

ECE 656: Fall 2009
Lecture 23 Homework
(Revised: 10/29/09)

- 1) Repeat the electron-phonon energy-momentum conservation arguments presented in L23, but this time assume electrons in graphene.

L23 (actually L24)

$$E' = E \pm \hbar \omega \quad (1)$$

$$\vec{p}' = \vec{p} \pm \hbar \vec{\beta} \quad (2)$$

$$E = \hbar v_F k$$

$$= v_F p$$

$$E^2 = v_F^2 p^2$$

$$E = \frac{p^2}{(E/v_F^2)}$$

$$m^* = E/v_F^2 \text{ graphene}$$

E

(1) becomes

$$E = p^2/m^* \quad (3)$$

$$\frac{p'^2}{m^*} = \frac{p^2}{m^*} \pm \hbar \omega \beta \quad (1')$$

$$(2) \text{ becomes } \vec{p}' \cdot \vec{p}' \rightarrow p'^2 = p^2 \pm 2\hbar \vec{p} \cdot \vec{\beta} + \hbar^2 \beta^2$$

$$\frac{p'^2}{m^*} = \frac{p^2}{m^*} \pm \frac{2\hbar \vec{p} \cdot \vec{\beta}}{m^*} + \frac{\hbar^2 \beta^2}{m^*} \text{ use (1')}$$

$$\pm \hbar \omega \beta = \pm \frac{2\hbar \vec{p} \cdot \vec{\beta}}{m^*} + \frac{\hbar^2 \beta^2}{m^*} \quad (4)$$

$$\pm \hbar \omega \beta = \pm \frac{2\hbar p \beta \cos \theta}{m^*} + \frac{\hbar^2 \beta^2}{m^*}$$

1)

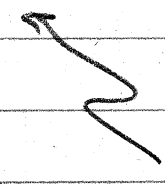
$$\frac{\hbar^2 \beta^2}{m^*} = \mp \frac{2\hbar p \beta}{m^*} \cos \theta \pm \hbar \omega \beta$$

$$\hbar \beta = \mp 2p \cos \theta \pm \frac{m^* \hbar \omega \beta}{\hbar \beta}$$

$$\hbar \beta = 2p \left[\mp \cos \theta \pm \frac{m^* \omega \beta}{2p \beta} \right]$$

$$\frac{m^*}{p} = \frac{E/V_F^2}{E/V_F} = \frac{1}{V_F}$$

$$\hbar \beta = 2p \left[\mp \cos 2\theta \pm \frac{\omega \beta}{2\beta V_F} \right] \quad (5)$$


 almost the same
 as the result for
 parabolic bands
 except for the factor
 of 2.

all results and conclusions will be very similar.

for example, AP EMS

$$\hbar\beta_{\max} = 2p \left[1 - \frac{v_s}{2v_F} \right]$$

Since $v_F \gg v_s$ all electrons can emit AP

OP EMS

(5) gives

$$\hbar\beta|_{\max} = 2p \left[1 - \frac{\omega_0}{2\beta v_F} \right]$$

$$\hbar\beta|_{\max} = 2p - \frac{2p\omega_0}{2\beta v_F}$$

$$2\hbar v_F \beta^2 - 4v_F p \beta + 2p\omega_0 = 0$$

$$\beta|_{\max}^2 - \frac{2p}{\hbar} \beta|_{\max} + \frac{p\omega_0}{\hbar v_F} = 0$$

$$\beta|_{\max} = \frac{+2p/\hbar \pm \sqrt{\frac{4p^2}{\hbar^2} - \frac{4\omega_0 p}{\hbar v_F}}}{2} = \frac{p}{\hbar} \pm \frac{p}{\hbar} \sqrt{1 - \frac{4\hbar^2 \omega_0 p}{4p^2 \hbar v_F}}$$

$$\hbar\beta_{\max} = p \left[1 + \sqrt{1 - \frac{\hbar\omega_0}{p v_F}} \right] = p \left[1 + \sqrt{1 - \frac{\hbar\omega_0}{E}} \right]$$