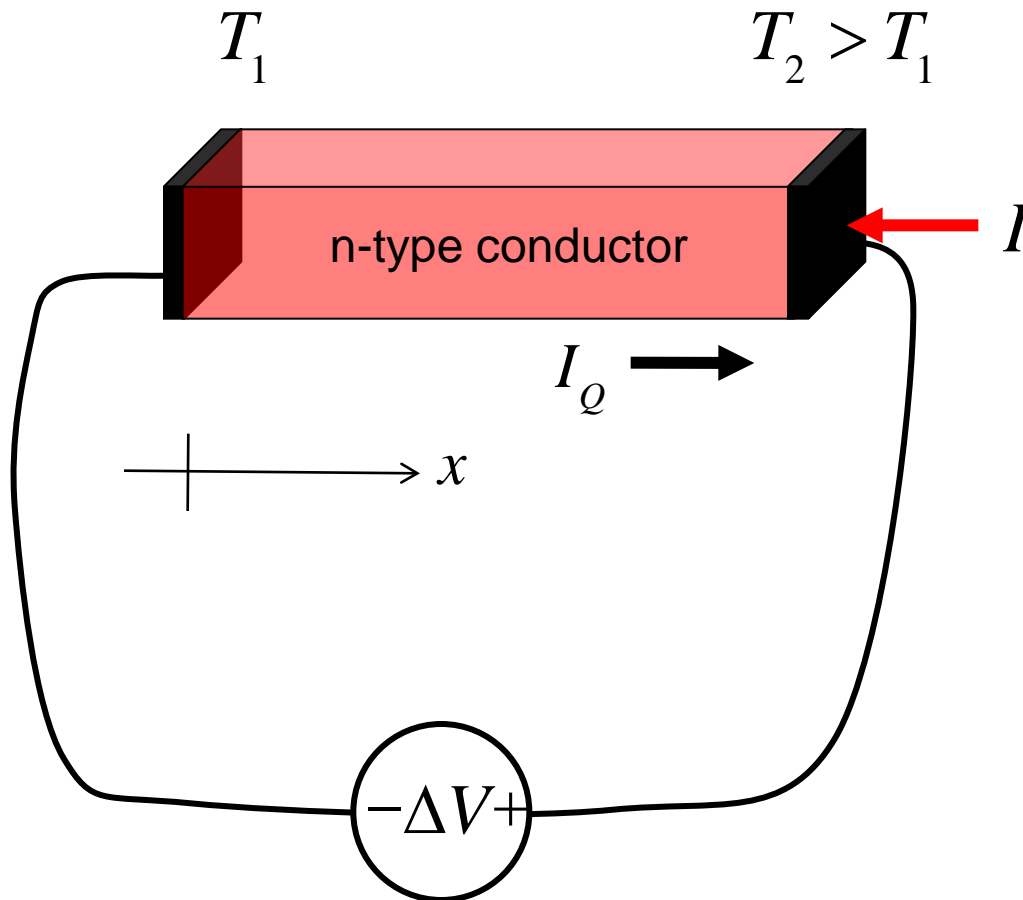
$$I = G\Delta V - [SG]\Delta T$$

$$I_Q = T[SG]\Delta V - K_0\Delta T$$

$$G > 0$$

$$[SG] > 0$$

$$K_0 > 0$$

(electronic) thermal conductance for $\Delta V = 0$

conductance vs. conductivity

T_1

$T_2 > T_1$

n-type conductor

I

I_Q

x

$-\Delta V+$

$$I = G\Delta V - [SG]\Delta T$$

$$I_Q = T[SG]\Delta V - K_0 \Delta T$$

3D conductors

$$G = \sigma \frac{A}{L}$$

$$K_0 = \kappa_0 \frac{A}{L}$$

etc.

temperature gradients

$$I \propto \left(f_1 - f_2 \right) = \left(f_1 - f_2 \right)_T + \left(f_1 - f_2 \right)_{E_F}$$

$$\left(f_1 - f_2 \right)\big|_T = -\left(\frac{\partial f_0}{\partial E_F} \right) \Delta E_F = \left(-\frac{\partial f_0}{\partial E} \right) (q\Delta V)$$

$$\left(f_1 - f_2 \right)\big|_{E_F} = \left(-\frac{\partial f_0}{\partial \Theta} \right) \Delta \Theta$$

$$= k_B T \left(-\frac{\partial f_0}{\partial E} \right) \times \frac{\left(E - E_F \right)}{k_B} \Delta \left(\frac{1}{T} \right)$$

$$f_0 = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

$$f_0 = \frac{1}{1 + e^{\Theta}}$$

$$\Theta = \left(E - E_F \right)/k_B T$$

The proper driving forces are changes in the electrochemical potential and in the <u>inverse</u> temperature.

coupled flows

$$I = L_{11}\Delta(E_F) + L_{12}\Delta\left(\frac{1}{T}\right)$$

$$I_Q = L_{21}\Delta E_F + L_{22}\Delta\left(\frac{1}{T}\right)$$

$$I_1 = L_{11}(B)F_1 + L_{12}(B)F_2$$

$$I_2 = L_{21}(B)F_1 + L_{22}(B)F_2$$

I_1, I_2 "generalized fluxes"

F_1, F_2 "generalized forces"

$L_{12} = L_{21}$ Onsager relation

example

temperature differences produce heat currents

pressure differences produce matter currents

heat flow per pressure difference = matter flow per temperature difference

Lars Onsaeger, Nobel Prize in Chemistry, 1968.

http.en.wikipedia.org/wiki/Onsaeger_reciprocal_relations

for more about this topic

A.C. Smith, J.F. Janak, and R.B. Adler, *Electronic Conduction in Solids*, McGraw-Hill, New York, 1967.

Irreversible thermodynamics: Chapter 2

Onsager relations: Chapter 3

outline

1) Onsager relations

2) **Measurement considerations**

3) Thermoelectric devices

measurements

$$I = G\Delta V - [SG]\Delta T$$

$$I_Q = T[SG]\Delta V - K_0\Delta T$$

$$\Delta V = RI - S\Delta T$$

$$I_Q = -\pi I - K_e\Delta T$$

two forms of the equations

$$I = G\Delta V - [SG]\Delta T$$

$$I_Q = T[SG]\Delta V - K_0\Delta T$$

$$\Delta V = RI - S\Delta T$$

$$I_Q = -\pi I - K_e\Delta T$$

$$\pi = TS < 0$$

$$K_e = K_0 - T\frac{[SG]^2}{G} > 0$$

electronic thermal
conductance for $I = 0$

$$\Delta V = \frac{1}{G}I + \frac{[SG]}{G}\Delta T$$

$$R = 1/G \quad S = -[SG]/G$$

$$\Delta V = RI - S\Delta T$$

$$I_Q = T[SG](RI - S\Delta T) - K_0\Delta T$$

$$I_Q = T\frac{[SG]}{G}I - (K_0 + T[SG]S)\Delta T$$

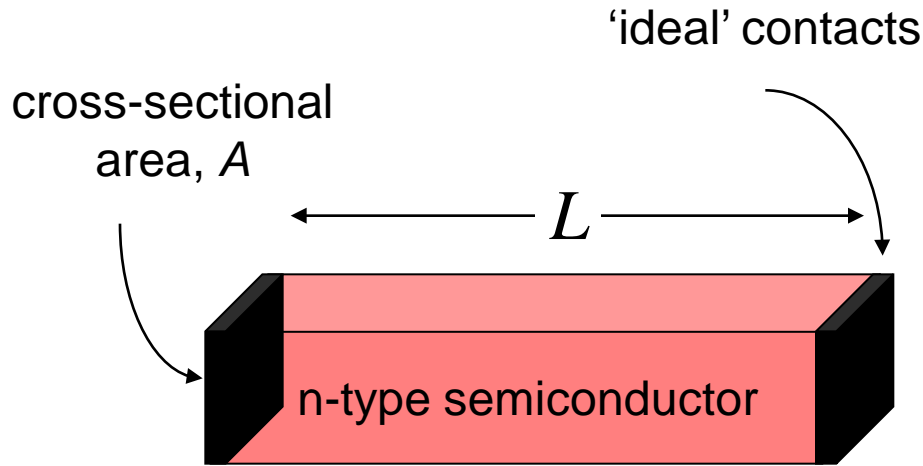$$I_Q = -TSI - \left(K_0 - T\frac{[SG]^2}{G}\right)\Delta T$$

resistivity / conductivity measurements

$$I = G\Delta V - [SG]\Delta T$$

$$I_Q = T[SG]\Delta V - K_0\Delta T$$

$$\Delta V = RI - S\Delta T$$

$$I_Q = -\pi I - K_e\Delta T$$

People generally measure **resisitivity** (or **conductivity**) because for bulk materials, these parameters depend on material properties and not on the length of the resistor or its width or cross-sectional area.

resistance and resistivity

'ideal' contacts

cross-sectional area, A

L

n-type semiconductor

How does the resistivity depend on the parameters of the semiconductor?

$$I = V / R$$

$$R \propto \frac{L}{A} \; \Omega$$

$$R = \rho \frac{L}{A} \; \Omega$$

resistivity:

ρ ½-m

conductance and conductivity

'ideal' contacts

cross-sectional area, A

L

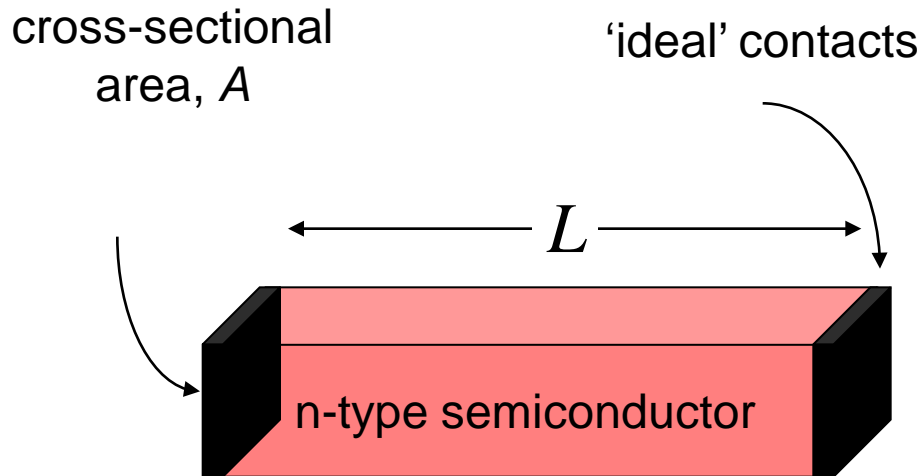n-type semiconductor
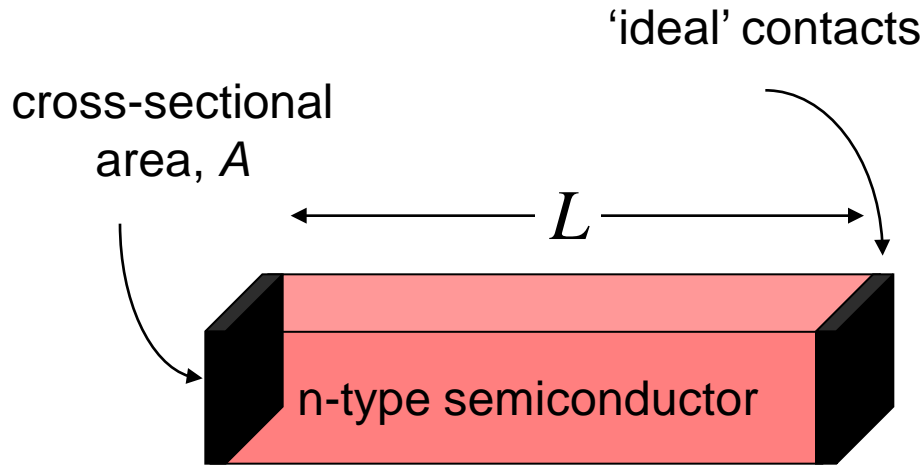
$$I = GV = V/R$$

$$G = \frac{1}{R} \propto \frac{A}{L} \ \text{S} \left(\Omega\right)^{-1}$$

$$G = \sigma \frac{A}{L}$$

conductivity:

$$\sigma = \frac{1}{\rho} \ \text{S/m}$$

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traditional conductance and conductivity

cross-sectional area, A

'ideal' contacts

n-type semiconductor

$\langle \upsilon \rangle$

$Q = (qn)AL$

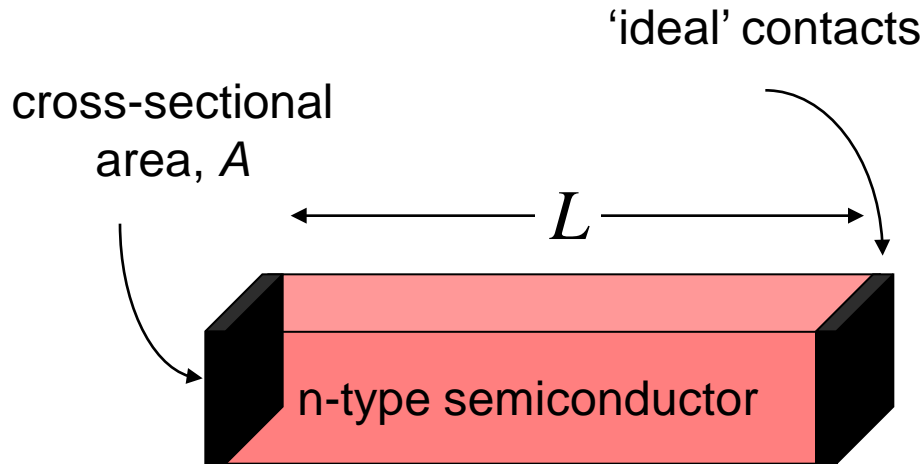$$I = GV \equiv \frac{Q}{\tau} \text{ Amperes}$$

$$\langle \upsilon \rangle = \mu_n E = \mu_n \frac{V}{L} \text{ cm/sec}$$

$$\tau = \frac{L}{\langle \upsilon \rangle} = \frac{L^2}{\mu_n V}$$

$$I = \left\{ (nq\mu_n) \frac{A}{L} \right\} V = GV$$

$$G = \sigma \frac{A}{L} \qquad \sigma = nq\mu_n$$

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Landauer conductance and conductivity

‘ideal’ contacts

cross-sectional
area, A

L

n-type semiconductor

$$I = GV$$

$$G \propto \frac{A}{L} \text{ S} \left(\Omega \right)^{-1}$$

$$G = \sigma \frac{A}{L} \qquad \sigma = \frac{1}{\rho} \text{ S/m}$$

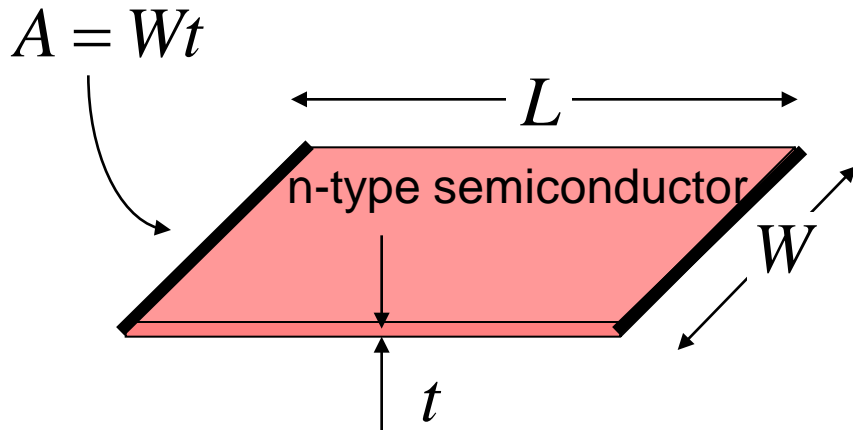$$G = \frac{2q^2}{h} \left(\int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \right)$$

$$\sigma = \frac{G}{A/L} = \frac{2q^2}{h} \left(\int \lambda(E) \frac{M(E)}{A} \left(-\frac{\partial f_0}{\partial E} \right) dE \right) \qquad \left(T(E) \approx \lambda(E)/L \right)$$

sheet conductance

$A = Wt$

L

n-type semiconductor

W

t

Top view

W

L
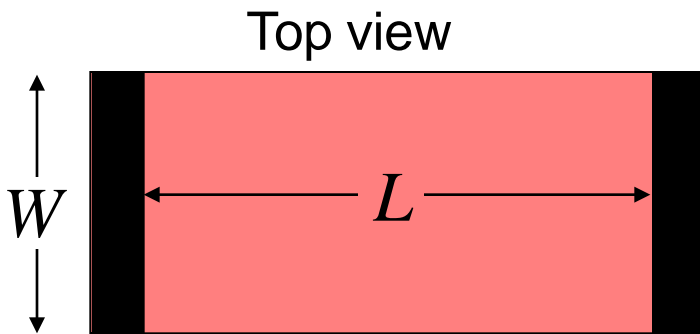
$I = GV$
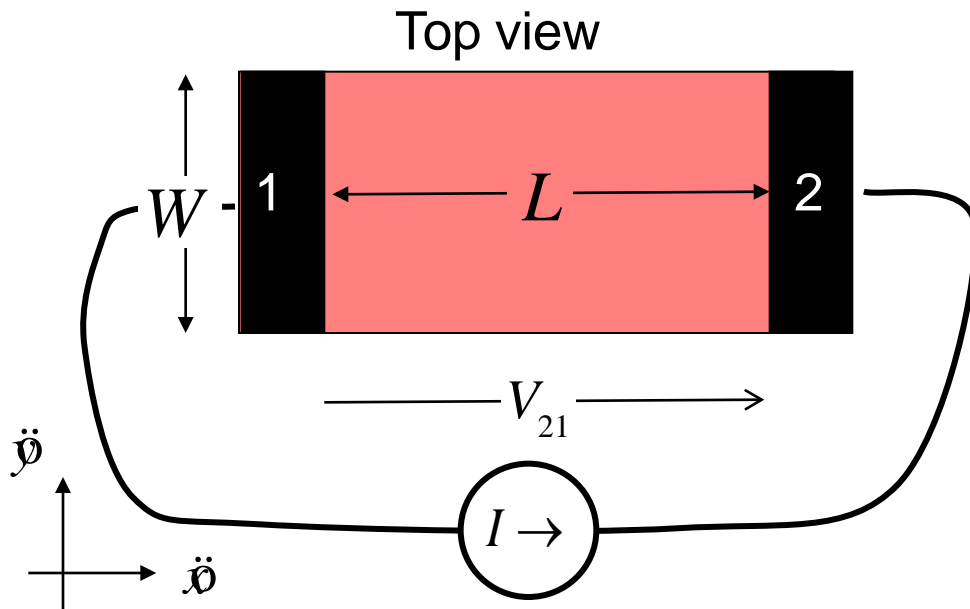
$$G = \sigma \frac{A}{L} \qquad \sigma = nq\mu_n$$

$$G = \sigma \left(\frac{Wt}{L} \right) = nq\mu_n t \left(\frac{W}{L} \right)$$

$$G = \sigma_S \left(\frac{W}{L} \right)$$

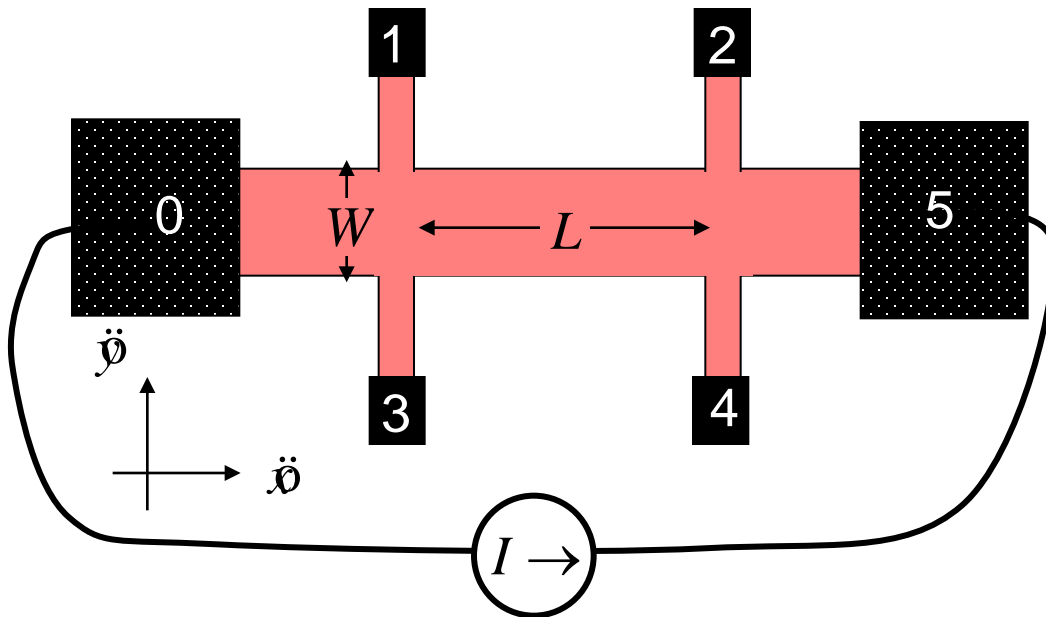$$\sigma_S = n_S q \mu_n$$

'sheet conductance'

2-probe measurements

Top view

$$R_{DEV} = \rho_S \frac{L}{W}$$

$$V_{21} = I\left(2R_C + R_{DEV}\right)$$

$$R_{DEV} \neq \frac{V_{21}}{I}$$
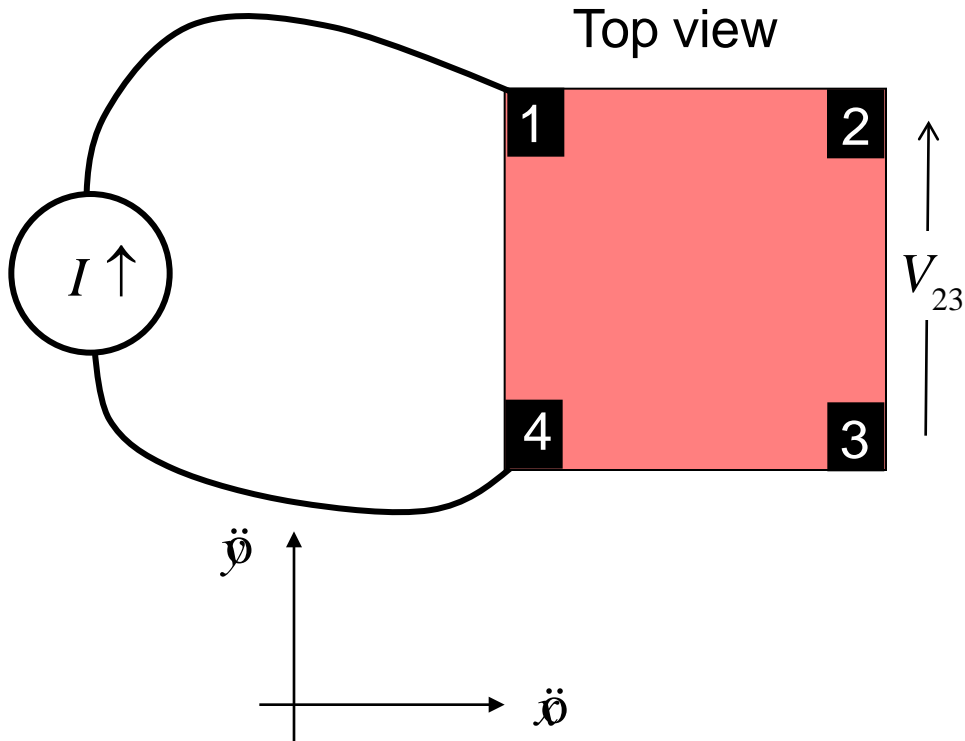
Lundstrom EE-656 F09

4-probe measurements

Top view

$$R_{DEV} = \rho_S \frac{L}{W}$$

$$V_{21} = I\left(R_{DEV}\right)$$

(high impedance voltmeter)

"Hall bar geometry"
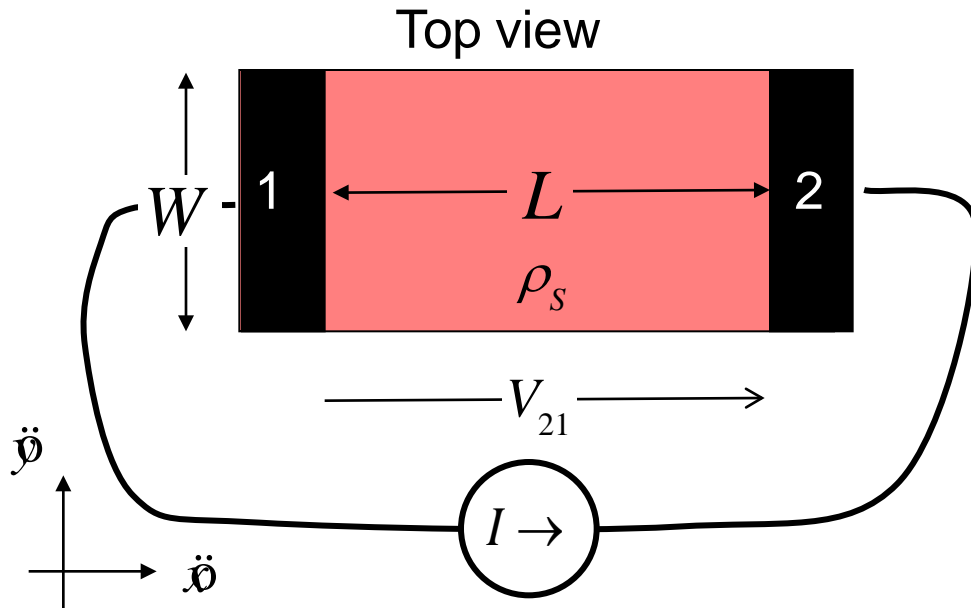
van der Pauw technique

Top view

$$R_{1,4/2,3} \equiv \frac{V_{23}}{I_{14}} \qquad R_{2,1/3,4} \equiv \frac{V_{34}}{I_{21}}$$

$$e^{-\frac{\pi}{\rho_S}R_{1,4/2,3}} + e^{-\frac{\pi}{\rho_S}R_{2,1/3,4}} = 1$$

$$\Rightarrow \rho_S$$

Force *I* through two contacts, measure *V* between the other two contacts.

isothermal vs. adiabatic

Top view

W

1

L

ρ_S

2

V_{21}

$I \rightarrow$

$$R_{DEV} = \rho_S \frac{L}{W}$$

$$V_{21} = R_{DEV}I - S\Delta T$$

$$I_Q = -\pi I - K_e \Delta T$$

i) $\Delta T = 0$

$$V_{21} = R_{DEV}I \rightarrow R_{DEV} = V_{21}/I$$

ii) $I_Q = 0$

$$\Delta T = \left(-\pi/K_e\right)I$$

$$V_{21} = R_{DEV}I + \left(S^2 T/K_e\right)I$$

$$V_{21}/I = R_{DEV} + \left(S^2 T/K_e\right)$$

for more about low-field measurments

D.C. Look, *Electrical Characterization of GaAs Materials and Devices*, John Wiley and Sons, New York, 1989.
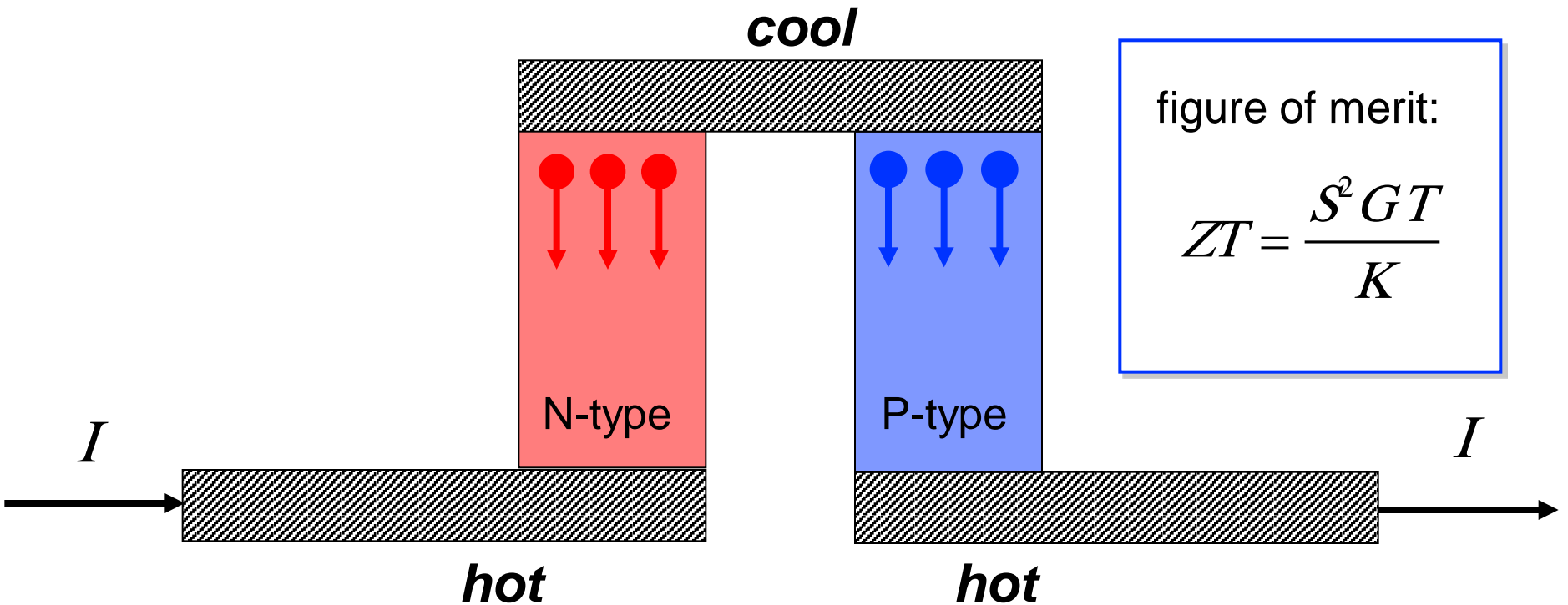
L.J. van der Pauw, "A method of measuring specific resistivity and Hall effect of discs of arbitrary shape," *Phillips Research Reports*, vol. 13, pp. 1-9, 1958.
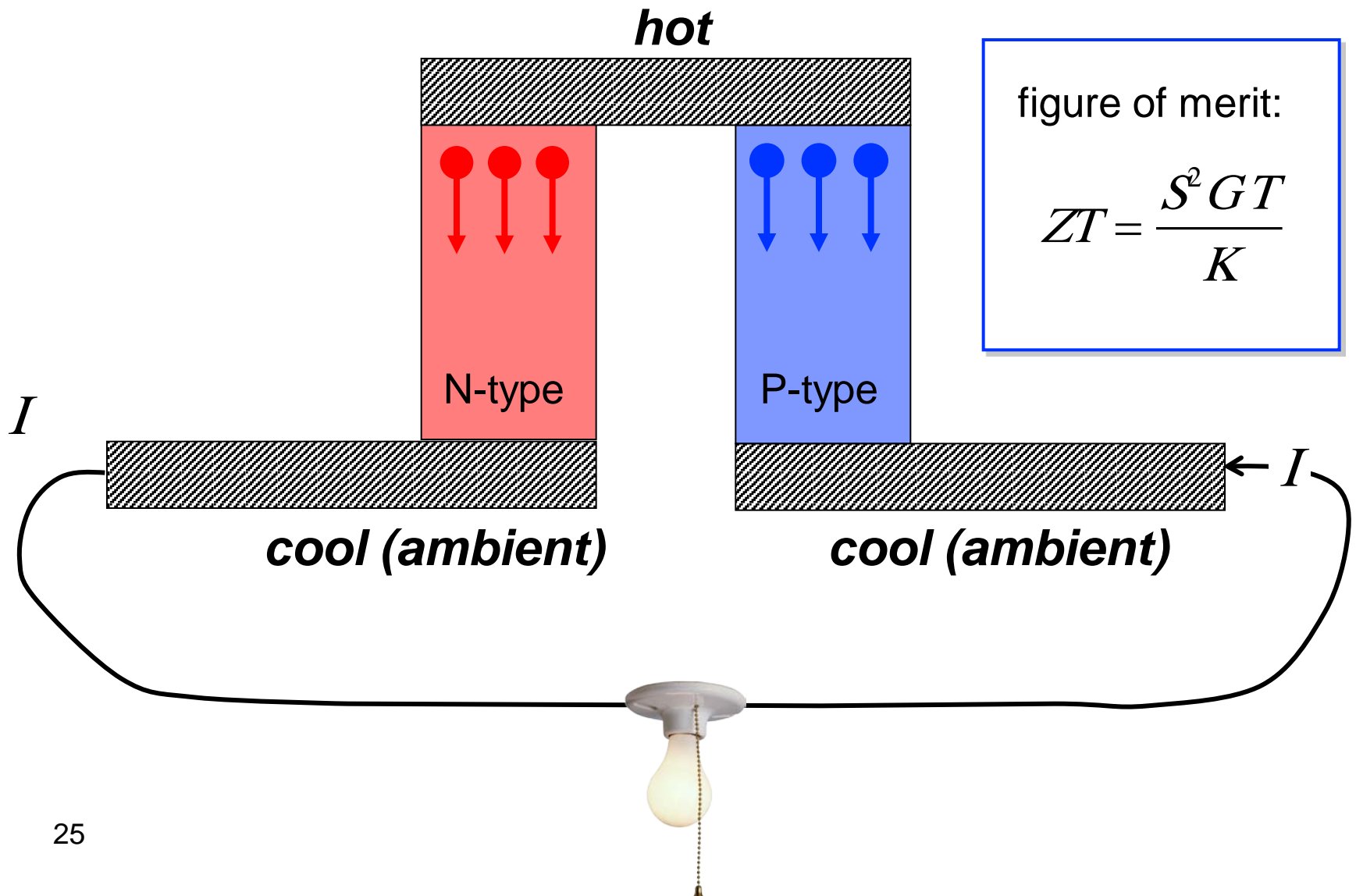
Lundstrom, Chapter 4, Sec. 7

outline

1) Onsager relations

2) Measurement considerations

3) **Thermoelectric devices**

thermoelectric devices: cooling

cool

N-type

P-type

I

I

hot

hot

figure of merit:

$$ZT = \frac{S^2 G T}{K}$$

thermoelectric devices: power generation

hot

N-type

P-type

I

I

cool (ambient)   **cool (ambient)**

figure of merit:

$$ZT = \frac{S^2 GT}{K}$$

the TE industry: 2009

1) NASA: Radioisotope Thermoelectric Generators for deep-space missions.

2) DOD: cooling for lasers, night vision, sensors, guidance systems, etc.

1) World market for commercial TE power generation: $30-50M/yr

2) World market for commercial TE cooling: $200-250M/yr

Automobile market (seat cooler/heater) is growing.   Vehicle waste heat recovery to improve fuel efficiency is a potential 'killer-app.'

Lundstrom EE-656 F09

TE heat engine
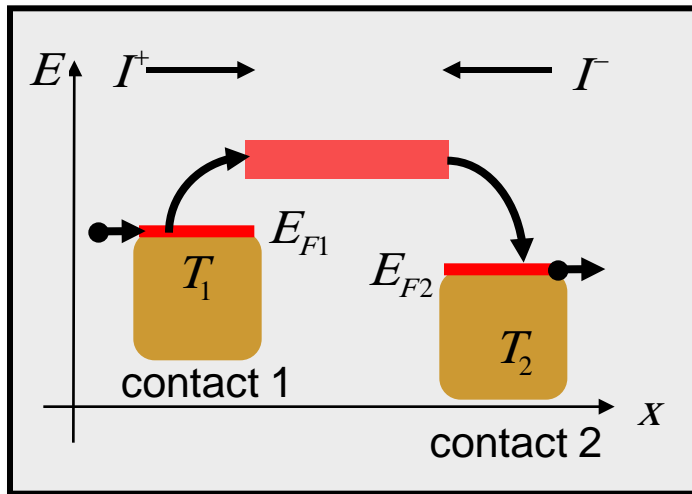
If we heat contact 1, and keep contact 2 cool, what is the maximum efficiency with which we can heat into useful work?
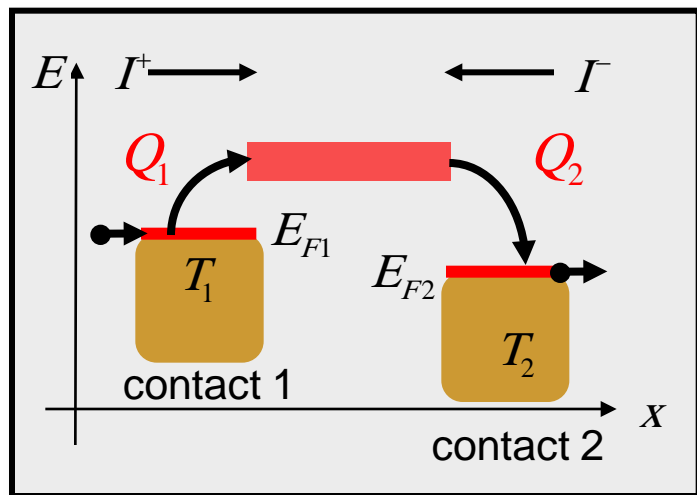
Answer:

Maximum efficiency:

$$\eta_{\max} = 1 - \frac{T_2}{T_1} = 1 - \frac{T_{COLD}}{T_{HOT}}$$

Carnot's theorem

what's necessary for current to flow?

$$I(E) = \frac{2q}{h} T(E) M(E) \left(f_1 - f_2 \right)$$

Current flows when there is a difference in Fermi levels.

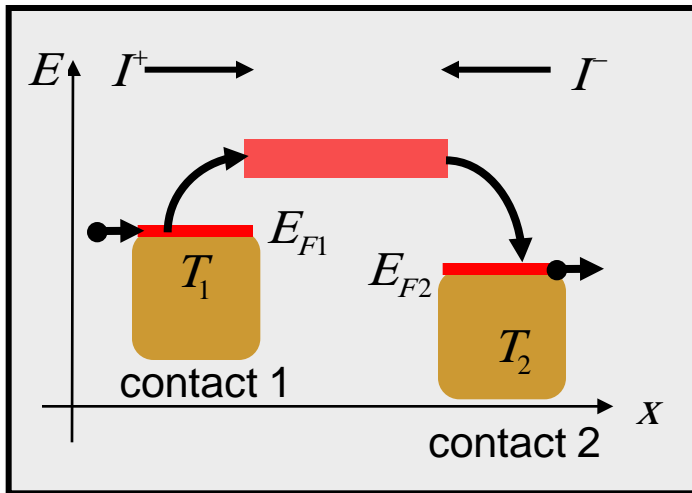$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T_1}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T_2}}$$

$$\boxed{\frac{Q_1}{T_1} < \frac{Q_2}{T_2}}$$

for current to flow

$$I(E) > 0 \Rightarrow f_1 > f_2$$

$$\Rightarrow \frac{\left(E - E_{F1} \right)}{k_B T_1} < \frac{\left(E - E_{F2} \right)}{k_B T_2}$$

maximum efficiency of a TE heat engine

$$Q_1 / T_1 < Q_2 / T_2$$

Maximum efficiency:

$$\eta_{\max} = 1 - \frac{T_2}{T_1} = 1 - \frac{T_{COLD}}{T_{HOT}}$$

Carnot's theorem

Thermal energy absorbed from contact 1:

$$Q_1 = \left(E - E_{F1} \right)$$

Thermal energy dissipated into contact 2:
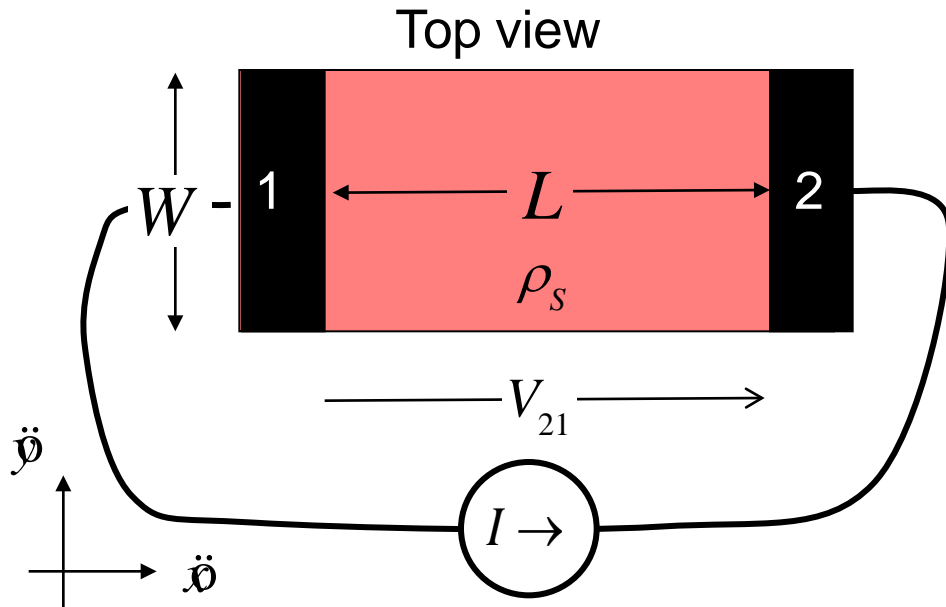
$$Q_2 = \left(E - E_{F2} \right)$$

Net thermal energy absorbed from contact 1 that can be converted into useful work:

$$Q_1 - Q_2$$

Efficiency:

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{Q_2 / T_2}{Q_1 / T_1} \left(\frac{T_2}{T_1} \right)$$

maximum temperature difference

Top view

W ⊣ | 1 | ←———— L ————→ | 2

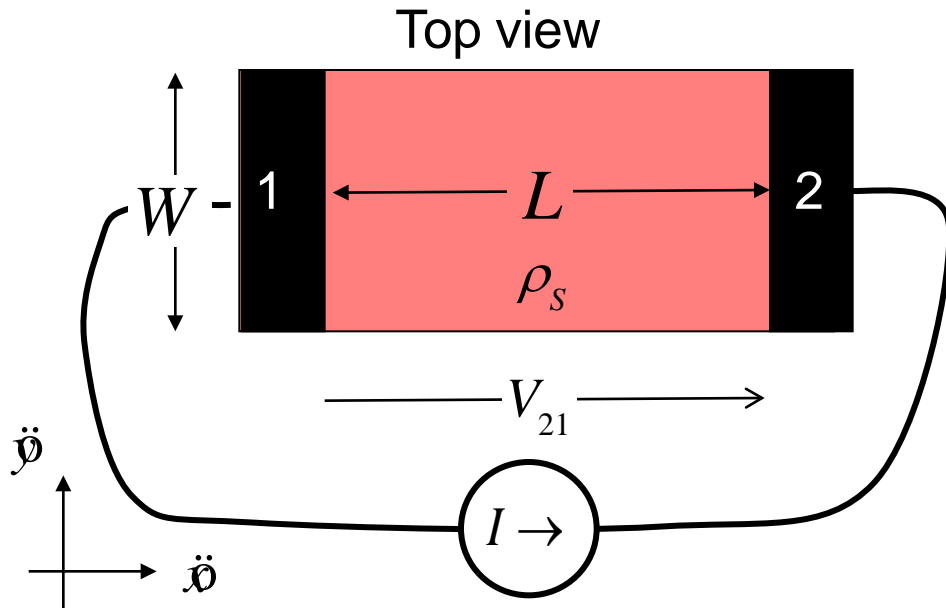ρ_S

———— V_{21} ————→

$I \rightarrow$

$$\Delta V = RI - S\Delta T$$

$$I_Q = -\pi I - K_e \Delta T$$

If we force a current through this thermoelectric device, what is the maximum temperature difference, $\Delta T = T_2 - T_1$, that we can produce?

maximum temperature difference

Top view

$$\Delta V = RI - S\Delta T$$

$$I_Q = -\pi I - K_e \Delta T$$

$$I_{Q1} = -TSI - K_e \Delta T - \frac{1}{2}I^2 R$$

For a fixed current, the temperature will rise until $I_Q = 0$

$$I_{opt} = -TS/R$$

$$\Delta T = -\frac{\left(TSI + I^2 R/2\right)}{K_e}$$

$$\Delta T\big|_{max} = \frac{1}{2}\frac{T^2 S^2 G}{K}$$

$$\frac{\partial \Delta T}{\partial I} = -\frac{\left(TS + IR\right)}{K_e} = 0$$

maximum temperature difference

$$\Delta T\big|_{\text{max}} = \frac{1}{2}\frac{T^2 S^2 G}{K}$$

$$\frac{\Delta T}{T}\bigg|_{\text{max}} = \frac{1}{2}\frac{S^2 GT}{K} = \frac{1}{2}ZT$$

$$ZT = \frac{S^2 GT}{K_e + K_L}$$ dimensionless figure of merit

K_e: electronic heat conductance

K_L: lattice heat conductance

figure of merit

Power factor:
   -large Seebeck coefficient
   -high conductance
   -depends on material parameters
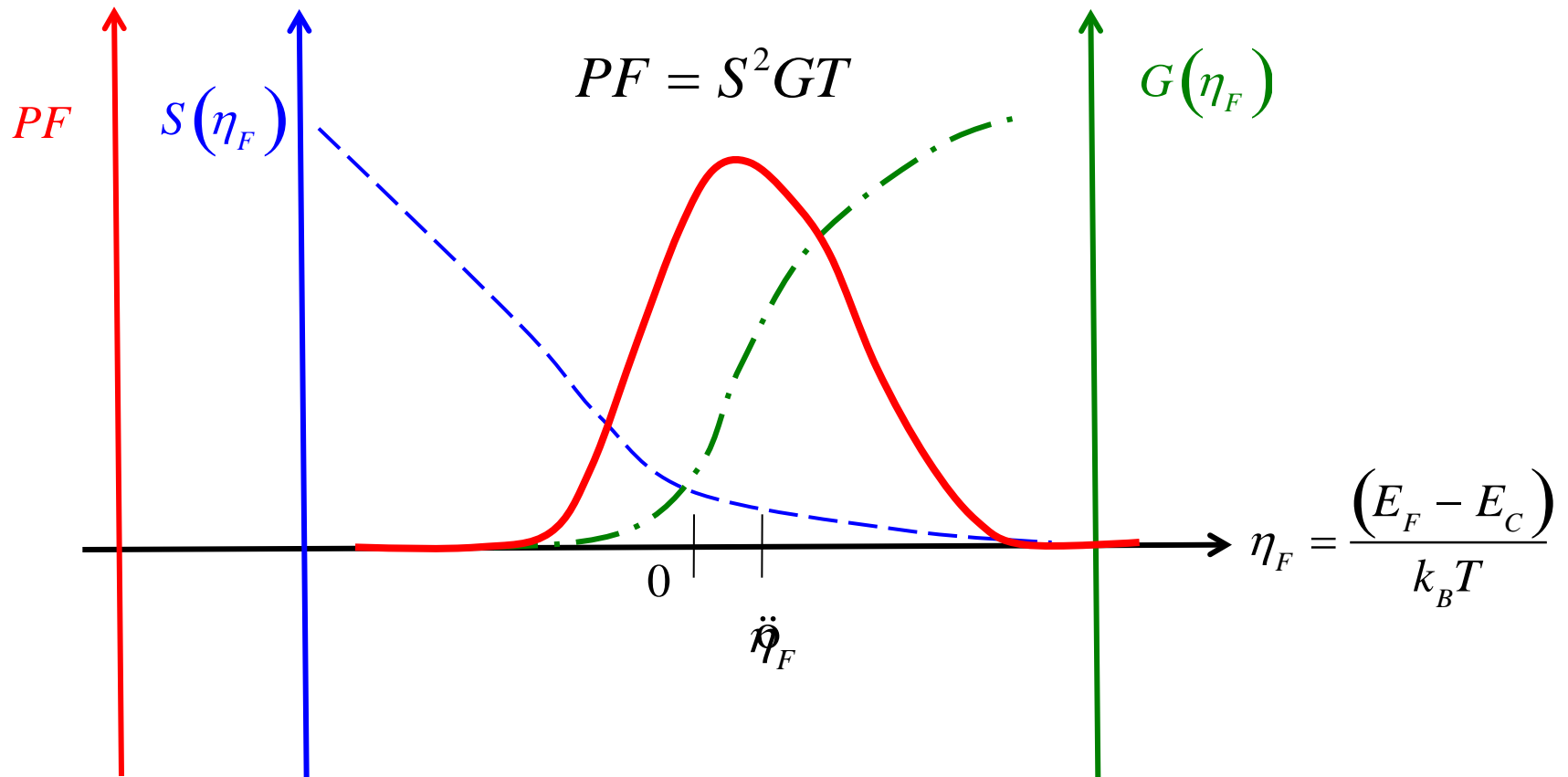
$$ZT = \frac{S^2 G T}{K_e + K_L}$$

thermal conductance:
   -$K_L > K_e$
   -need short mfp for phonons
    (alloys with large mass difference –
      e.g. Bi_2Te_3, SiGe)
   -nanostructuring for reducing mfp
   -"phonon glass electron crystal"

G.D. Mahan and J.O. Sofo, "The Best Thermoelectric," *Proc. Natl. Acad. Sci.*, **93**, 7436, 1996
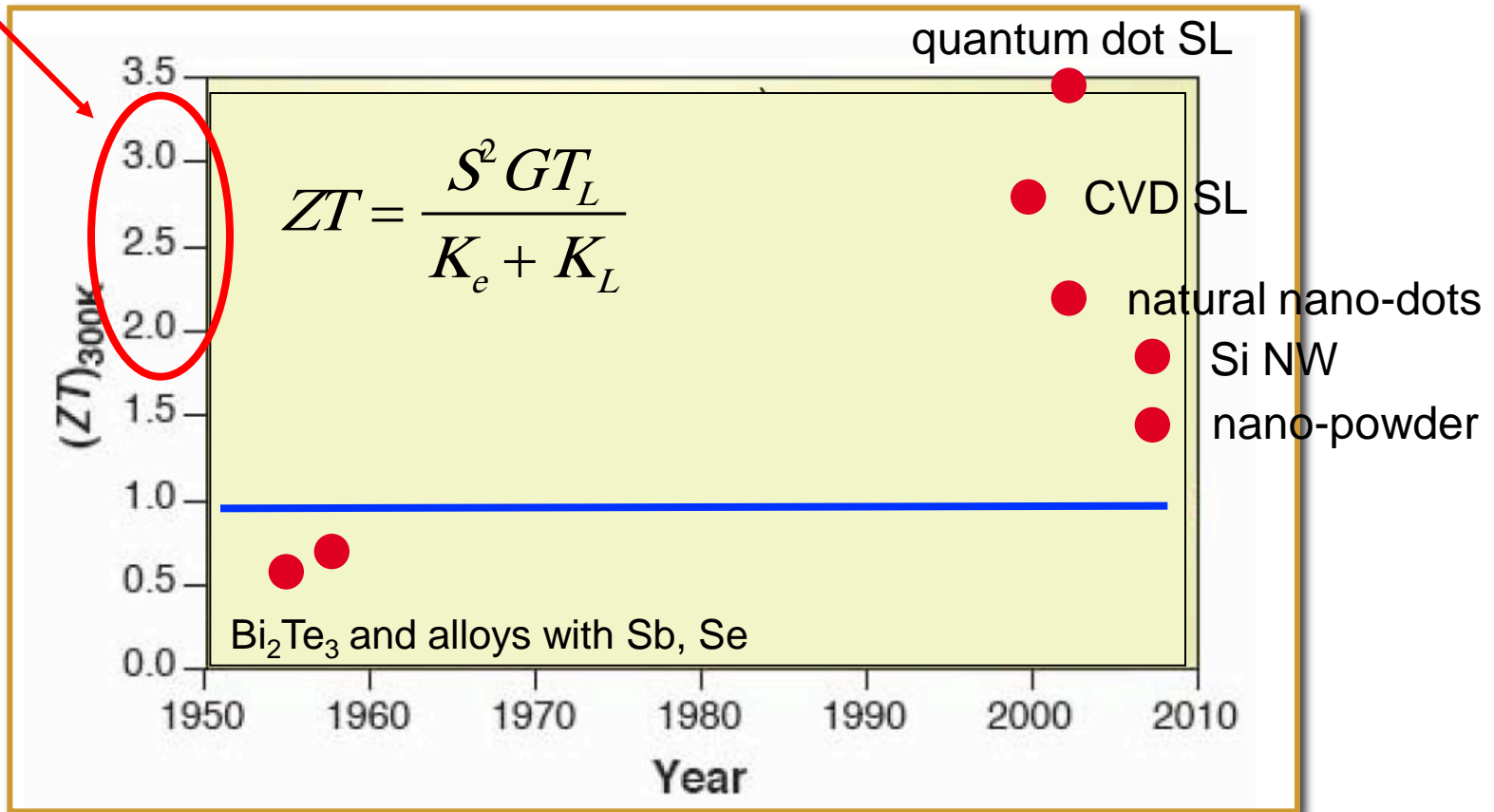
optimal power factor

$$PF = S^2 G T$$

The optimum power factor occurs when E_F is near E_C.

history and prospects

holy grail

nanostructured thermoelectrics

quantum dot SL

$$ZT = \frac{S^2 G T_L}{K_e + K_L}$$

CVD SL

natural nano-dots

Si NW

nano-powder

Bi_2Te_3 and alloys with Sb, Se

$(ZT)_{300K}$

Year

questions

1) Onsager relations

2) Measurement considerations

3) Thermoelectric devices