Fundamentals of Nanoelectronics

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ECE 453
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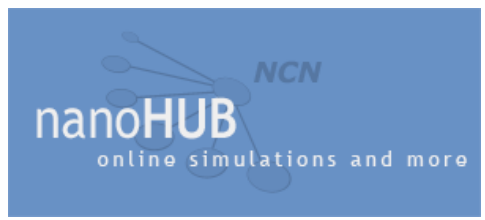
12.08.2004

Lecture 39: Coulomb Blockade

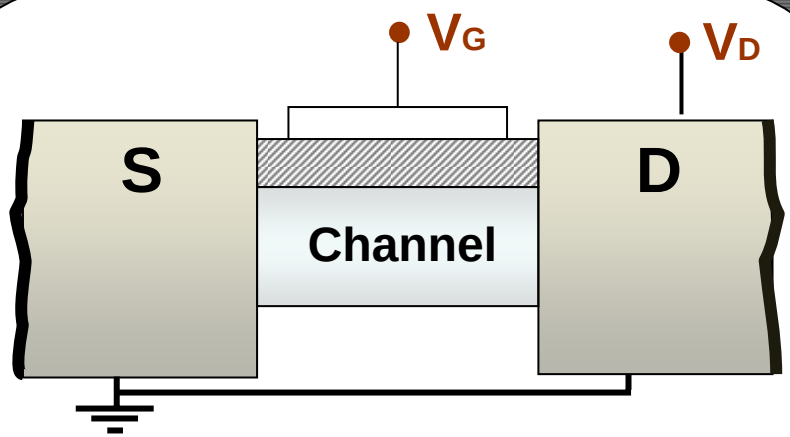
Ref. Chapter 3.4

Network for Computational Nanotechnology



Preview

00:05



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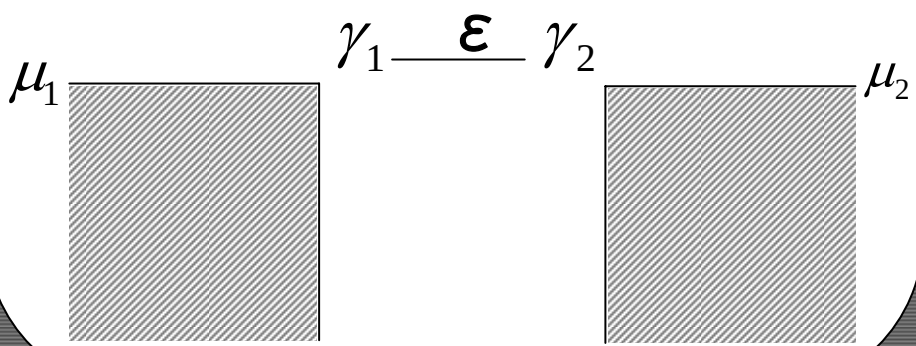


- Today we want to talk about coulomb blockade which is a very important concept for small devices.
- We started the course with a one level device model:

$$I_1 = q\frac{\gamma_1}{\hbar}(f_1 - N)$$

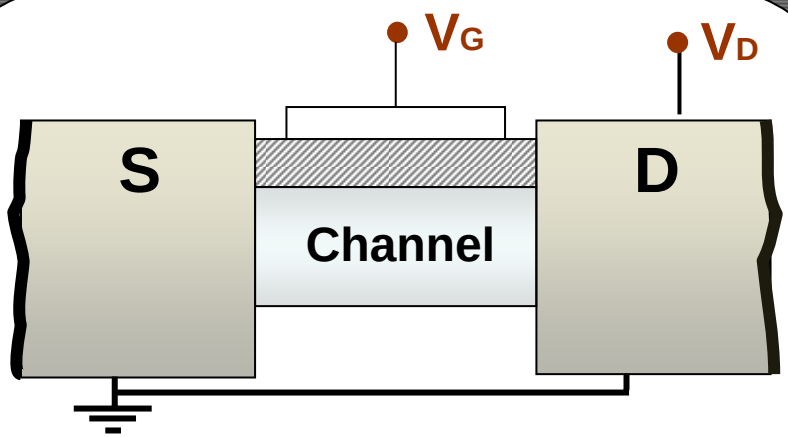
$$I_2 = q\frac{\gamma_2}{\hbar}(N - f_2)$$

$$N = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$

$$I = \frac{q}{\hbar}\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2}(f_1 - f_2)$$

Single Electron Charging U0

02:00



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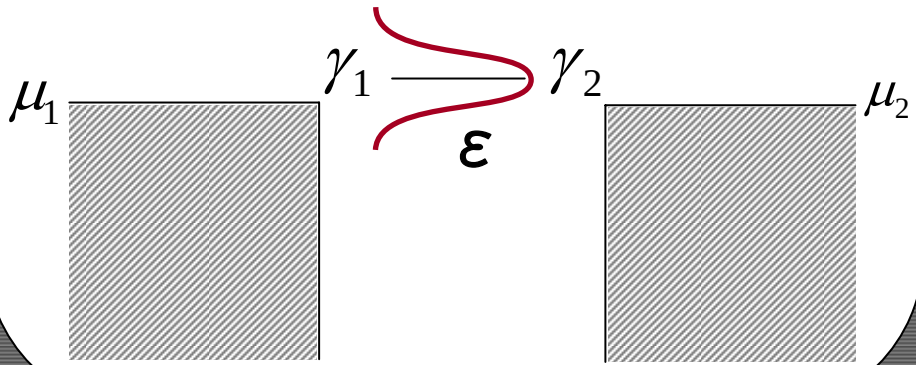


- The first thing we'd like to know is under what conditions coulomb blockade is important and has to be accounted for in modeling.
- Coulomb blockade becomes important when the coupling to the contacts and kT become much less than the single electron charging energy:

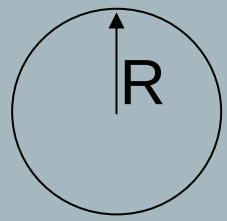
$$\gamma = \gamma_1 + \gamma_2$$

$$\gamma, k_B T << U_0$$

- To begin with notice that the absence or presence of an electron in the channel results in raising or lowering of the level. The way we can account for this effect is: $U = U_L + U_0 \Delta n$
- U0 tells how much the potential changes if the number of electrons changes by 1.

Estimation of U0

• To get a feel for how large U0 is, we can consider a spherical device and calculate its potential when one electron is distributed over it:



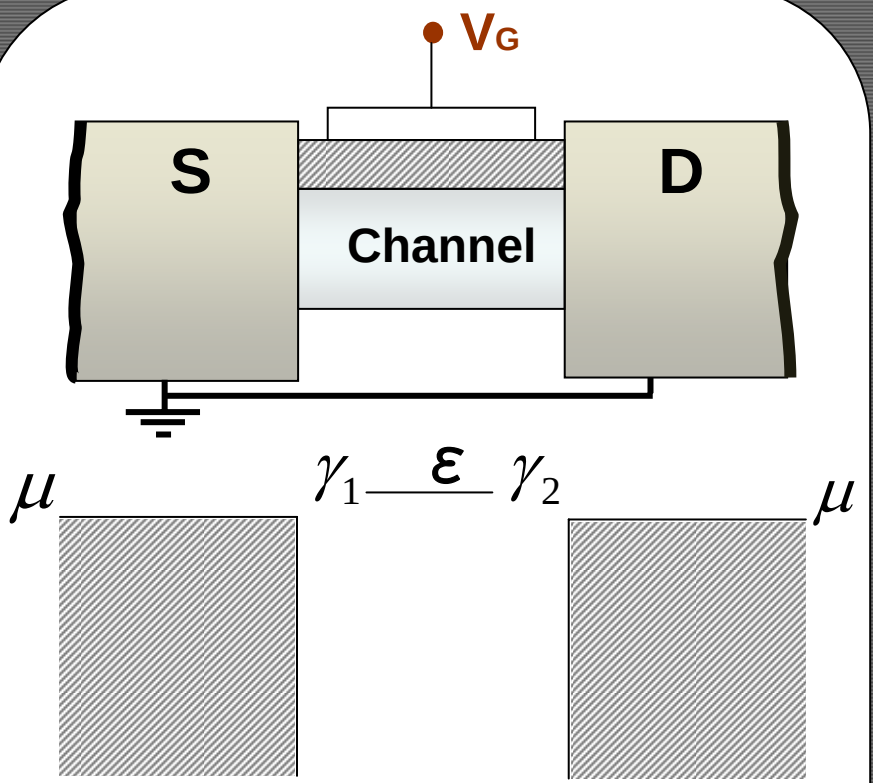
$$U_0 = \frac{q^2}{4\pi\varepsilon R} = \frac{q^2}{4\pi\varepsilon(100nm)} \approx 10's \text{ of milivolts}$$

• For a device with the size above, U0 becomes observable. i.e.

$$\gamma, k_B T << U_0$$

• So the question becomes this: Is U0 large enough for the device under consideration? If yes, then coulomb blockade has to be accounted for.

Filling Up The Level At Equilibrium

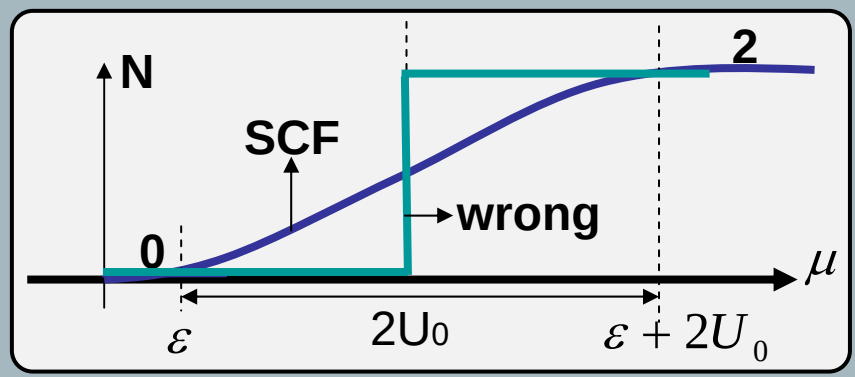


• Consider the equilibrium problem where the drain voltage is 0 and we apply a positive gate voltage. This causes the level ε to move down and as it moves below the Fermi energy, it will get filled.

• An alternative way of thinking about this is to held the channel potential fixed and apply a negative voltage to source and drain. This time ε stays put and the Fermi levels in drain and source move up.

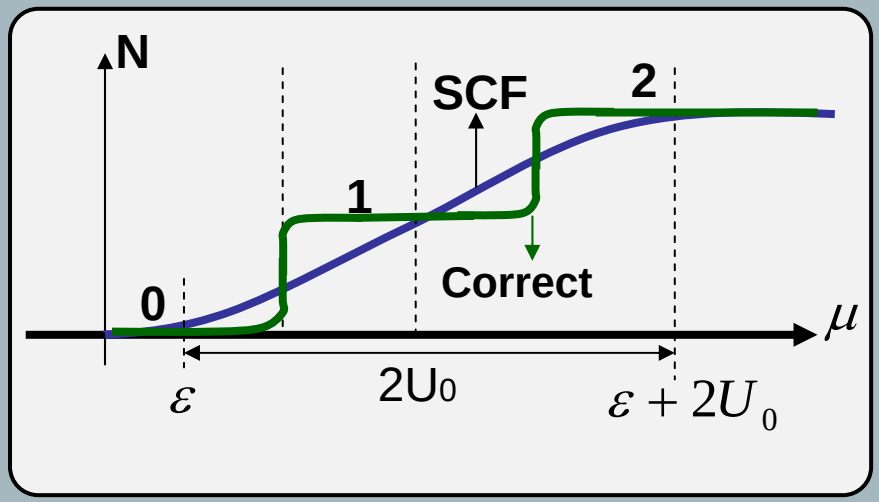
• How does the number of electrons in the level change as we raise μ?

• Based in what we know thus far:



What Happens if kT Is Comparable to U0?

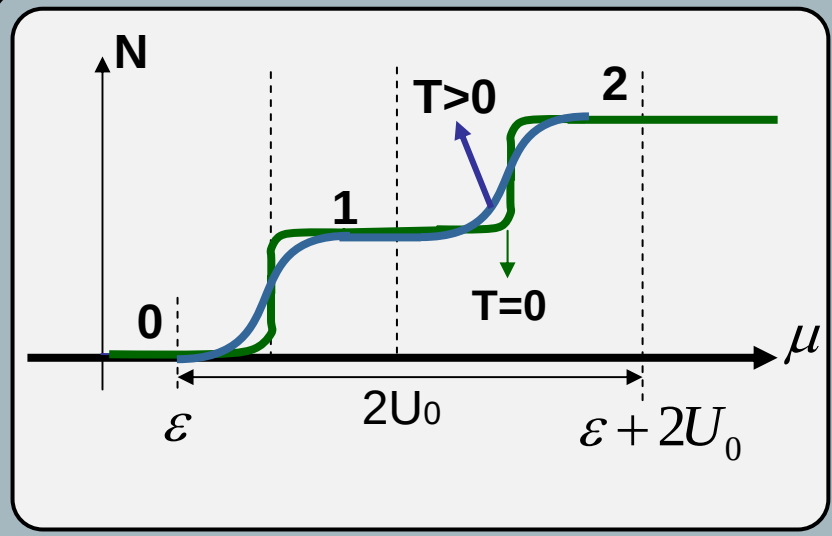
• How ever under the conditions under which coulomb blockade is dominant experiment shows that:



• Notice that this difference would not be noticeable unless we are this regime:

$$\gamma, k_B T << U_0$$

• First consider the temperature. The correct curve above is drawn for 0k. For higher temperatures we have:

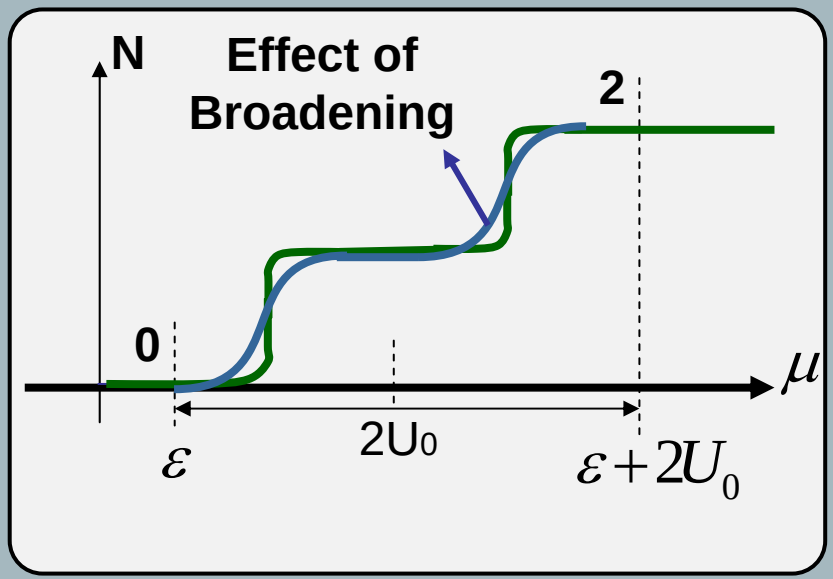


• You can see from the figure above that when T is large (for example room temperature) the smoothed curve above will actually be very close to the Self Consistent Field (SCF) curve drawn on the left figure. Under this conditions coulomb blockade cannot be observed.

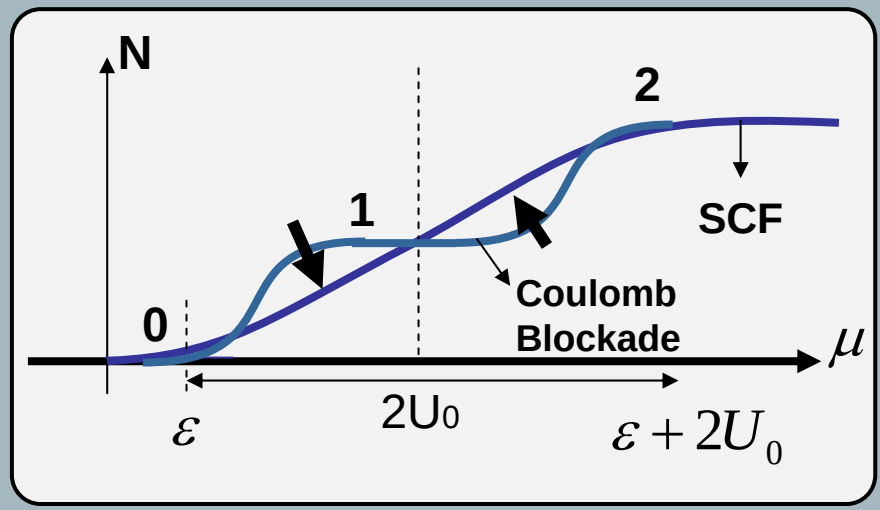
What Happens If Broadening Is Comparable To U0?

14:47

- Second, think about Gamma. When we couple to a level, we broaden it. So the level is not a sharp delta function any more and broadens out. As the result the sharp steps tend to smooth out because of broadening.



- Again what happen it one wouldn't be able to tell the difference between the SCF curve and the one based on coulomb blockade.
- The two thick arrows show indicate that as broadening is increased the coulomb blockade curves converges onto the Self Consistent Field (SCF) curve.



- So as long as $\gamma, k_B T << U_0$ is **NOT** the case we get the correct results if we use the SCF method as before.

Self Interaction Correction (SIC)

15:55

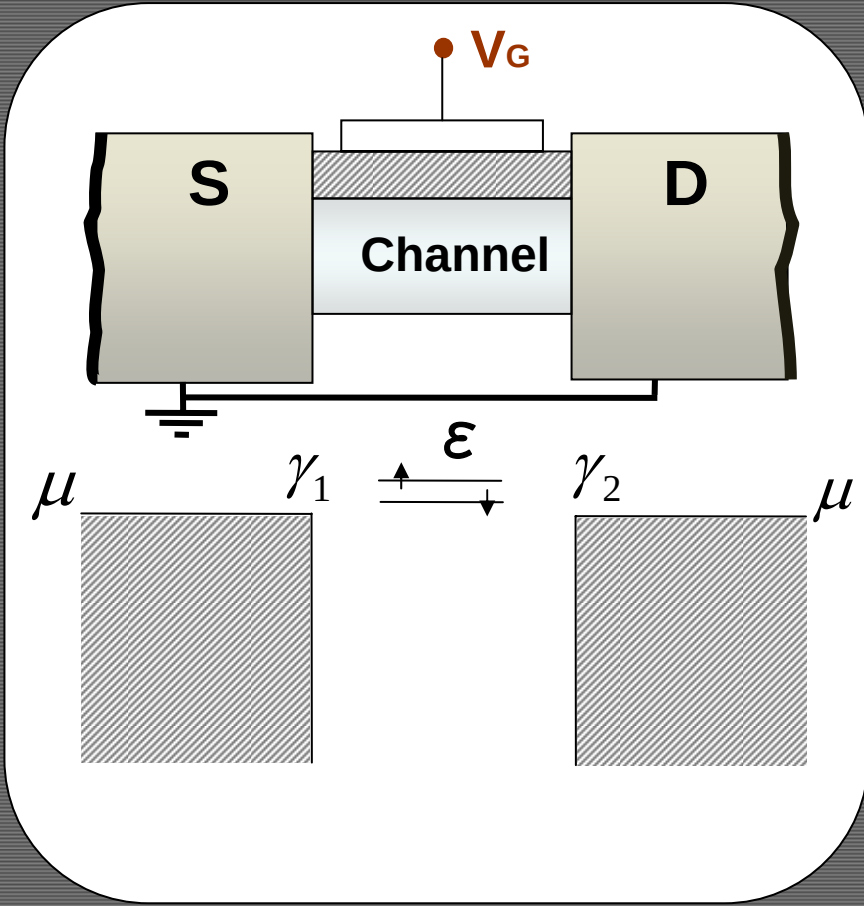
• Under the condition $\gamma, k_B T << U_0$ we should consider coulomb blockade. The next step is to model the correct behavior. There are two ways of doing this. One is easier to understand and more tractable but does not always give the correct results. The other method is exact but it is very hard to implement for practical problems. So most of the time the approximate method is used. Let's first see how the approximate method works.

• Consider the SCF potential:

$$U = U_L + U_0 \Delta n$$

• The potential above does not take into account the non existence of self interaction. Since an electron does not feel anything due to itself, the more correct way is to subtract this contribution that is overlooked in above formalism.

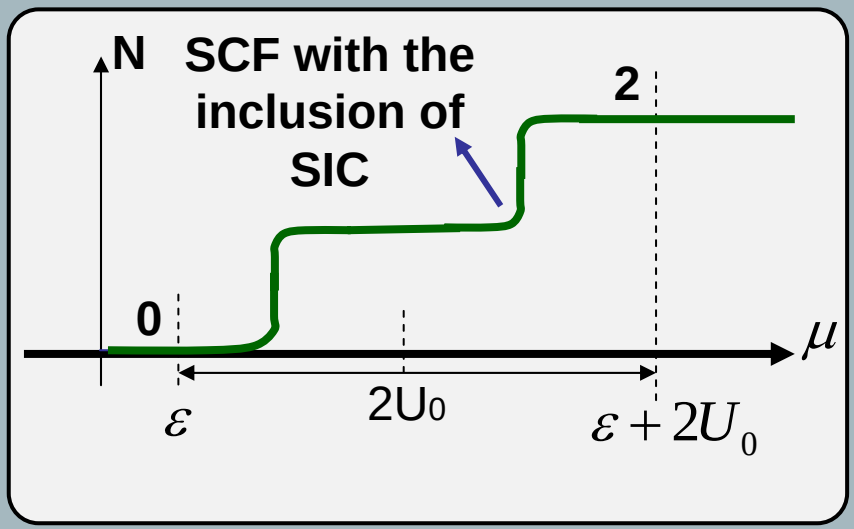
SIC: $U_i = U_L + U_0(\Delta n - \Delta V_i)$



Correct Results Using SIC For One Level Device

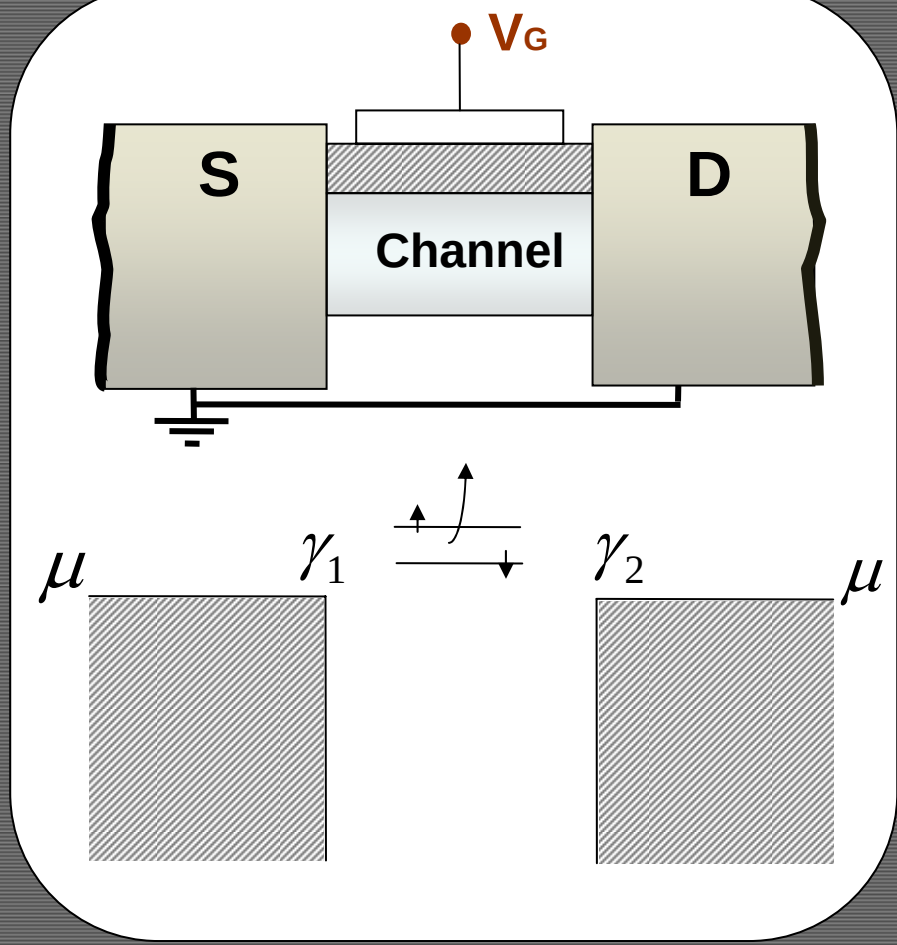
19:00

- If we include Self Interaction Correction (SIC) then we get something that looks like the star curse curve in the curve.



SCF with the inclusion of SIC

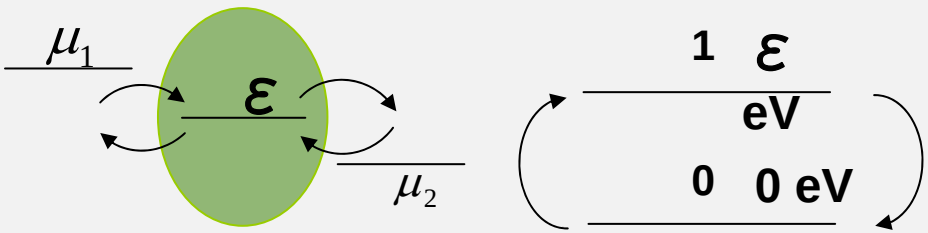
$$U_i = U_L + U_0(\Delta n - \Delta V_i)$$



Level To System

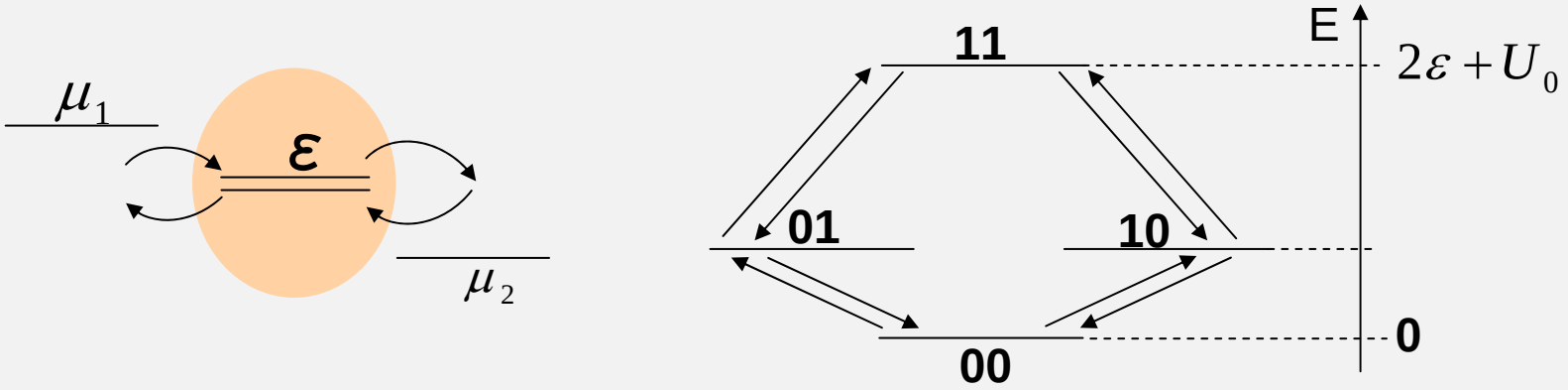
22:25

System with only one level



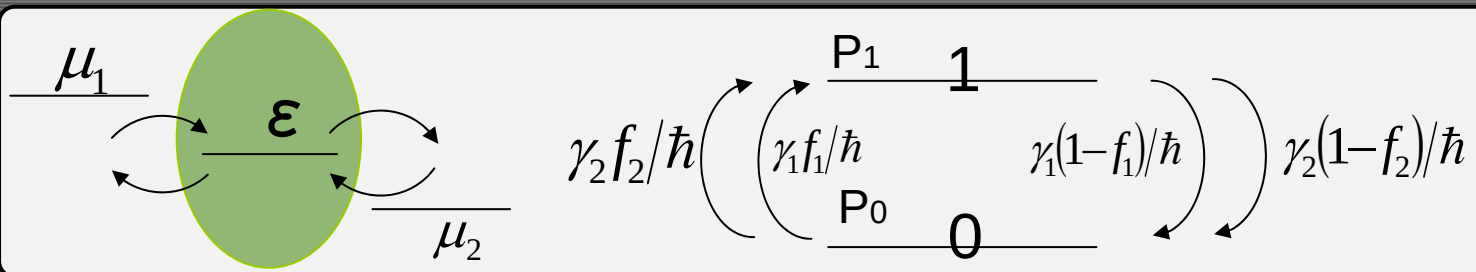
• When an electron comes in, the system goes form state 0 to state 1. When an electron goes out, the system goes from state 1 to state 0.

System with two levels



• In general we have 2^N sates for multi electron system where N is the number of allowed energy levels for the electrons in the device.

Rate Equations



P1+ P0 = 1

- At steady state, the rate at which an electron comes in must be equal to the rate at which electron goes out; therefore: $P_0 R_a = P_1 R_b$
- Where: $R_a = (\gamma_2 f_2 + \gamma_1 f_1)/\hbar$ $\quad R_b = [\gamma_2(1-f_2)+\gamma_1(1-f_1)]/\hbar$

$$\frac{P_1}{P_0} = \frac{R_a}{R_b} = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1(1-f_1)+\gamma_2(1-f_2)}$$

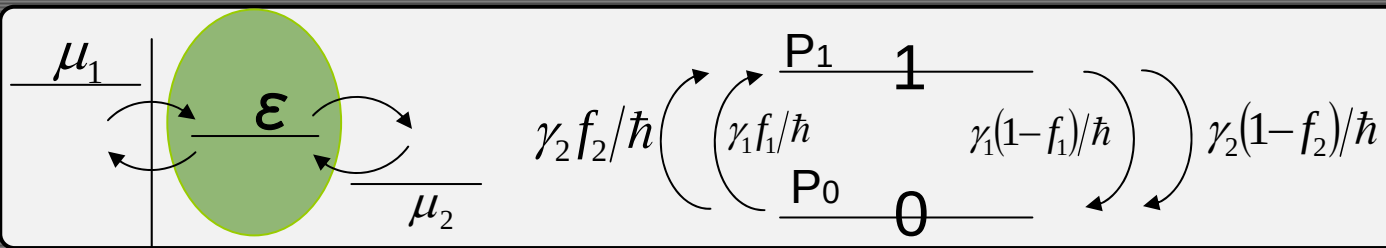
Two equations, two unknowns

P1+ P0 = 1

- Algebraic trick: $\frac{a}{b}=\frac{c}{d} \Rightarrow \frac{a}{a+b}=\frac{c}{c+d} \Rightarrow \frac{P_1}{P_0+P_1} = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1+\gamma_2}$
- This is exactly what we would expect. We got the same answer from two different viewpoints.

$$P_1 = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1+\gamma_2} \qquad P_0 = \frac{\gamma_1(1-f_1)+\gamma_2(1-f_2)}{\gamma_1+\gamma_2}$$

Current



• Algebraic sum of the current flowing from contact 1 into the channel and the current from channel into the contact will give us the net current flow for the left interface.

• Current for the left interface:

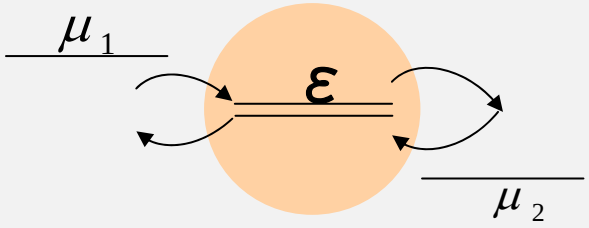
$$I_1 = q\left[\frac{\gamma_1}{\hbar} f_1 P_0 - \frac{\gamma_1}{\hbar}(1-f_1)P_1\right] = q\frac{\gamma_1}{\hbar}\left(f_1 P_0 - (1-f_1)P_1\right) = q\frac{\gamma_1}{\hbar}(f_1 - P_1)$$

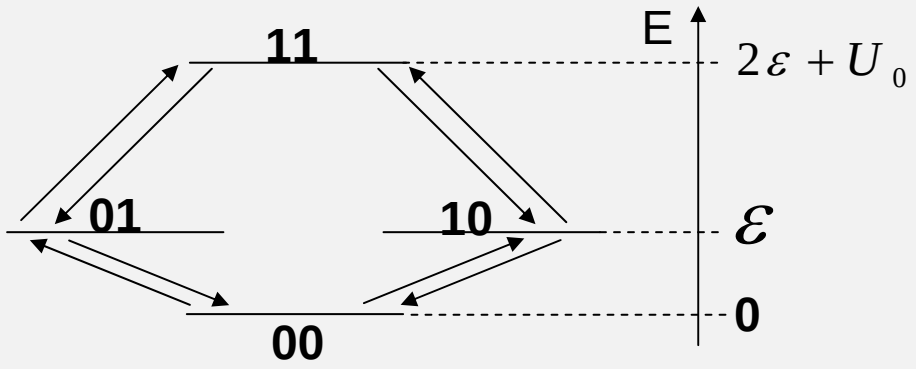
$$P_1 = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} = N$$

• Doing a similar calculation for I2 and finding the current through the device we get exactly what we had before.

$$\therefore I = \frac{q}{\hbar}\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}(f_1 - f_2)$$

Two Level And Multi Level Systems





- In principle one can set up the rate equations to get the current using the viewpoint of multi electron systems.
- Are there any issues or problems with using this method?
- Yes; the size of it. The number of possible states goes as 2^N where N is the number of states. It can easily be seen that this quickly goes out of hand. This is one of the reasons we don't use this method.
- Is there any other problem with it?
- Yes, we don't know how to incorporate broadening into this picture. For small broadening it will be fine but for large broadening you can't use it.
- This is why people try to come up with a simple one particle picture. One thought is to modify the self consistent field such that it would mimic the correct results. See next page for an example.