ECE 656: Fall 2009
Lecture 22 Homework

1) Problem 2.2 in Lundstrom

L22 : Lundstrom 2.2

from (2.36b) Lundstrom

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} \cdot \frac{N_I q^4}{K_S^2 \epsilon_0^2 \Omega} \sum_{\vec{p}', \uparrow} \frac{\delta(E-E')}{16(p/\hbar)^4 \sin^2(\alpha/2)}$$

$$\sin^2(\alpha/2) = \frac{1}{2}(1-\cos\alpha) \quad (\text{let } \alpha = \theta)$$

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} \frac{N_I q^4}{K_S^2 \epsilon_0^2 \Omega} \cdot \frac{\Omega}{8\pi^3 \hbar^3} \frac{\hbar^4}{p^4} \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \int_{-1}^{\cos\theta_{min}} \frac{d(\cos\theta)}{(1-\cos\theta)^2} \times \frac{1}{16 \cdot 1/4} \int_0^{\infty} \delta(E-E') p'^2 dp'$$

$$\frac{1}{\tau} = \frac{q^4 N_I}{8 K_S^2 \epsilon_0^2 p^4 \cdot \pi} I_1 \times I_2 \quad (*)$$

$$I_1 = \int_2^{1-\cos\theta_{min}} \frac{-dx}{x^2} = \frac{1}{1-\cos\theta_{min}} - \frac{1}{2} = \frac{1}{2}\left(\frac{1}{\sin^2\theta_{min}/2} - 1\right)$$

$$= \gamma_{cw}^2/2 \quad \text{(2.43) Lundstrom}$$

1)

$I_2 = ?$

$$\frac{p'^2}{2m^*} = E' \rightarrow p'^2 = 2mE'$$

$$p' = \sqrt{2mE'}$$
$$dp' = \frac{\sqrt{2m}}{2}(E')^{-1/2}$$
$$p'^2 dp' = \sqrt{2}\, m^{3/2} \sqrt{E'}$$

$$I_2 = \sqrt{2}\,(m^*)^{3/2} \sqrt{E}$$

insert in (*)

$$\frac{1}{\tau} = \frac{q^4 N_I}{8 K_S^2 \epsilon_0^2 p^4 \cdot \pi} \cdot \frac{\gamma_{cw}^2}{2} \cdot \sqrt{2}\,(m^*)^{3/2} \sqrt{E}$$

use (2.44) for $\gamma_{cw}^2/2$

result

$$\frac{1}{\tau} = N_I (\pi b_{max})^2 \sqrt{2m^* E / m^*}$$

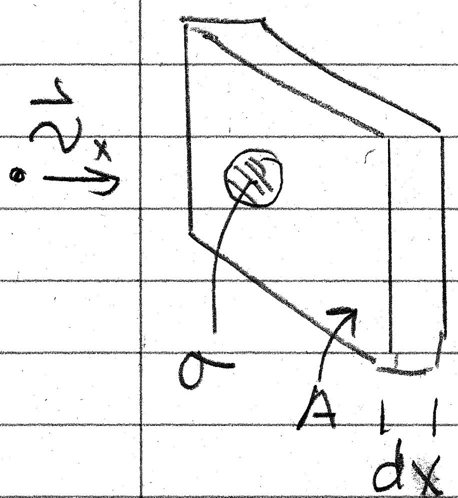
$$\frac{1}{\tau} = N_I (\pi b_{max})^2 \cdot v$$

how do we interpret?

2)

2.2b.

recall the meaning of cross-section



area of scatterer σ cm^2

# of scatterers in slab: $N_I A dx$

area obscured: $(N_I A dx)\sigma$

fraction of area obscured

$= N_I dx \sigma$

prob of scattering $= dt \times \frac{1}{\tau} = N_I dx \sigma$

$$\frac{1}{\tau} = N_I \sigma \frac{dx}{dt}$$

$$\boxed{\frac{1}{\tau} = N_I \sigma v} = N_I (\pi b_{max}^2) v$$

$\sigma = \pi b_{max}^2$ ✓

3)

2.2c

using (2.46) and the result of 2.2a

$$\frac{\tau_m}{\tau} = 2\left(\frac{4\pi K_s \epsilon_0 E}{q^2}\right)^2 \cdot b_{max}^2 \times \frac{1}{\ln(1+\gamma_{cw}^2)}$$

N_I (cm^{-3})	b_{max} (nm)	γ_{cw}	τ_m/τ
10^{14}	110	75	326
10^{15}	50	35	85
10^{16}	23	16	24
10^{17}	11	7.5	7
10^{18}	5nm	3.5	2.4

$\tau_m > \tau$ because small χ's are favored

$\tau_m \to \tau$ as N_D increases because b_{max} decreases so θ_{min} decreases.

4)