ECE 656: Fall 2009
Lecture 16 Homework
(Revised 10/27/09)

Follow the approach of Lecture 16 to compute the magneto-conductivity tensor for grapene, which has an $E(k)$ given by:

$$E(k) = \pm\hbar\upsilon_F\sqrt{k_x^2 + k_y^2}$$

1) Show that the approach used in Lecture 16, DOES NOT work for graphene, and explain why.

2) See if you can find a way to solve the BTE for graphene when a small magnetic field is present. Alternatively, use the simpler approach of Lecture 18.

1)

$$\left(-q\vec{\mathcal{E}} - q\vec{v}\times\vec{B}\right)\cdot\nabla_p f = -f_A/\tau_f$$

i) $\vec{B}=0 \quad \vec{\mathcal{E}}\neq 0$

$$-q\vec{\mathcal{E}}\cdot\nabla_p f_s = -f_A'/\tau_f$$

$$\nabla_p f_s = \frac{\partial f_s}{\partial E}\vec{v}$$

$$f_A' = -q\tau_f\left(-\frac{\partial f_s}{\partial E}\right)\vec{v}\cdot\vec{\mathcal{E}}$$

ii) $\vec{B}\neq 0 \quad \vec{\mathcal{E}}=0$

$$(-q\vec{v}\times\vec{B})\cdot\nabla_p(f_s+f_A') = -f_A''/\tau_f$$

$$f_A'' = q\tau_f(\vec{v}\times\vec{B})\cdot\nabla_p f_A'$$

$$\nabla_p f_A' = \nabla_p\left\{-q\tau_f\left(-\frac{\partial f_s}{\partial E}\right)\vec{v}\cdot\vec{\mathcal{E}}\right\}$$

only term:

$$\nabla_p f_A' = \left\{-q\tau_f\left(-\frac{\partial f_s}{\partial E}\right)\right\}\left[\nabla_p(\vec{v}\cdot\vec{\mathcal{E}})\right]$$

1)

$\vec{\nabla}_p(\vec{v}\cdot\vec{\mathcal{E}}) = ?$

$$\frac{\partial}{\partial p_x}(v_x \mathcal{E}_x)\hat{x} + \text{etc.}$$

$$v_x = \frac{\partial E}{\partial p_x} \qquad \frac{\partial v_x}{\partial p_x} = 0 \text{ for graphene}$$

so $\nabla_p(\vec{v}\cdot\vec{\mathcal{E}}) = 0$ for graphene

therefore $\nabla_p f_A' = 0 \rightarrow f_A'' = 0$

no effect of $\vec{B}$!

so the method used in L16 does not work for graphene

2)

2)

instead of solving the BTE, use the simpler approach of L18 Sec. 2

$$\vec{F}_e = -q\vec{\mathcal{E}} - q\vec{v}\times\vec{B} = d\vec{p}/dt$$

$$\vec{p} = -q\tau\vec{\mathcal{E}} - q\tau\vec{v}\times\vec{B}$$

assume $\vec{B} = B_z\hat{z}$

$$p_x = -q\tau\mathcal{E}_x - q\tau v_y B_z \quad (1)$$
$$p_y = -q\tau\mathcal{E}_y + q\tau v_x B_z \quad (2)$$

$$E = \hbar v_F\sqrt{k_x^2+k_y^2}$$

$$v_x = \frac{1}{\hbar}\frac{\partial E}{\partial k_x} = v_F\frac{k_x}{\sqrt{k_x^2+k_y^2}}$$ now solve for $\hbar k_x = p_x$

$$\hbar k_x = \hbar v_x\frac{\sqrt{k_x^2+k_y^2}}{v_F} = v_x\frac{\hbar v_F\sqrt{k_x^2+k_y^2}}{v_F^2} = v_x\frac{E}{v_F^2}$$

return to (1) and (2)

$$v_x = \frac{-q\tau}{(E/v_F^2)}\mathcal{E}_x - \frac{q\tau}{(E/v_F^2)}v_y B_z \quad (3)$$

3)

$$v_y = \frac{-q\tau \mathcal{E}_x}{(E/v_F^2)} + \frac{q\tau}{(E/v_F^2)} v_x B_z \quad (4)$$

Eqs. (3) and (4) are exactly the same as in L18 if

$$m^* \rightarrow (E/v_F^2)$$

so the final result is

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_n}{1+(\mu_n B_z)^2} \begin{bmatrix} 1 & -\mu_n B_z \\ +\mu_n B_z & 1 \end{bmatrix} \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix}$$

$$\mu_n = \frac{q\tau}{(E/v_F^2)} \qquad \sigma_n = nq\mu_n$$

$$\mu_n B_z = \frac{q\tau}{(E/v_F^2)} B_z = \omega_c \tau \qquad \omega_c = \frac{qB_z}{(E/v_F)^2}$$

$\tau = \tau(E_F)$ because graphene is degenerate

4)

2)

Formally solving the BTE takes more work. See, for example:

Introduction to Semiconductor Theory, A. Anselm, Prentice Hall, 1981 pp. 500–506.

5)