ECE 656: Fall 2009
Lecture 23 Homework
(Revised: 10/29/09)

1) Repeat the electron-phonon energy-momentum conservation arguments presented in L23, but this time assume electrons in graphene.

L23 (actually L24)

$$E' = E \pm \hbar\omega \quad (1)$$
$$\vec{p}\,' = \vec{p} \pm \hbar\vec{\beta} \quad (2)$$

$E = \hbar v_F k$

$= v_F p$

$E^2 = v_F^2 p^2$

$E = \dfrac{p^2}{(E/v_F^2)}$

$m^* = E/v_F^2$ graphene

(1) becomes

$E = p^2/m^*$ (3)

$$\frac{p'^2}{m^*} = \frac{p^2}{m^*} \pm \hbar\omega_\beta \quad (1')$$

(2) becomes $\vec{p}\,'\cdot\vec{p}\,' \rightarrow p'^2 = p^2 \pm 2\hbar\vec{p}\cdot\vec{\beta} + \hbar^2\beta^2$

$$\frac{p'^2}{m^*} = \frac{p^2}{m^*} \pm \frac{2\hbar\vec{p}\cdot\vec{\beta}}{m^*} + \frac{\hbar^2\beta^2}{m^*} \text{ use } (1')$$

$$\pm\hbar\omega_\beta = \pm\frac{2\hbar\vec{p}\cdot\vec{\beta}}{m^*} + \frac{\hbar^2\beta^2}{m^*} \quad (4)$$

$$\pm\hbar\omega_\beta = \pm\frac{2\hbar p\beta\cos\theta}{m^*} + \frac{\hbar^2\beta^2}{m^*}$$

$$\frac{\hbar^2 \beta^2}{m^*} = \mp \frac{2\hbar p \beta}{m^*} \cos\theta \pm \hbar \omega_\beta$$

$$\hbar\beta = \mp 2p\cos\theta \pm \frac{m^* \hbar \omega_\beta}{\hbar \beta}$$

$$\hbar\beta = 2p\left[\mp \cos\theta \pm \frac{m^* \omega_\beta}{2p\beta}\right]$$

$$\frac{m^*}{p} = \frac{E/v_F^2}{E/v_F} = \frac{1}{v_F}$$

$$\hbar\beta = 2p\left[\mp \cos\theta \pm \frac{\omega_\beta}{2\beta v_F}\right] \qquad (5)$$

almost the same as the result for parabolic bands except for the factor of 2.

all results and conclusions will be very similar.

2)

for example, AP EMS

$$\hbar\beta_{max} = 2p\left[1 - \frac{v_s}{2v_F}\right]$$

since $v_F \gg v_s$ all electrons can emit AP

OP EMS

(5) gives

$$\hbar\beta\Big|_{max} = 2p\left[1 - \frac{\omega_0}{2\beta v_F}\right]$$

$$\hbar\beta\Big|_{max} = 2p - \frac{2p\omega_0}{2\beta v_F}$$

$$2\hbar v_F\beta^2 - 4v_F p\beta + 2p\omega_0 = 0$$

$$\beta\Big|_{max}^2 - \frac{2p}{\hbar}\beta\Big|_{max} + \frac{p}{\hbar}\frac{\omega_0}{v_F} = 0$$

$$\beta\Big|_{max} = \frac{+2p/\hbar \pm \sqrt{\frac{4p^2}{\hbar^2} - \frac{4\omega_0 p}{\hbar v_F}}}{2} = \frac{p}{\hbar} \pm \frac{p}{\hbar}\sqrt{1 - \frac{4\hbar^2\omega_0 p}{4p^2\hbar v_F}}$$

37 $$\hbar\beta_{max} = p\left[1 + \sqrt{1 - \frac{\hbar\omega_0}{p^2 v_F}}\right] = p\left[1 + \sqrt{1 - \frac{\hbar\omega_0}{E}}\right]$$