ECE-656: Fall 2009

Lecture 24: Phonon Scattering II

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outline

1) **Review**

2) Energy-momentum conservation

3) Mathematical formulation

4) Example

5) Summary

electron-phonon scattering

$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\left|H_{p,p'}\right|^2 \delta\left(E' - E \mp \hbar\omega\right)$$ (weak scattering)

$$H_{p',p} = \frac{1}{\Omega}\int_{-\infty}^{+\infty} e^{-i\vec{p}\,'\bullet\vec{r}/\hbar}\, U_S(\vec{r})\, e^{i\vec{p}\bullet\vec{r}/\hbar} d\vec{r}$$ (plane waves: overlap integral = 1)

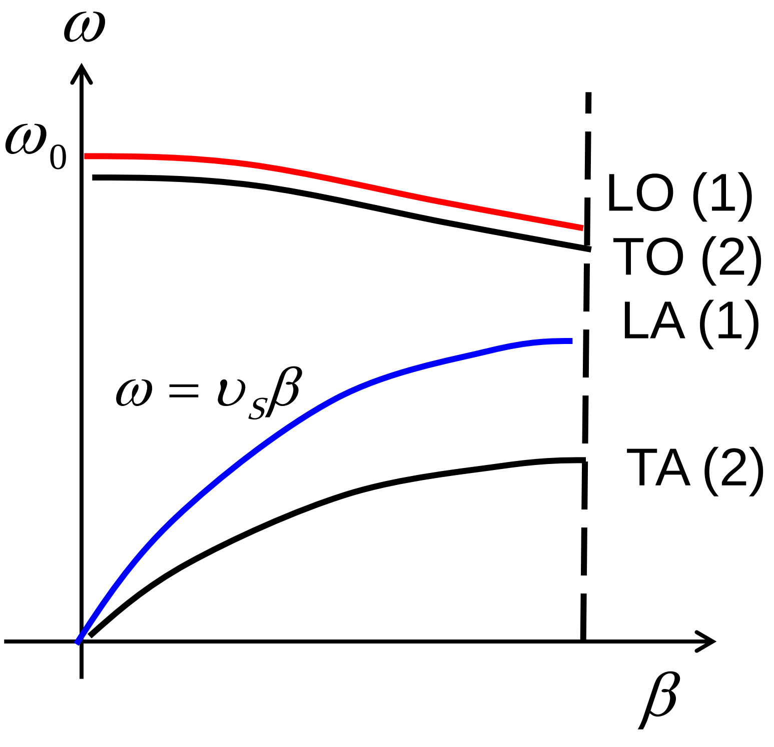
$$U_S(\vec{r}) = K_\beta u_\beta \qquad u_\beta(\vec{r},t) = A_\beta e^{\pm i\left(\vec{\beta}\bullet\vec{r} - \omega_\beta t\right)}$$ (electron-phonon coupling)

$$\left|A_\beta\right|^2 = \frac{\hbar}{2\Omega\rho\,\omega}\left(N_\omega + \frac{1}{2} \mp \frac{1}{2}\right) \qquad \begin{pmatrix}\text{ABS}\\ \text{EMS}\end{pmatrix}$$

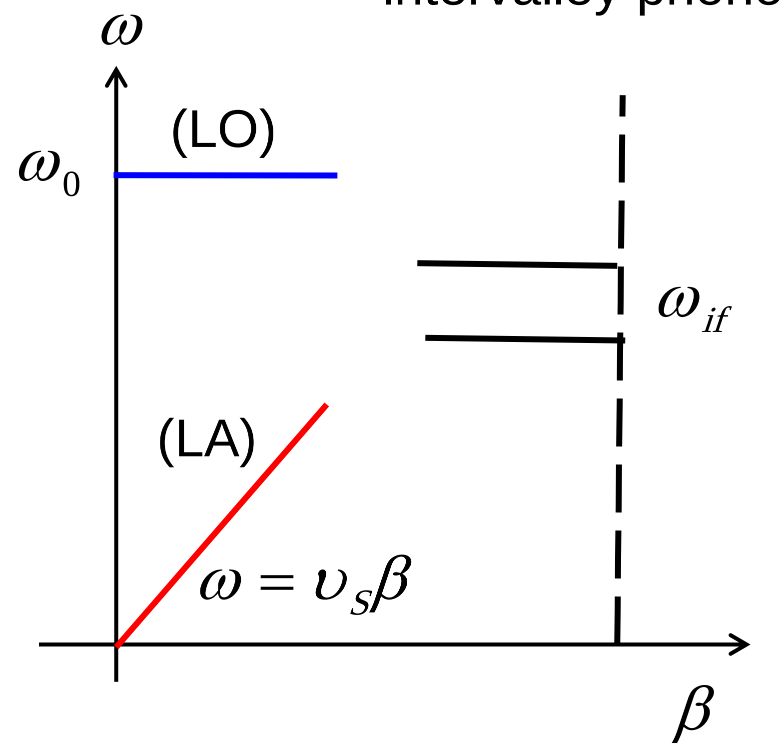
elastic: static potentials

inelastic: time-varying (harmonic) potentials with $\hbar\omega << \langle E \rangle$

phonon dispersion



"intervalley phonons"



scattering potentials

$$U_S = K_\beta u_\beta \qquad u_\beta(\vec{r},t) = A_\beta e^{\pm i\left(\vec{\beta}\bullet\vec{r}-\omega_\beta t\right)}$$

ADP $\left|K_\beta\right|^2 = \beta^2 D_A^2$

ODP $\left|K_\beta\right|^2 = D_0^2$

PZ $\left|K_\beta\right|^2 = \left(qe_{PZ}/\kappa_S\varepsilon_0\right)^2$

POP $\left|K_\beta\right|^2 = \dfrac{\rho q^2\omega_0^2}{\beta^2\kappa_0\varepsilon_0}\left(\dfrac{\kappa_0}{\kappa_\infty}-1\right)$

other scattering potentials

1) Neutral impurity

2) Alloy scattering

3) Surface / edge roughness scattering

4) Plasmon scattering

5) Electron-electron scattering

6) Electron-hole

7) …

outline

1) Review

2) **Energy-momentum conservation**

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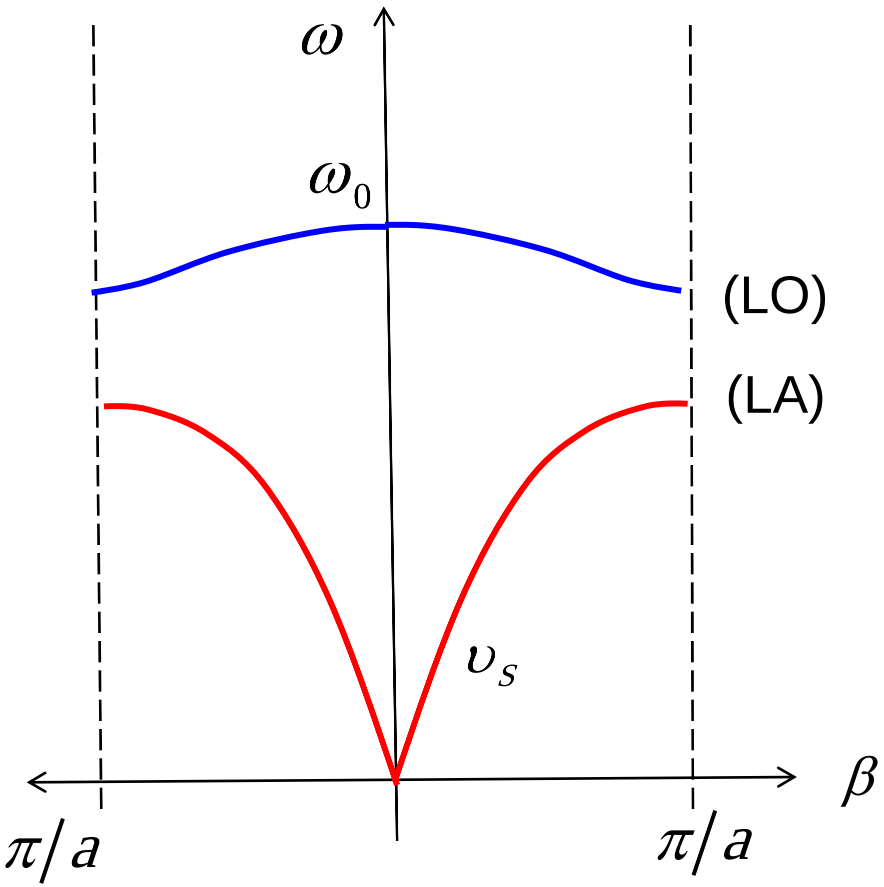
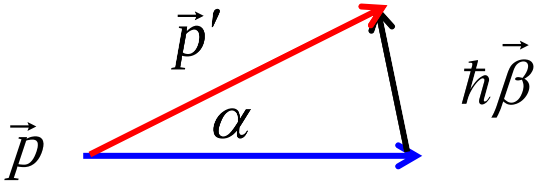
which phonons scatter?





$$E' = E \pm \hbar\omega_\beta$$

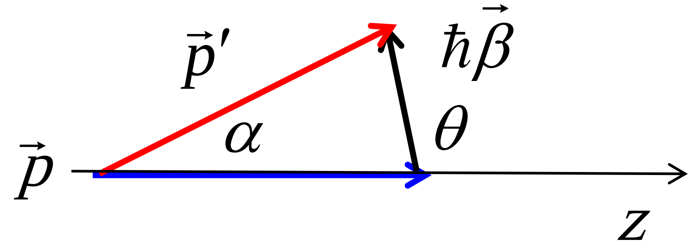
$$\vec{p}\,' = \vec{p} \pm \hbar\vec{\beta}$$

which phonons scatter?

$$\frac{p'^2}{2m^*} = \frac{p^2}{2m^*} \pm \hbar\omega_\beta$$

$$\vec{p}\,' = \vec{p} \pm \hbar\vec{\beta}$$

intravalley
scattering



$$\vec{p}\,'\bullet\vec{p}\,' = \left(\vec{p} \pm \hbar\vec{\beta}\right)\bullet\left(\vec{p} \pm \hbar\vec{\beta}\right)$$

$$p'^2 = p^2 \pm 2\hbar\,\vec{p}\bullet\vec{\beta} + \hbar^2\beta^2$$

$$\pm\hbar\omega_\beta = \pm\hbar\frac{\vec{p}\bullet\vec{\beta}}{m^*} + \frac{\hbar^2\beta^2}{2m^*}$$

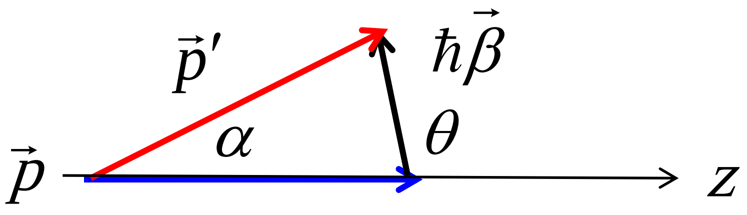
$$\hbar\beta = 2p\left[\mp\cos\theta \pm \frac{\omega_\beta}{\beta\upsilon}\right]$$

top sign: ABS
bottom sign: EMS

maximum beta: ABS

$$\frac{p'^2}{2m^*} = \frac{p^2}{2m^*} \pm \hbar\omega_\beta$$

$$\vec{p}\,' = \vec{p} \pm \hbar\vec{\beta}$$



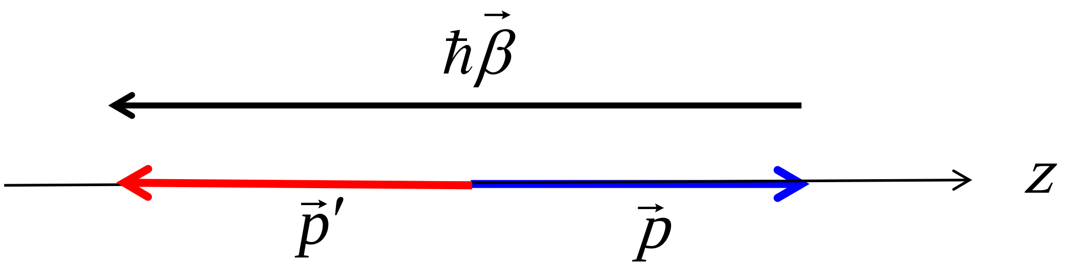
$$\hbar\beta = 2p\left[\mp\cos\theta \pm \frac{\omega_\beta}{\beta\upsilon}\right]$$

top sign: ABS

ABS: $\hbar\beta = 2p\left[-\cos\theta + \dfrac{\omega_\beta}{\beta\upsilon}\right]$

$$\hbar\beta_{\max} = 2p\left[1 + \frac{\omega_\beta}{\beta_{\max}\upsilon}\right] \quad \theta = \pi$$

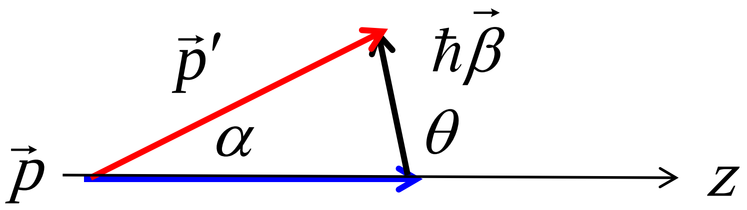
$$\vec{p}\,' = \vec{p} + \hbar\vec{\beta}$$



maximum beta: EMS

$$\frac{p'^2}{2m^*} = \frac{p^2}{2m^*} \pm \hbar\omega_\beta$$

$$\vec{p}\,' = \vec{p} \pm \hbar\vec{\beta}$$



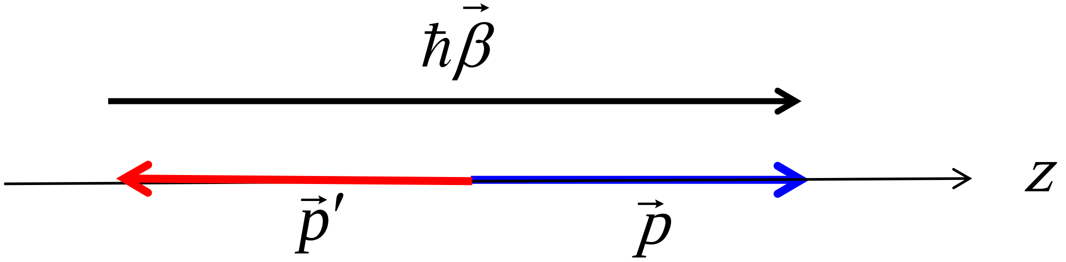
$$\hbar\beta = 2p\left[\mp\cos\theta \pm \frac{\omega_\beta}{\beta\upsilon}\right]$$

top sign: ABS

EMS: $\hbar\beta = 2p\left[+\cos\theta - \frac{\omega_\beta}{\beta\upsilon}\right]$

$$\hbar\beta_{max} = 2p\left[1 - \frac{\omega_\beta}{\beta_{max}\upsilon}\right] \quad \theta = 0$$

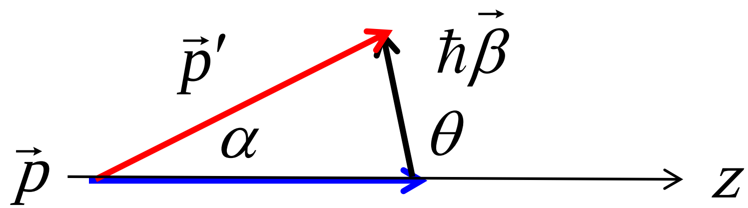
$$\vec{p}\,' = \vec{p} - \hbar\vec{\beta}$$



maximum beta: acoustic phonons

$$\frac{p'^2}{2m^*} = \frac{p^2}{2m^*} \pm \hbar\omega_\beta$$

$$\vec{p}' = \vec{p} \pm \hbar\vec{\beta}$$



$$\hbar\beta = 2p\left[\mp\cos\theta \pm \frac{\omega_\beta}{\beta\upsilon}\right]$$

top sign: ABS

$$\hbar\beta_{\max} = 2p\left[1 \pm \frac{\omega_\beta}{\beta_{\max}\upsilon}\right]$$

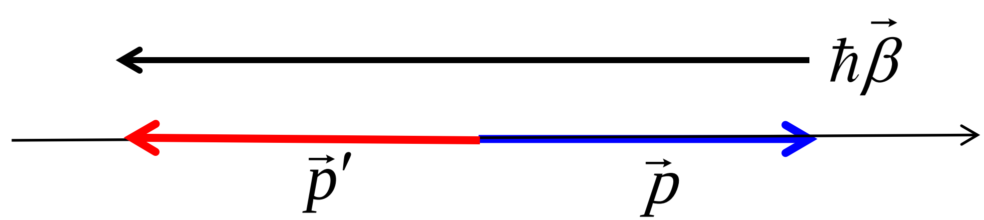
$$\omega_\beta = \upsilon_S\beta$$

$$\hbar\beta_{\max} = 2p\left[1 \pm \frac{\upsilon_S}{\upsilon}\right]$$

To emit an AP, the electron's velocity must be greater than the phonon's.

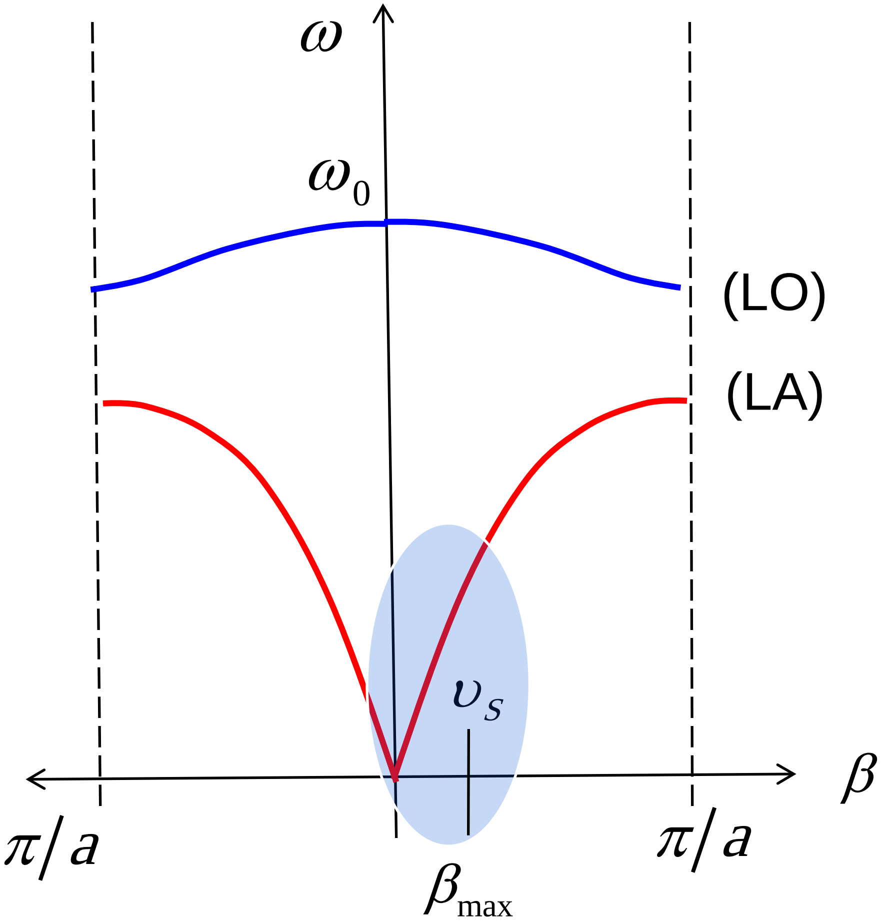
$$\upsilon_S < 10^6 \text{ cm/s} \qquad \langle\upsilon\rangle \approx 10^7 \text{ cm/s}$$

$$\hbar\beta_{\max} \approx 2p$$ (nearly elastic)



Near room temperature, intravalley acoustic phonon scattering is nearly elastic.

maximum beta: acoustic phonons



$$\hbar\beta_{\max} \approx 2p$$

$$\frac{\beta_{\max}}{\pi/a} = \frac{2p}{\hbar\,\pi/a} \approx \frac{1}{4}$$

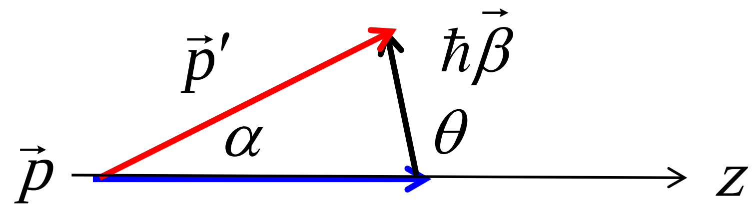
$$\left(p = m_0\upsilon,\ a = 5\ \text{Ang}\right)$$

$$\Delta E_{\max} = \hbar\omega_{\max} = \hbar\beta_{\max}\upsilon_S \approx 10^{-3}\ \text{eV}$$

optical phonons

$$\frac{p'^2}{2m^*} = \frac{p^2}{2m^*} \pm \hbar\omega_\beta$$

$$\vec{p}' = \vec{p} \pm \hbar\vec{\beta}$$



$$\hbar\beta = 2p\left[\mp\cos\theta \pm \frac{\omega_\beta}{\beta\upsilon}\right]$$

top sign: ABS

$$\omega_\beta \approx \omega_0$$

$$\hbar\beta = 2p\left[\mp\cos\theta \pm \frac{\omega_0}{\beta\upsilon}\right]$$

$$\hbar\beta = p\left[-\cos\theta + \sqrt{\cos^2\theta \pm \frac{\hbar\omega_0}{E}}\right]$$

$$\hbar\beta_{\max} = p\left[1 + \sqrt{1 \pm \frac{\hbar\omega_0}{E}}\right]$$

(inelastic)

Any electron can absorb an optical phonon, but to emit an optical phonon, its energy must be > optical phonon energy.

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transition rate

$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\left|H_{p,p'}\right|^2 \delta(E' - E \mp \hbar\omega)$$

$$H_{p',p} = \frac{1}{\Omega}\int_{-\infty}^{+\infty} e^{-i\vec{p}\,'\bullet\vec{r}/\hbar}\, U_S(\vec{r})\, e^{i\vec{p}\bullet\vec{r}/\hbar} d\vec{r}$$

$$U_S(\vec{r}) = K_\beta\, u_\beta \qquad u_\beta(\vec{r}) = A_\beta e^{\pm i\vec{\beta}\bullet\vec{r}}$$

$$H_{p',p} = \frac{1}{\Omega}\int_{-\infty}^{+\infty} e^{-i\vec{p}\,'\bullet\vec{r}/\hbar}\left(K_\beta A_\beta e^{\pm i\vec{\beta}\bullet\vec{r}}\right) e^{i\vec{p}\bullet\vec{r}/\hbar} d\vec{r}$$

$$H_{p',p} = \frac{1}{\Omega}\int_{-\infty}^{+\infty} K_\beta A_\beta e^{-i\left(\vec{p}\,' - \vec{p} \mp \hbar\vec{\beta}\right)\bullet\vec{r}/\hbar}\, d\vec{r}$$

$$H_{p',p} = K_\beta\, A_\beta\, \delta_{\vec{p}\,',\vec{p}\pm\vec{\beta}}$$

$$\left|H_{p',p}\right|^2 = \left|K_\beta\right|^2 \left|A_\beta\right|^2 \delta_{\vec{p}\,',\vec{p}\pm\vec{\beta}}$$

$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\left|K_\beta\right|^2 \left|A_\beta\right|^2 \delta_{\vec{p}\,',\vec{p}\pm\vec{\beta}}\, \delta(E' - E \mp \hbar\omega)$$

energy-momentum conservation

$$S\left(\vec{p},\vec{p}'\right)=\frac{2\pi}{\hbar}\left|K_\beta\right|^2\left|A_\beta\right|^2\delta_{\vec{p}',\vec{p}\pm\vec{\beta}}\,\delta\left(E'-E\mp\hbar\omega\right)$$

$$\vec{p}'=\vec{p}\pm\hbar\vec{\beta}$$

$$E'=E\pm\hbar\omega_\beta$$

$$\vec{p}'\bullet\vec{p}'=\left(\vec{p}\pm\hbar\vec{\beta}\right)\bullet\left(\vec{p}\pm\hbar\vec{\beta}\right)$$

$$p'^2=p^2\pm2\hbar\,\vec{p}\bullet\vec{\beta}+\hbar^2\beta^2$$

$$\frac{p'^2}{2m^*}-\frac{p^2}{2m^*}=\pm\frac{\hbar\,p\beta}{2m^*}\cos\theta+\frac{\hbar^2\beta^2}{2m^*}$$

$$E'-E\mp\hbar\omega_\beta=\pm\frac{\hbar\,p\beta}{2m^*}\cos\theta+\frac{\hbar^2\beta^2}{2m^*}\mp\hbar\omega$$

$$E'-E\mp\hbar\omega_\beta=\hbar\upsilon\beta\left(\pm\cos\theta+\frac{\hbar\beta}{2p}\mp\frac{\omega}{\upsilon\beta}\right)$$

$$\delta_{\vec{p}',\vec{p}\pm\vec{\beta}}\,\delta\left(E'-E\mp\hbar\omega_\beta\right)\rightarrow$$

$$\frac{1}{\hbar\upsilon\beta}\delta\left(\pm\cos\theta+\frac{\hbar\beta}{2p}\mp\frac{\omega_\beta}{\upsilon\beta}\right)$$

phonon amplitude

$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\left|K_\beta\right|^2\left|A_\beta\right|^2\frac{1}{\hbar\upsilon\beta}\,\delta\left(\pm\cos\theta + \frac{\hbar\beta}{2p} \mp \frac{\omega}{\upsilon\beta}\right)$$

$$u_\beta(\vec{r},t) = A_\beta e^{\pm i(\vec{\beta}\bullet\vec{r}-\omega t)} \qquad u_\beta(t) = A_\beta e^{-i\omega t} + A_\beta^* e^{i\omega t} \qquad u_\beta(t) = 2\left|A_\beta\right|\cos(\omega t+\phi)$$

$$\omega = \sqrt{K/M}$$

$$KE = \frac{1}{2}M\left(\frac{du}{dt}\right)^2 = \frac{1}{2}M\omega^2 4\left|A_\beta\right|^2\sin^2(\omega t+\phi)$$

$$PE = \frac{1}{2}Ku^2 = \frac{1}{2}K4\left|A_\beta\right|^2\cos^2(\omega t+\phi) = \frac{1}{2}M\omega^2 4\left|A_\beta\right|^2\cos^2(\omega t+\phi)$$

$$E = KE + PE = 2M\omega^2\left|A_\beta\right|^2$$

$$E = N_\omega\hbar\omega$$

$$\left|A_\beta\right|^2 \rightarrow \frac{\hbar\omega}{2M\omega^2}N_\omega = \frac{\hbar}{2\rho\Omega\omega}N_\omega$$

(almost)

S. Datta, *Atom to Transistor*, Sec. 10.4

absorption vs. emission (L21)

$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\left|K_\beta\right|^2 \left|A_\beta\right|^2 \frac{1}{\hbar\upsilon\beta}\delta\left(\pm\cos\theta + \frac{\hbar\beta}{2p} \mp \frac{\omega}{\upsilon\beta}\right)$$

$$\left|A_\beta\right|^2 \to \frac{\hbar}{2\rho\Omega\omega}N_\omega$$

(almost)

$$S^{ABS}(\vec{p}\,',\vec{p}) \sim N_\omega$$

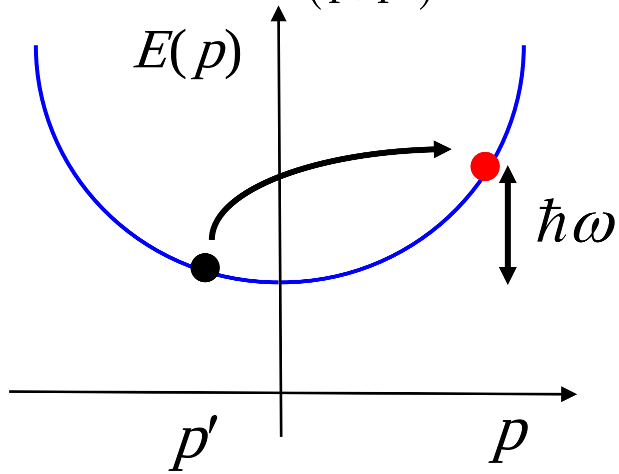
$$S^{EMS}(\vec{p},\vec{p}\,') \sim N_\omega + 1$$

$$N_\omega = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

$$\left|A_\beta^{ABS}\right|^2 \to \frac{\hbar}{2\rho\Omega\omega}N_\omega$$

$$\left|A_\beta^{EMS}\right|^2 \to \frac{\hbar}{2\rho\Omega\omega}\left(N_\omega + 1\right)$$

$$\frac{S^{ABS}(\vec{p}\,',\vec{p})}{S^{EMS}(\vec{p},p')} = e^{-\hbar\omega/k_B T}$$

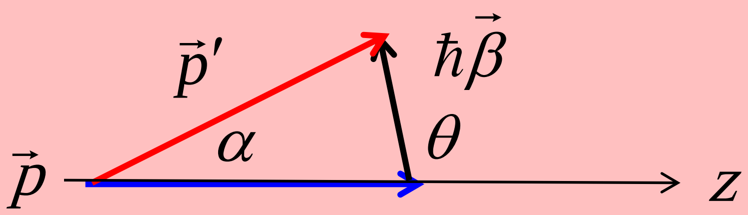


final answer

$$S\left(\vec{p},\vec{p}'\right)=\frac{2\pi}{\hbar}\left|K_\beta\right|^2\frac{\hbar}{2\rho\Omega\omega}\frac{1}{\hbar\upsilon\beta}\left(N_\omega+\frac{1}{2}\mp\frac{1}{2}\right)\delta\left(\pm\cos\theta+\frac{\hbar\beta}{2p}\mp\frac{\omega}{\upsilon\beta}\right)$$

$$S\left(\vec{p},\vec{p}'\right)=\frac{1}{\Omega}C_\beta\left(N_\omega+\frac{1}{2}\mp\frac{1}{2}\right)\delta\left(\pm\cos\theta+\frac{\hbar\beta}{2p}\mp\frac{\omega}{\upsilon\beta}\right)$$

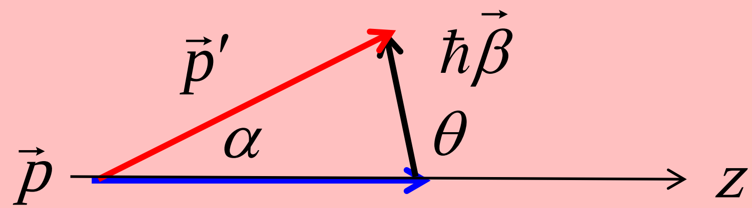
$$C_\beta=\frac{\pi}{\hbar\rho\upsilon\omega\beta}\left|K_\beta\right|^2 \qquad N_\omega=\frac{1}{e^{\hbar\omega/k_B T}-1}$$



scattering rate

$$S(\vec{p},\vec{p}\,') = \frac{1}{\Omega} C_\beta \left(N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) \delta\left(\pm\cos\theta + \frac{\hbar\beta}{2p} \mp \frac{\omega}{\upsilon\beta} \right)$$

$$C_\beta = \frac{\pi}{\hbar\rho\upsilon\omega\beta} \left| K_\beta \right|^2 \qquad N_\omega = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$



$$\frac{1}{\tau} = \sum_{\vec{p}\,',\uparrow} S(\vec{p},\vec{p}\,') = \sum_{\vec{\beta},\uparrow} S(\vec{p},\vec{p}\,') \qquad \vec{p}\,' = \vec{p} \pm \hbar\vec{\beta}$$

scattering rate (ii)

$$\frac{1}{\tau} = \sum_{\vec{\beta},\uparrow} S\left(\vec{p},\vec{p}'\right) = \frac{\Omega}{\left(2\pi\right)^3} \int_0^{2\pi} d\phi \int_0^{\infty} \beta^2 d\beta \int_{-1}^{+1} S\left(\vec{p},\vec{\beta}\right) d\left(\cos\theta\right)$$

$$\frac{1}{\tau} = \frac{1}{8\pi^3} \int_0^{2\pi} d\phi \int_0^{\infty} C_\beta \left(N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) \beta^2 \, d\beta \int_{-1}^{+1} \delta\left(\pm\cos\theta + \frac{\hbar\beta}{2p} \mp \frac{\omega}{\upsilon\beta} \right) d\left(\cos\theta\right)$$

integrate delta function first….

$$\int_{x_1}^{x_2} \delta\left(x - x_0\right) dx = 1 \quad \text{if} \quad x_1 < x_0 < x_2$$

$$= 0 \quad \text{otherwise}$$

$$x \rightarrow \cos\theta$$

$$x_0 = f\left(\beta\right)$$

Integration of the delta function simply restricts β to those values that satisfy energy and momentum conservation.

$$\beta_{\min} < \beta < \beta_{\max}$$

scattering rate (iii)

$$\frac{1}{\tau} = \frac{1}{8\pi^3} \int_0^{2\pi} d\phi \int_{\beta_{\min}}^{\beta_{\max}} C_\beta \left(N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) \beta^2 \, d\beta$$

$$\frac{1}{\tau} = \frac{1}{4\pi^2} \int_{\beta_{\min}}^{\beta_{\max}} C_\beta \left(N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) \beta^2 \, d\beta$$

$$C_\beta = \frac{\pi}{\hbar \rho \upsilon \omega \beta} \left| K_\beta \right|^2 \qquad N_\omega = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

General expression for phonon scattering.

momentum relaxation rate

$$\frac{1}{\tau_m} = \sum_{\vec{p}',\uparrow} S\left(\vec{p},\vec{p}'\right)\left(1-\frac{p'}{p}\cos\alpha\right)$$

$$\left(1-\frac{p'}{p}\cos\alpha\right) = 1-\frac{\vec{p}\bullet\vec{p}'}{p^2} = 1-\frac{\vec{p}\bullet\left(\vec{p}\pm\hbar\vec{\beta}\right)}{p^2} = 1-\frac{p^2 \pm p\hbar\beta\cos\theta}{p^2} = \frac{\mp\hbar\beta\cos\theta}{p}$$

$$\frac{1}{\tau_m} = \sum_{\vec{\beta},\uparrow} S\left(\vec{p},\vec{\beta}\right)\left(\frac{\mp\hbar\beta\cos\theta}{p}\right)$$

$$\frac{1}{\tau_m} = \frac{1}{8\pi^3}\int_0^{2\pi} d\phi \int_0^{\infty} C_\beta\left(N_\omega+\frac{1}{2}\mp\frac{1}{2}\right)\beta\, d\beta^2 \underbrace{\int_{-1}^{+1}\left(\frac{\mp\hbar\beta\cos\theta}{p}\right)\delta\left(\pm\cos\theta+\frac{\hbar\beta}{2p}\mp\frac{\omega_\beta}{\upsilon\beta}\right)d\left(\cos\theta\right)}_{\text{Integration over delta function is a little different.}}$$

momentum relaxation rate (ii)

$$\int_{-1}^{+1}\left(\frac{\mp\hbar\beta\cos\theta}{p}\right)\delta\left(\pm\cos\theta+\frac{\hbar\beta}{2p}\mp\frac{\omega_\beta}{\upsilon\beta}\right)d\left(\cos\theta\right)$$

$$\int_{x_1}^{x_2} f(x)\delta\left(x-x_0\right)dx = f\left(x_0\right) \quad \text{if} \quad x_1 < x_0 < x_2$$

$$= 0 \quad \text{otherwise}$$

$$x \to \cos\theta$$

$$f\left(x\right)\to\left(\frac{\mp\hbar\beta\cos\theta}{p}\right)$$

$$x_0 \to -\frac{\hbar\beta}{2p}\pm\frac{\omega_\beta}{\upsilon\beta}$$

$$\frac{1}{\tau_m}=\frac{1}{4\pi^2}\int_{\beta_{\min}}^{\beta_{\max}} C_\beta\left(N_\omega+\frac{1}{2}\mp\frac{1}{2}\right)\left(\frac{\hbar\beta}{2p}\mp\frac{\omega_\beta}{\upsilon\beta}\right)\frac{\hbar\beta^3}{p}d\beta$$

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acoustic phonon scattering

$$\frac{1}{\tau} = \frac{1}{4\pi^2} \int_{\beta_{\min}}^{\beta_{\max}} C_\beta \left(N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) \beta^2 \, d\beta$$

$$C_\beta = \frac{\pi m^* D_A^2}{\hbar \rho \upsilon_s p}$$ (Lundstrom, p. 79)

$$N_\omega = \frac{1}{e^{\hbar\omega_s/k_B T} - 1} \qquad \hbar\omega_s << k_B T \rightarrow N_{\omega_s} \approx \frac{k_B T}{\hbar\omega_s} \approx N_{\omega_s} + 1$$

"equipartition"

$$\frac{1}{\tau_{abs}} = \frac{1}{\tau_{ems}} = \frac{1}{4\pi^2} \int_{\beta_{\min}}^{\beta_{\max}} \frac{\pi m^* D_A^2}{\hbar \rho \upsilon_s p} \frac{k_B T}{\hbar\omega_s} \beta^2 \, d\beta$$

acoustic phonon scattering (ii)

$$\frac{1}{\tau_{abs}} = \frac{1}{\tau_{ems}} = \frac{m^* D_A^2 k_B T}{4\pi\hbar^2 \rho \upsilon_s p} \int_{\beta_{\min}}^{\beta_{\max}} \frac{\beta}{\omega_s} \beta \, d\beta \qquad \upsilon_s = \sqrt{c_l/\rho}$$

$$\frac{1}{\tau_{abs}} = \frac{1}{\tau_{ems}} = \frac{m^* D_A^2 k_B T}{4\pi\hbar^2 c_l p} \int_{\beta_{\min}}^{\beta_{\max}} \beta \, d\beta = \frac{m^* D_A^2 k_B T}{4\pi\hbar^2 c_l p} \left(\frac{\beta_{\max}^2}{2} - \frac{\beta_{\min}^2}{2} \right)$$

elastic scattering:

$$\hbar\beta_{\max} = 2p \qquad \hbar\beta_{\min} = 0$$

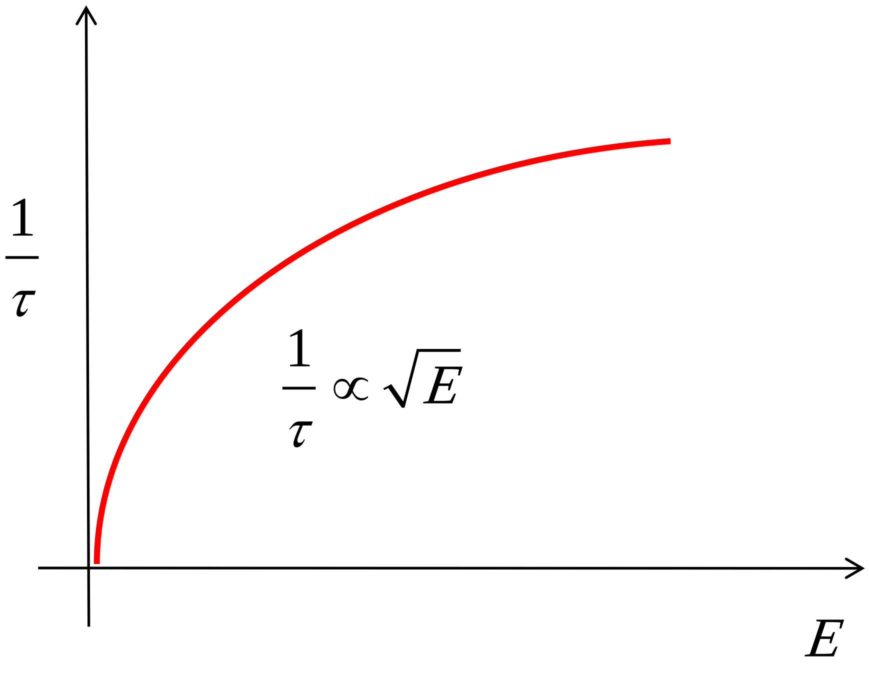
$$\frac{1}{\tau_{abs}} = \frac{1}{\tau_{ems}} \qquad \frac{1}{\tau} = \frac{1}{\tau_{abs}} + \frac{1}{\tau_{ems}}$$

acoustic phonon scattering (iii)

$$\frac{1}{\tau} = \frac{1}{\tau_{abs}} + \frac{1}{\tau_{ems}} = \frac{2\pi}{\hbar}\left(\frac{D_A^2 k_B T}{c_l}\right)\frac{D_{3D}\left(E\right)}{2}$$

$$\frac{1}{\tau} = \frac{1}{\tau_m}$$

(isotropic)



Exercise: We have assumed that ADP scattering is elastic, which is generally a good assumption near room temperature. Repeat the derivation of the scattering and momentum relaxation rates without this assumption.

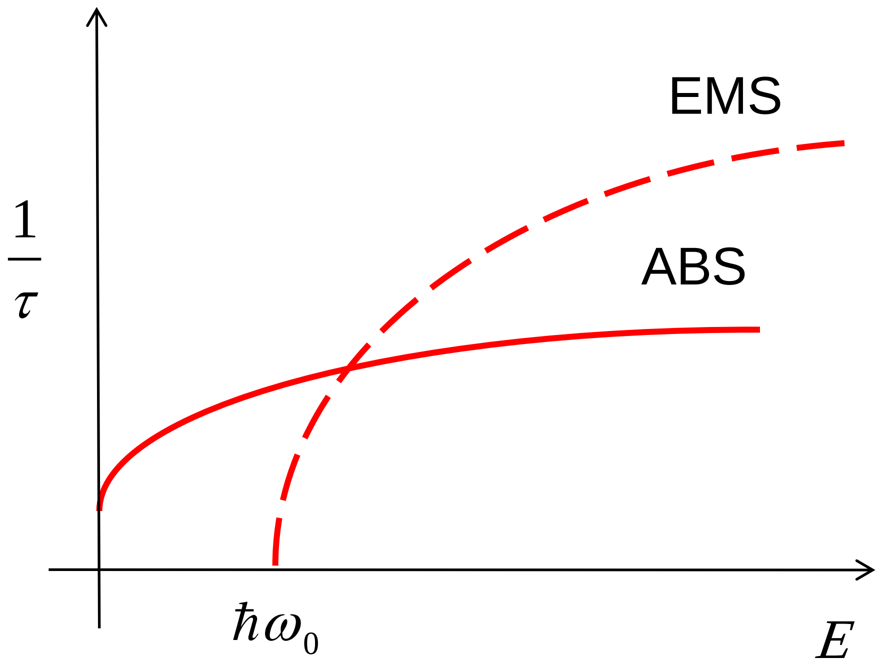
optical deformation potential scattering

$$\frac{1}{\tau}=\frac{1}{4\pi^2}\int_{\beta_{\min}}^{\beta_{\max}} C_\beta\left(N_\omega+\frac{1}{2}\mp\frac{1}{2}\right)\beta^2\,d\beta$$

$$C_\beta=\frac{\pi m^* D_O^2}{\hbar\rho\omega_0\beta p}$$

(Lundstrom, p. 79)

$$N_0=\frac{1}{e^{\hbar\omega_o/k_B T}-1}$$



$$\frac{1}{\tau}=\frac{1}{\tau_m}=\frac{2\pi}{\hbar}\left(\frac{\hbar D_O^2}{2\rho\omega_0}\right)\left(N_0+\frac{1}{2}\mp\frac{1}{2}\right)\frac{D_{3D}\left(E\pm\hbar\omega_0\right)}{2}$$

$$\frac{1}{\tau_{abs}}\neq\frac{1}{\tau_{ems}}$$

one final point

$$\frac{1}{\tau} = \frac{1}{4\pi^2} \int_{\beta_{\min}}^{\beta_{\max}} C_\beta \left(N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) \beta^2 \, d\beta \qquad C_\beta = \frac{\pi m^* D_O^2}{\hbar \rho \omega_0 \beta p}$$

How to we determine whether scattering is isotropic?

Go back to the matrix element:

$$\left| H_{p',p} \right|^2 = \frac{1}{\Omega} \left| K_\beta \right|^2 \frac{\hbar}{2\rho\, \omega_0} \left(N_0 + \frac{1}{2} \mp \frac{1}{2} \right) \delta_{\vec{p}', \vec{p} \pm \vec{\beta}}$$

$$\left| K_\beta \right|^2 = D_0^2 \qquad N_0 = \frac{1}{e^{\hbar\omega_o / k_B T} - 1}$$

No dependence on β → isotropic scattering.

questions

1) Review

2) Energy-momentum conservation

3) Mathematical formulation

4) Example

5) Summary