ECE-656: Fall 2009

Lecture 11: Discussion

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outline

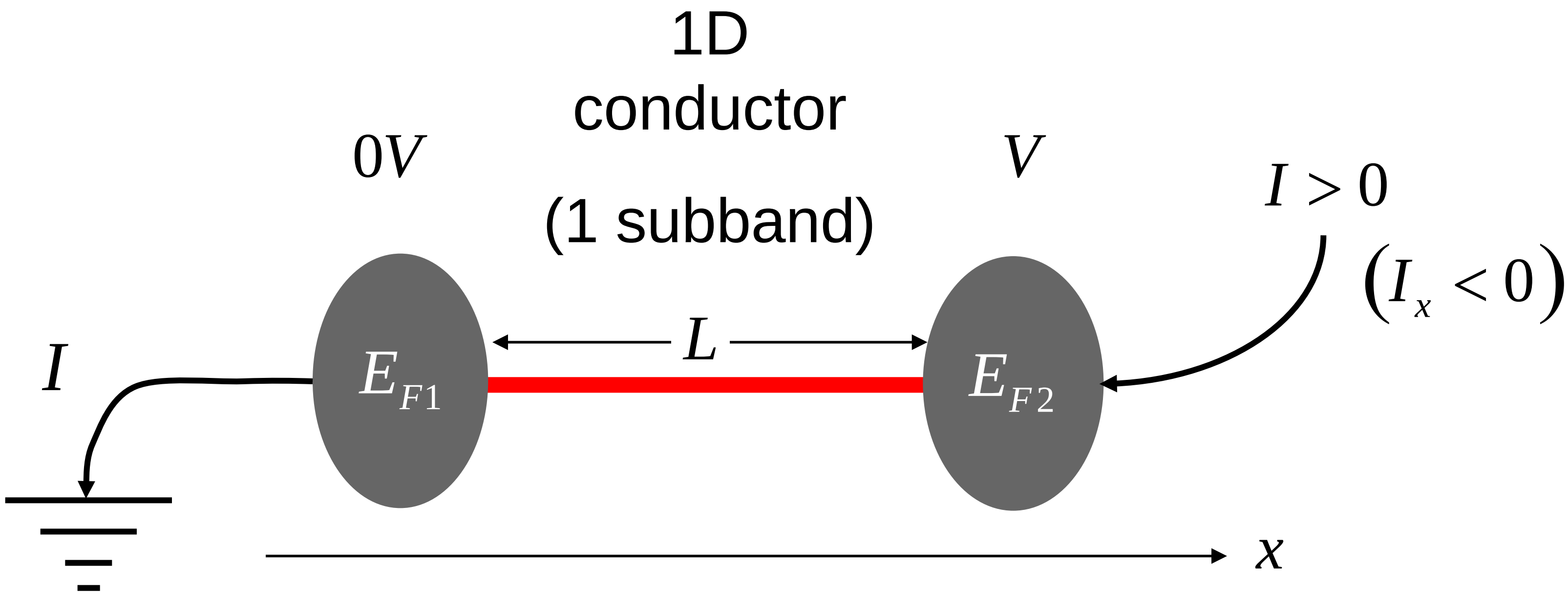
1) P-type conductors and thermoelectrics

2) ZT figure of merit

3) Maximizing the “power factor”

4) TE parameters for non-degenerate semiconductor

Lecture 5: 1D conductor



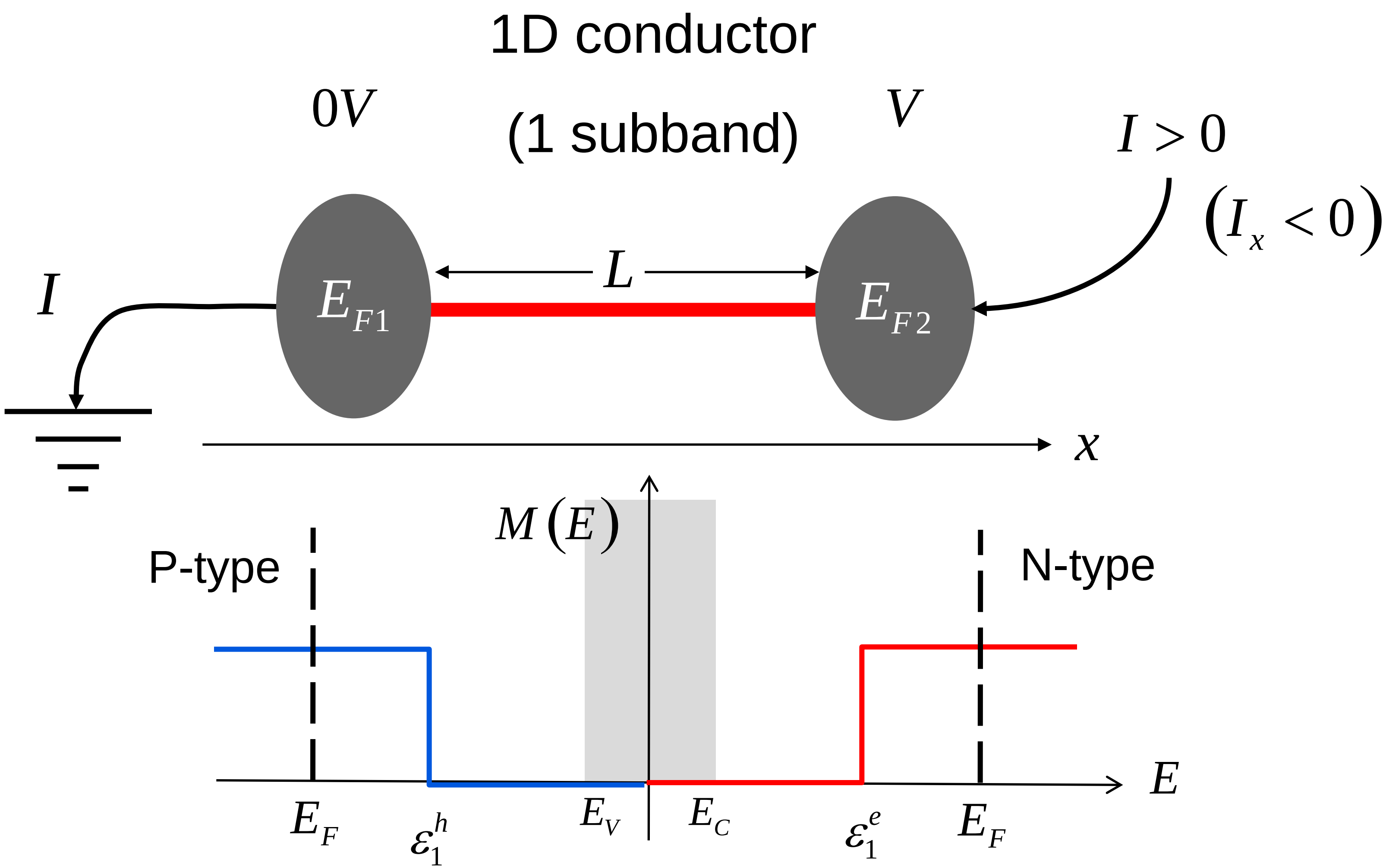
$$G = \frac{2q^2}{h}\int_{-\infty}^{+\infty} T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$T(E) \approx \frac{\lambda_0}{L}$$ (diffusive)

Does this apply to an n- or p-type conductor?

Answer: both

modes


1D conductor
(1 subband)
$0V$
V
$I > 0$
$(I_x < 0)$
I
E_{F1}
L
E_{F2}
x
$M(E)$
P-type
N-type
E
E_F
ε_1^h
E_V
E_C
ε_1^e
E_F

case i): n-type, T = 0K

$$G_{1D} = \frac{2q^2}{h}\int_{\varepsilon_1^e}^{+\infty} T(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$G_{1D} = \frac{2q^2}{h}\frac{\lambda(E_F)}{L}$$

$$G = n_L q \mu_n \frac{1}{L}$$

$$\mu_n = \frac{q\,\tau(E_F)}{m_n^*}$$

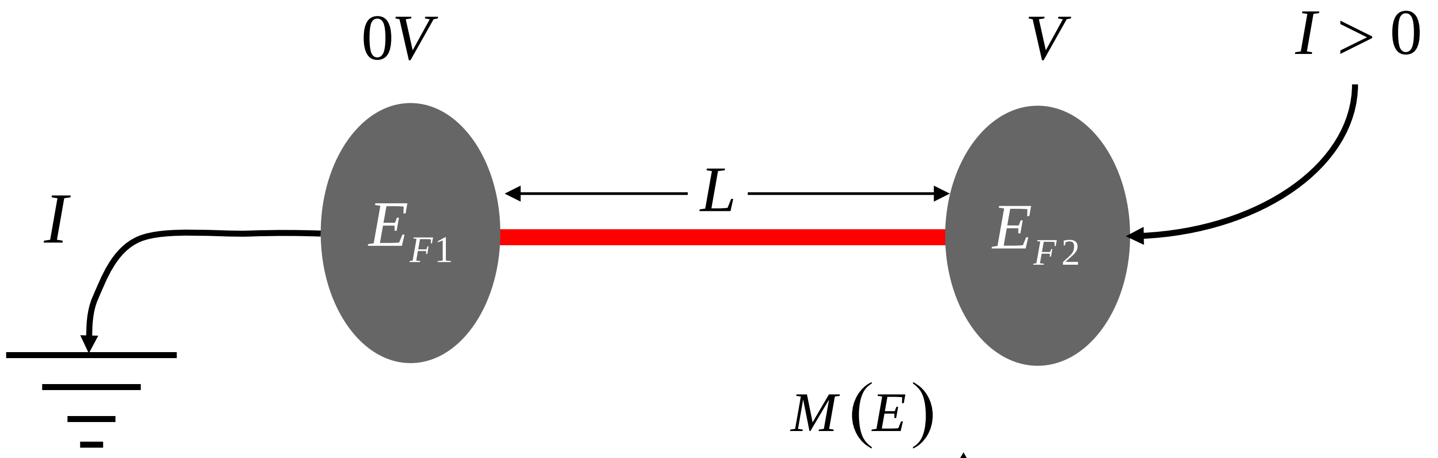
$$N = \frac{2k_F}{2\pi/L} \times 2$$

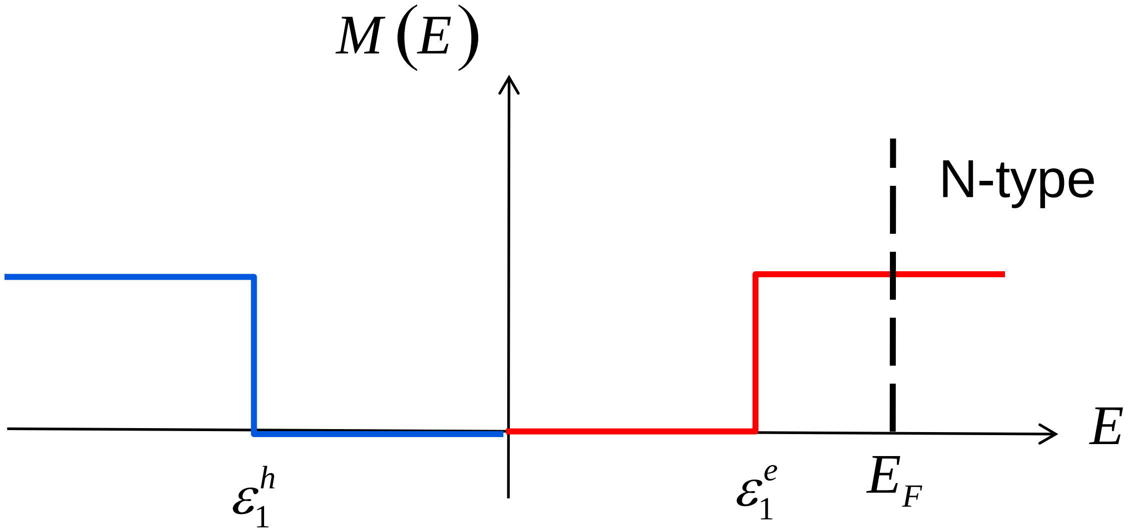
$$n_L = \frac{2k_F}{\pi} = \frac{2\sqrt{2m_n^*(E_F - \varepsilon_1)}}{\pi\hbar}$$

$$\lambda(E_F) = 2\upsilon(E_F)\tau(E_F)$$

$$\upsilon(E_F) = \sqrt{\frac{2(E_F - \varepsilon_1)}{m_n^*}}$$

case i): result





$$G_{1D} = \frac{2q^2}{h} \lambda(E_F)/L$$

$$G = n_L q \mu_n / L$$

$$\mu_n = q\,\tau(E_F)/m_n^*$$

$$V > 0 \Rightarrow I > 0$$

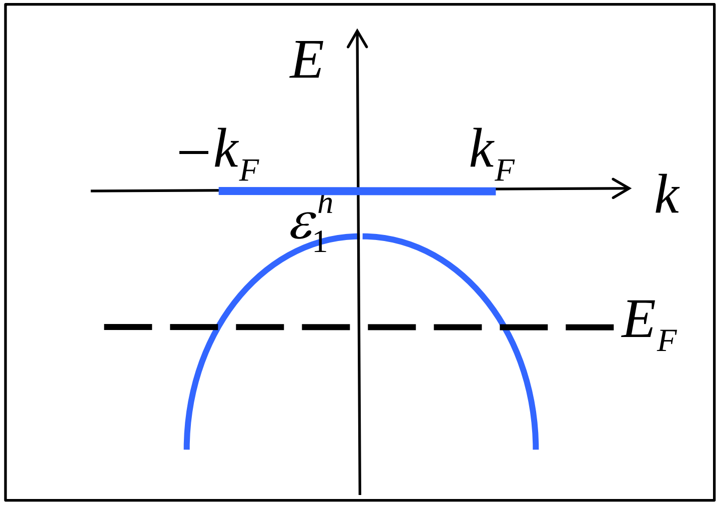
case ii): p-type, T = 0K

$$G_{1D} = \frac{2q^2}{h} \int_{-\infty}^{\varepsilon_1^h} T(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$G_{1D} = \frac{2q^2}{h} \frac{\lambda(E_F)}{L}$$

$$G = p_L q \mu_p \frac{1}{L}$$

$$\mu_p = \frac{q\,\tau(E_F)}{m_p^*}$$



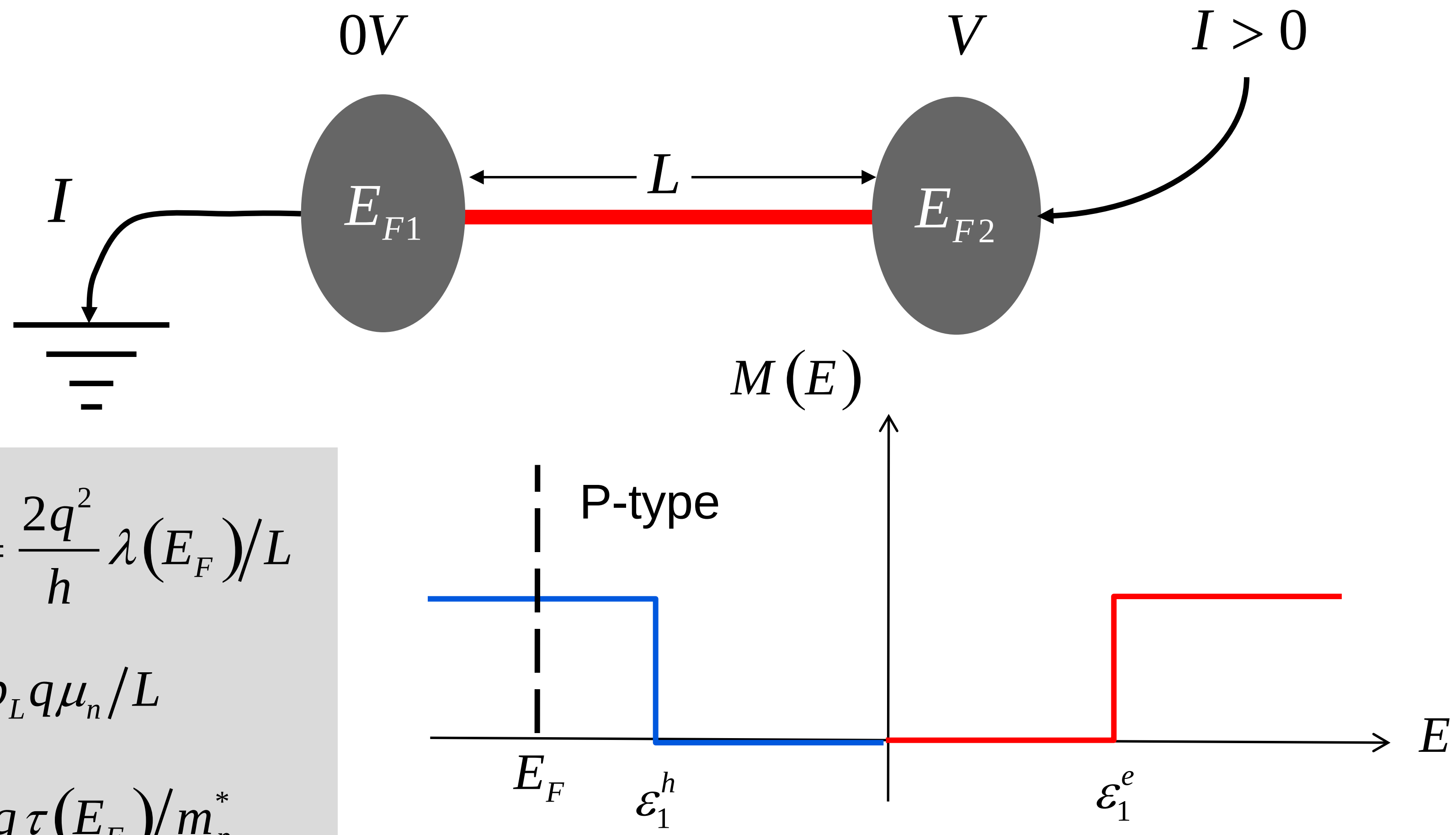
$$P = \frac{2k_F}{2\pi/L} \times 2$$

$$p_L = \frac{2k_F}{\pi} = \frac{2\sqrt{2m^*(\varepsilon_1^h - E_F)}}{\pi \mathrm{h}}$$

$$\lambda(E_F) = 2\upsilon(E_F)\tau(E_F)$$

$$\upsilon(E_F) = \sqrt{2\left(\varepsilon_1^h - E_F\right)/m^*}$$

case ii): result



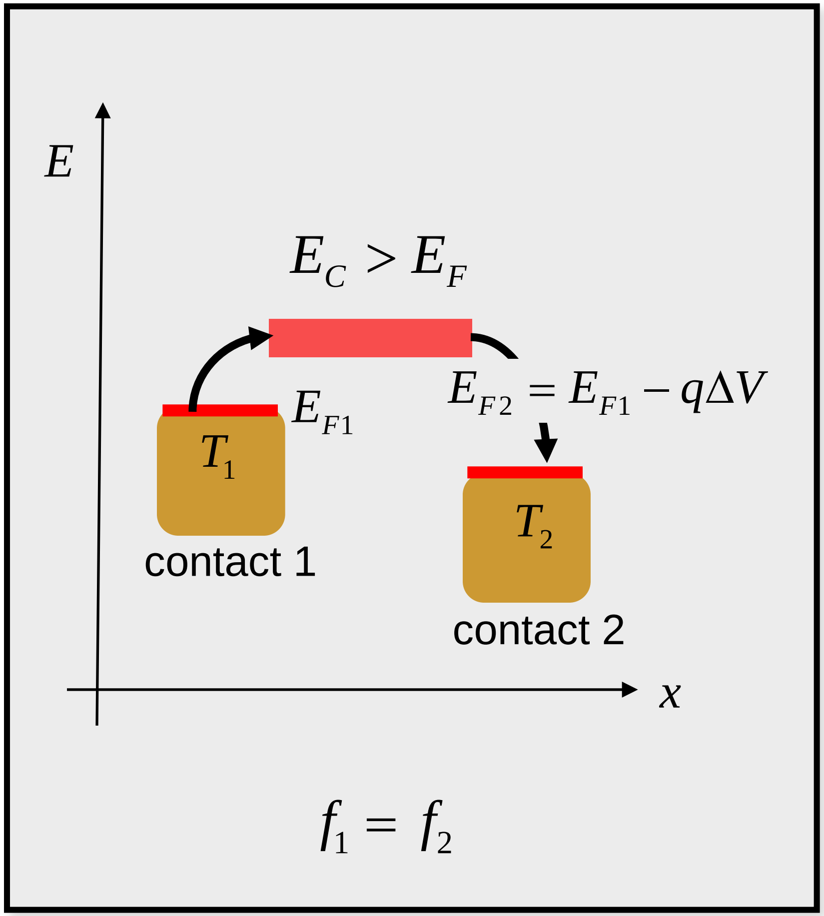
$$G_{1D} = \frac{2q^2}{h} \lambda(E_F)/L$$

$$G = p_L q \mu_n / L$$

$$\mu_n = q\tau(E_F)/m_p^*$$

$$V > 0 \Rightarrow I > 0$$

Seebeck coefficient: n-type

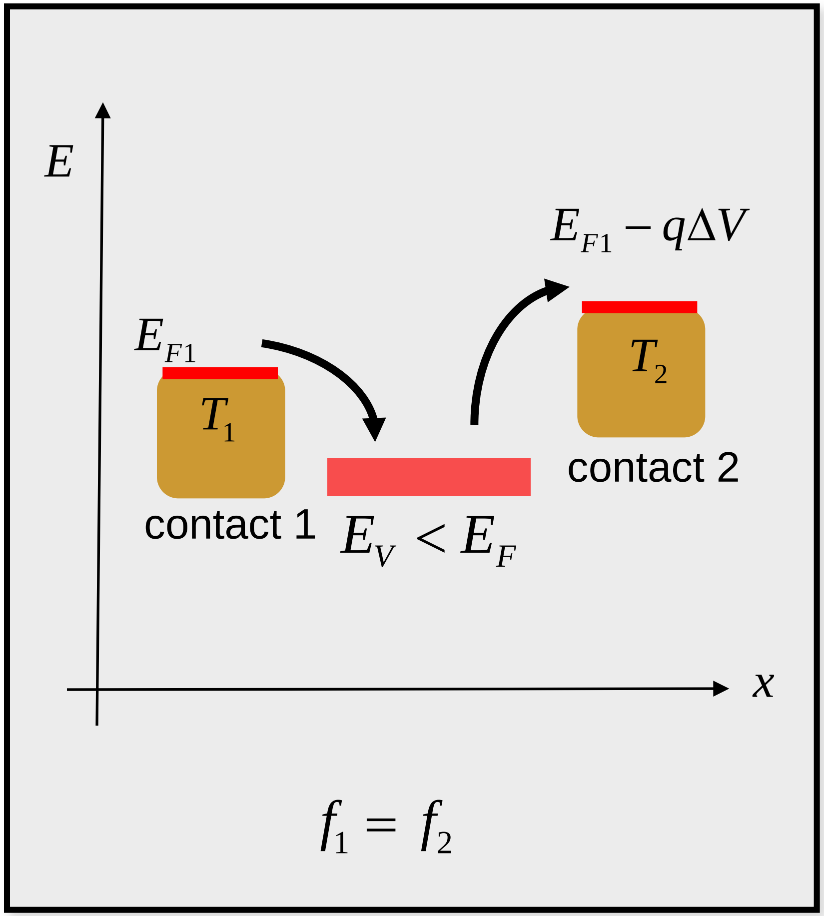


$$\Delta V = -S(E)\Delta T > 0$$

$$S(E) < 0$$

$$S(E) \approx \left(-\frac{k_B}{q}\right)\frac{(E_C - E_F)}{k_B T_L}$$

Seebeck coefficient: p-type



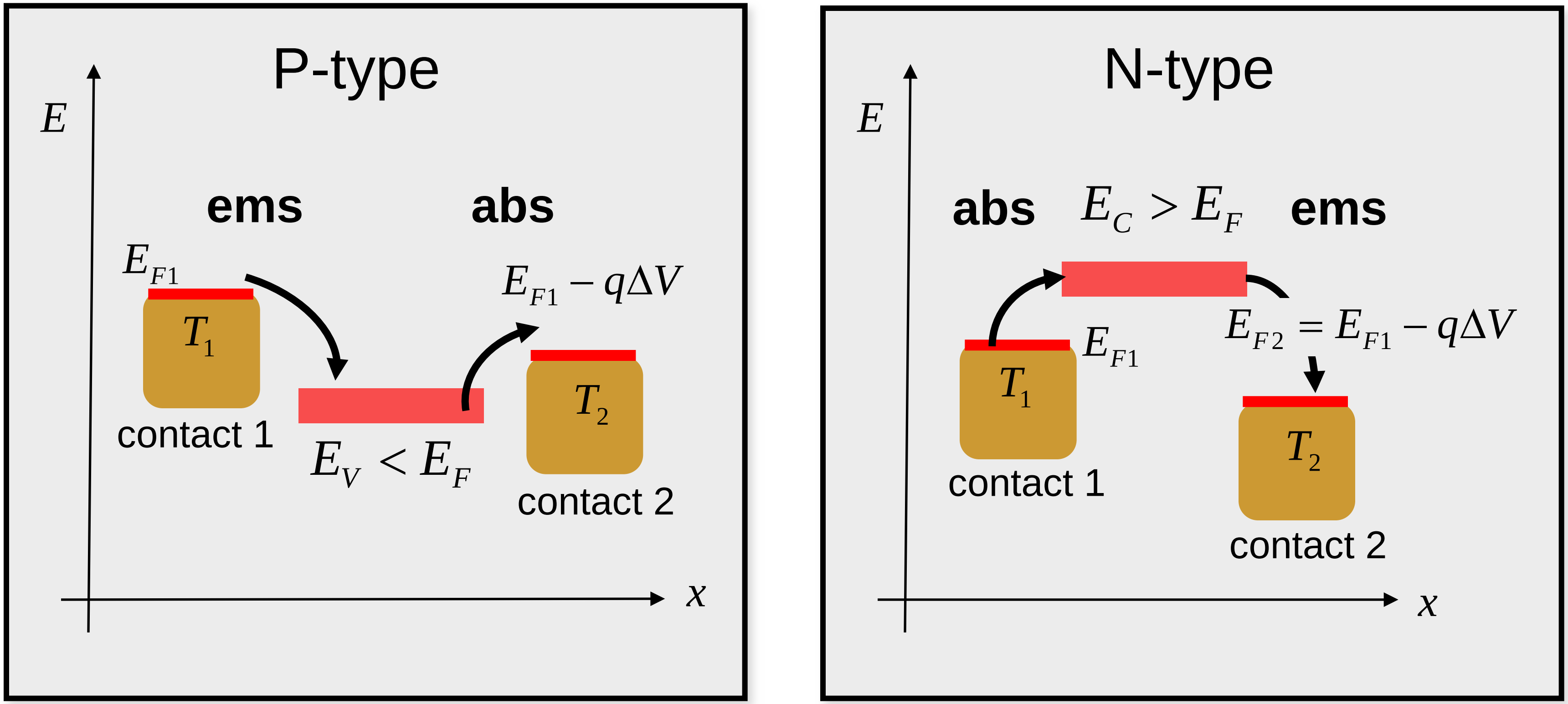
$$\Delta V = -S(E)\Delta T < 0$$

$$S(E) > 0$$

$$S(E) \approx \left(-\frac{k_B}{q}\right)\frac{(E_V - E_F)}{k_B T_L}$$

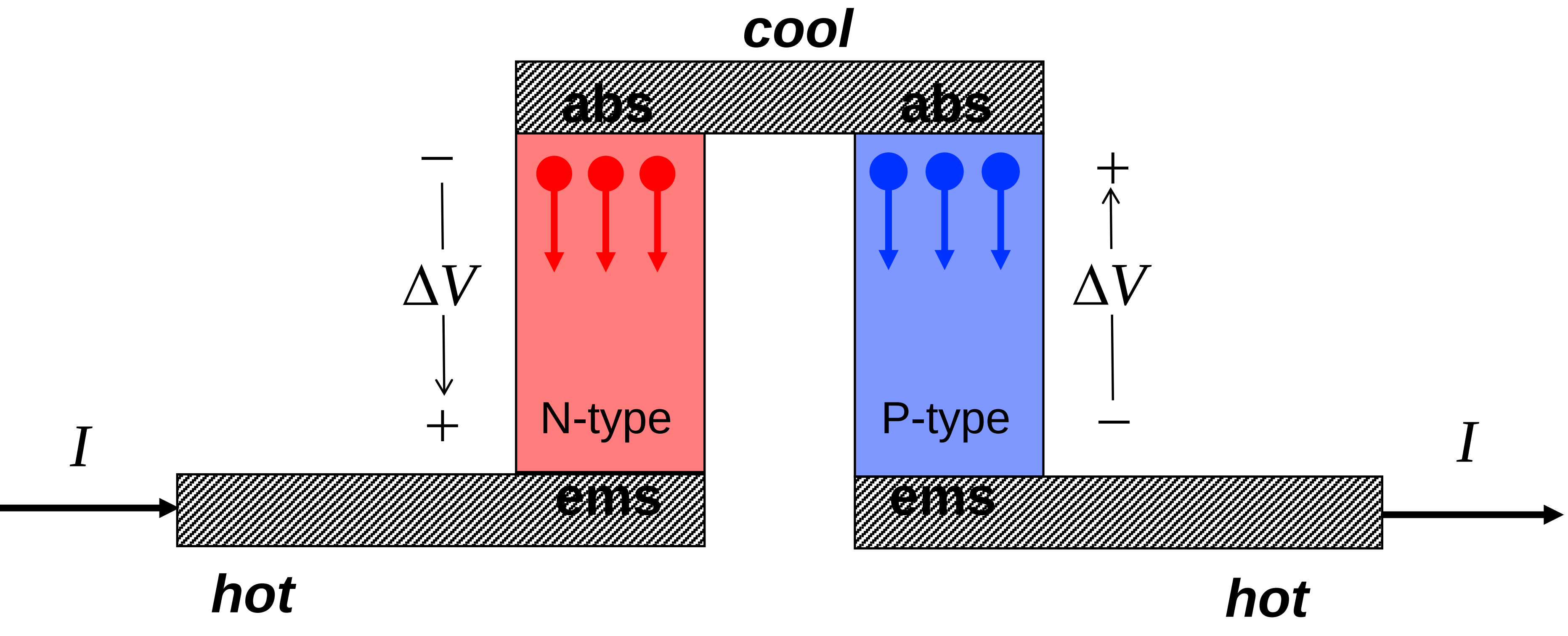
$$S(E) \approx \left(+\frac{k_B}{q}\right)\frac{(E_F - E_V)}{k_B T_L}$$

Peltier coefficient: p-type

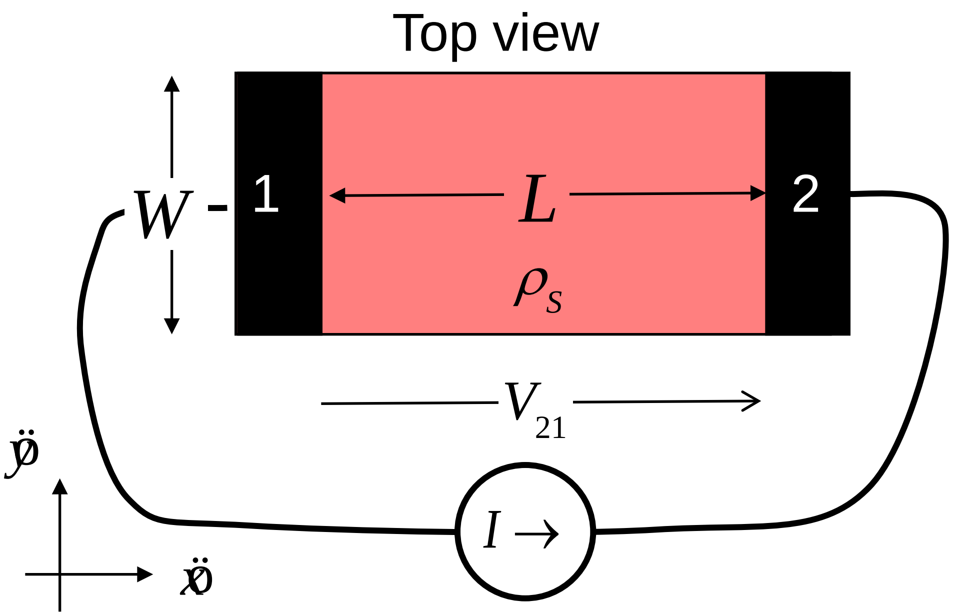


For the same sign voltage at contact 2, the contacts that absorb and emit heat are interchanged.

TE cooling



maximum temperature difference



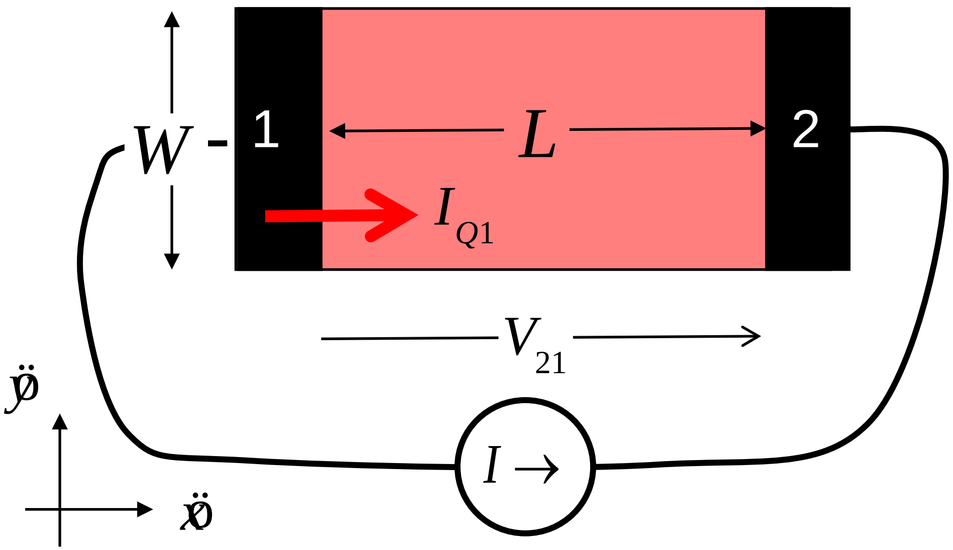
$$\Delta V = RI - S\Delta T$$

$$I_Q = -\pi I - K_e \Delta T$$

If we force a current through this thermoelectric device, what is the maximum temperature difference, $\Delta T = T_2 - T_1$, that we can produce?

maximum temperature difference

Top view



$$\Delta V = RI - S\Delta T$$

$$I_Q = -\pi I - K_e \Delta T$$

$$I_{Q1} = -T_1 SI - K_e \Delta T - \frac{1}{2} I^2 R$$

For a fixed current, the temperature will rise until $I_Q = 0$

$$\Delta T = -\frac{\left(T_1 SI + I^2 R/2\right)}{K_e}$$

$$\frac{\partial \Delta T}{\partial I} = -\frac{\left(T_1 S + IR\right)}{K_e} = 0$$

$$I_{opt} = -T_1 S/R$$

$$\Delta T\Big|_{\max} = \frac{1}{2}\frac{T_1^2 S^2 G}{K}$$

maximum temperature difference

$$\Delta T\big|_{max} = \frac{1}{2}\frac{T^2 S^2 G}{K}$$

$$\left.\frac{\Delta T}{T}\right|_{max} = \frac{1}{2}\frac{S^2 GT}{K} = \frac{1}{2} ZT$$

$$ZT = \frac{S^2 GT}{K_e + K_L}$$

dimensionless figure of merit

K_e: electronic heat conductance

K_L: lattice heat conductance

figure of merit

Power factor:

-large Seebeck coefficient

-high conductance

-depends on material parameters

$$ZT = \frac{S^2 GT}{K_e + K_L}$$

thermal conductance:

-$K_L > K_e$

-need short mfp for phonons

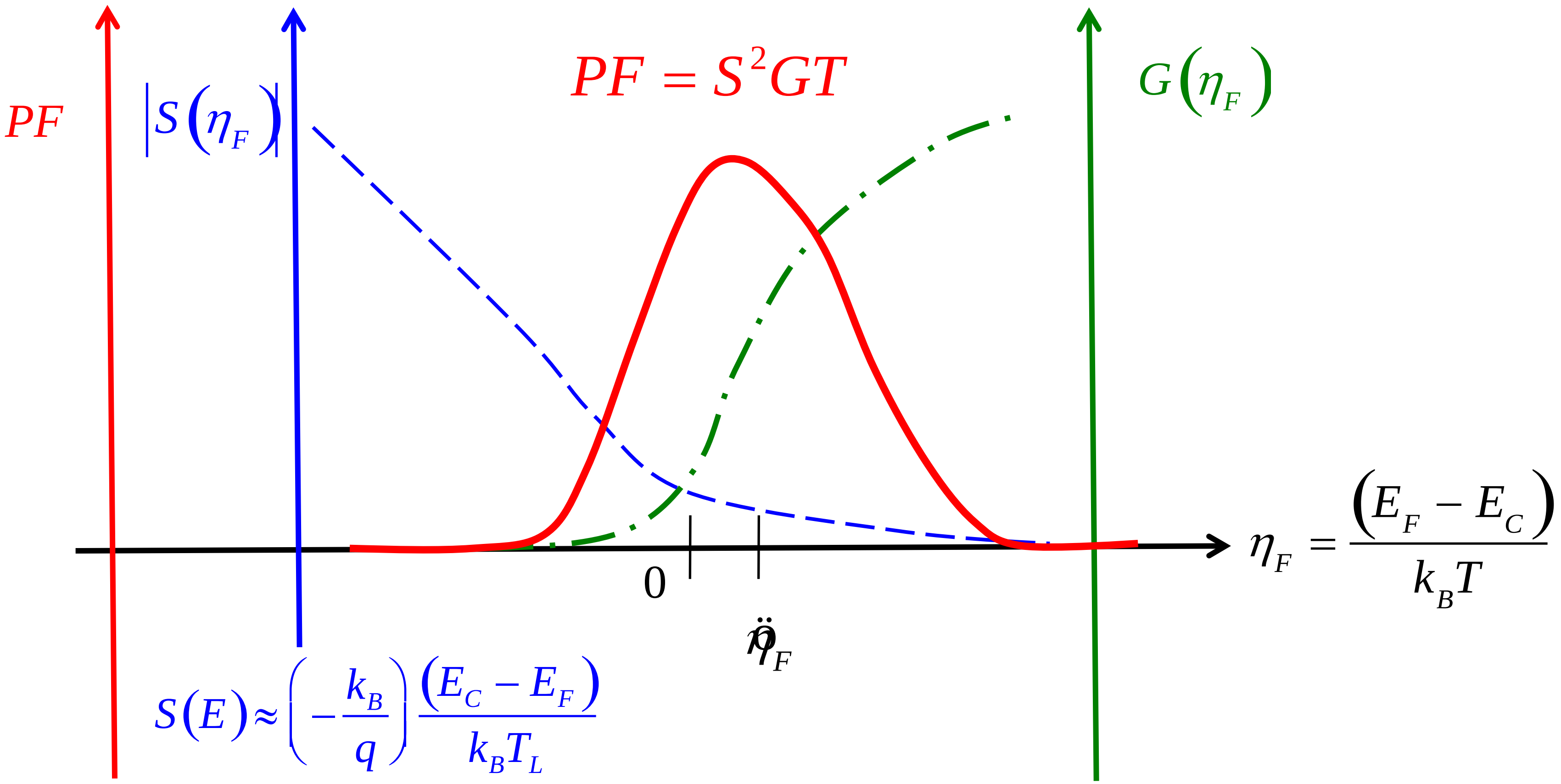
(alloys with large mass difference – e.g. Bi_2Te_3, SiGe)

-nanostructuring for reducing mfp

-“phonon glass - electron crystal”

G.D. Mahan and J.O. Sofo, “The Best Thermoelectric,” *Proc. Natl. Acad. Sci*., **93**, 7436, 1996

optimal power factor



The optimum power factor occurs when E_F is near E_C.

TE parameters for a non-degenerate semiconductor

$$\Delta V = RI - S\Delta T$$

$$I_Q = -\pi I - K_e \Delta T$$

$$\frac{\Delta V}{L} = \frac{RA}{L}\frac{I}{A} - S\frac{\Delta T}{L}$$

$$-\mathcal{E}_x = \rho\left(-J_x\right) - S\frac{dT}{dx}$$

$$\frac{I_Q}{A} = -\pi\frac{I}{A} - \frac{K_e L}{A}\frac{\Delta T}{L}$$

$$J_Q = -\pi\left(-J_x\right) - \kappa_e\frac{dT}{dx}$$

$$\mathcal{E}_x = \rho J_x + S\frac{dT}{dx}$$

$$J_Q = \pi J_x - \kappa_e\frac{dT}{dx}$$

What are the four TE parameters for a non-degenerate semiconductor?

TE parameters for a non-degenerate semiconductor

$$\Delta V = RI - S\Delta T$$

$$I_Q = -\pi I - K_e \Delta T$$

$$\mathcal{E}_x = \rho J_x + S\frac{dT}{dx}$$

$$J_Q = \pi J_x - \kappa_e \frac{dT}{dx}$$

$$G = \sigma \frac{A}{L}$$

$$K_e = \kappa_e \frac{A}{L}$$

$$T(E) = \frac{\lambda_0}{L}$$

$$G = 1/R = \left(2q^2/h\right) I_0$$

$$S = \left(-\frac{k_B}{q}\right)\frac{I_1}{I_0}$$

$$\pi = TS$$

$$K_e = \left(\frac{2k_B^2 T}{h}\right)\left[I_2 - \frac{I_1^2}{I_0}\right]$$

$$I_j = \int_{-\infty}^{+\infty} \left(\frac{E - E_F}{k_B T_L}\right)^j T(E) M(E) \left(-\frac{\partial f_0}{\partial E}\right) dE$$

TE parameters for a non-deg, 3D semiconductor

$$\mathcal{E}_x = \rho J_x + S\frac{dT}{dx}$$

$$J_Q = \pi J_x - \kappa_e \frac{dT}{dx}$$

$$n = 10^{15} \text{ cm}^{-3}$$

$$\mu_n = 1000 \text{ cm}^2/\text{V-s}$$

$$\rho_n \approx 6 \ \Omega\text{-cm}$$

$$\sigma_{3D} = nq\mu_n$$

$$n = N_C e^{\eta_F}$$

$$N_C = \frac{1}{4}\left(\frac{2m^* k_B T}{\pi \hbar^2}\right)^{3/2}$$

$$\eta_F = \left(\frac{E_F - E_C}{k_B T}\right)$$

$$\mu_n = \frac{\upsilon_T \lambda_0}{2}\frac{1}{(k_B T/q)}$$

TE parameters for a non-deg, 3D semiconductor

$$\mathcal{E}_x = \rho J_x + S\frac{dT}{dx}$$

$$J_Q = \pi J_x - \kappa_e \frac{dT}{dx}$$

$$n = 10^{15}\ \text{cm}^{-3}$$

$$N_C = 3.2 \times 10^{19}\,\text{cm}^{-3}$$

$$k_B/q \approx 86\ \ \mu V\,/\,\text{K}$$

$$\eta_F \approx -10$$

$$S \approx -1\ \ \text{mV/K}$$

$$S_{3D} = -\frac{k_B}{q}\left(2 - \eta_F\right)$$

$$n = N_C e^{\eta_F}$$

$$S_{3D} = -\frac{k_B}{q}\left(2 + \ln\left(N_C/n\right)\right)$$

$$\pi_{3D} = TS_{3D}$$

TE parameters for a non-deg, 3D semiconductor

$$\mathcal{E}_x = \rho J_x + S\frac{dT}{dx}$$

$$J_Q = \pi J_x - \kappa_e \frac{dT}{dx}$$

$$\kappa_e^{3D} = 2\left(\frac{k_B}{q}\right)\left(\frac{k_B T}{q}\right)\sigma_0$$

$$\kappa_e^{3D} = 2\left(\frac{k_B}{q}\right)^2 T\sigma_0 = L\,T\sigma_0$$

L = "Lorenz number"

$$\frac{\kappa_e^{3D}}{T\sigma_0} = 2\left(\frac{k_B}{q}\right)^2$$

"Wiedemann-Franz ratio"

for metals…..

$$\frac{\kappa_e^{3D}}{T\sigma_0} = \frac{\pi^2}{3}\left(\frac{k_B}{q}\right)^2$$