ECE 656: Fall 2009
Lecture 14 Homework SOLUTION

The purpose of this homework assignment is to solve the Boltzmann Transport Equation for a particle with charge $+Zq$, where Z is an integer > 1. This may occur in problems like the flow of ions through channels in cell walls or the flow of ions inside a battery.

1) Solve the BTE in the relaxation time approximation assuming a constant relaxation time, and a small electric field, but no concentration gradient. Use the result to derive an equation for the drift current.

2) Solve the BTE in the relaxation time approximation assuming a constant relaxation time, and a small concentration gradient, but no electric field. Use the result to derive an equation for the diffusion current.

3) Find the Einstein relation for these charged particles.

1)

$$\cancel{v_x \frac{\partial f}{\partial x}} + F_x \frac{\partial f}{\partial p_x} = -\frac{(f - f_0)}{\tau_f} \qquad F_x = +Zq\mathcal{E}_x$$

$$\tau_f = \tau_0$$

$$0$$

$$Zq\mathcal{E}_x \frac{\partial f}{\partial p_x} = -\frac{f_A}{\tau_0} \qquad f_A = -Zq\tau_0 \mathcal{E}_x \frac{\partial f_0}{\partial p_x}$$

$$f = f_0 - Zq\tau_0 \mathcal{E}_x \frac{\partial f_0}{\partial p_x} \rightarrow \text{displaced Maxwellian}$$

$$p_{dx} = +Zq\tau_0 \mathcal{E}_x$$

$$\langle v_x \rangle = \frac{p_{dx}}{m} = \frac{Zq\tau_0}{m^*}\mathcal{E}_x = \mu \mathcal{E}_x$$

$$\mu = \frac{Zq\tau_0}{m^*} \qquad J_{DRIFT} = nZq\left(\frac{Zq\tau_0}{m^*}\right)\mathcal{E}_x \checkmark$$

2) the BTE becomes:

$$v_x \frac{\partial f}{\partial x} = -\frac{f_A}{\tau_0} \qquad f_A = -v_x \tau_0 \frac{\partial f_0}{\partial x}$$

$$J_{DIFF} = \frac{1}{\Omega}\sum_{\vec{p}}{}' (Zq) v_x f_A = -\frac{Zq\tau_0}{\Omega}\sum_{\vec{k}}{}' v_x^2 \frac{\partial f_0}{\partial x}$$

1)

2)

$$J_{DIFF} = -Zq\tau_0 \frac{\partial}{\partial x}\left(\frac{1}{\Omega}\sum_{\vec{k}} v_x^2 f_0\right)$$

$$= -Zq\tau_0 \frac{\partial}{\partial x}\left(\frac{1}{\Omega}\sum \frac{m v_x^2}{2} f_0\right)\cdot\frac{2}{m}$$

$$v_x^2 = v^2/3$$

$$\sum_k m v^2/_2 f_0 = n\langle E_k\rangle = 3/_2 k_BT \times n$$

$$J_{DIFF} = -Zq\tau_0 \frac{\partial}{\partial x}\left(\frac{3}{2}k_BT\cdot n\right)\cdot\frac{2}{m}\cdot\frac{1}{3}$$

$$= -\frac{Zq\tau_0}{m}\cdot k_BT\,\frac{\partial n}{\partial x} \quad ✓ = Zq \times F_{DIFF}$$

$$F_{DIFF} = -\frac{q\tau_0}{m}\frac{\partial n}{\partial x}$$

3) according to (1) $\mu = Zq\tau_0/m$

according to (2) $D = \tau_0 k_BT/m$

$$\frac{D}{\mu} = \frac{k_BT}{Zq} \quad ✓$$

2)