ECE-656: Fall 2009

Lecture 25: Phonon Scattering III

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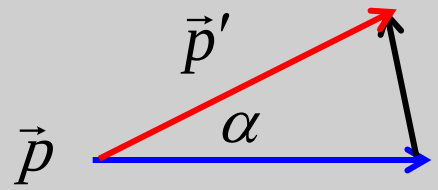
outline

1) **Review**
2) POP and IV scattering
3) Scattering in common semiconductors
4) Electron-electron scattering
5) Summary

electron-phonon scattering

$$S(\vec{p},\vec{p}') = \frac{2\pi}{\hbar}\left|H_{p,p'}\right|^2 \delta\left(E' - E \mp \hbar\omega\right)$$ (weak scattering)

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega}\left|K_\beta\right|^2 \frac{\hbar}{2\rho\,\omega}\left(N_\omega + \frac{1}{2} \mp \frac{1}{2}\right)\delta_{\vec{p}',\vec{p}\pm\vec{\beta}} \qquad N_\omega = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$



$\hbar\vec{\beta}$ $\qquad E' = E \pm \hbar\omega_\beta \qquad \vec{p}' = \vec{p} \pm \hbar\vec{\beta}$

Isotropic: no β dependence in the matrix element.

Elastic: approximately elastic when $\hbar\omega << \left\langle E(\vec{p})\right\rangle$

ADP scattering

$$S\left(\vec{p},\vec{p}'\right)\approx\frac{2\pi}{\hbar}\left|H_{p,p'}\right|^2\delta\left(E'-E\right)$$ (Nearly elastic at room temperature)

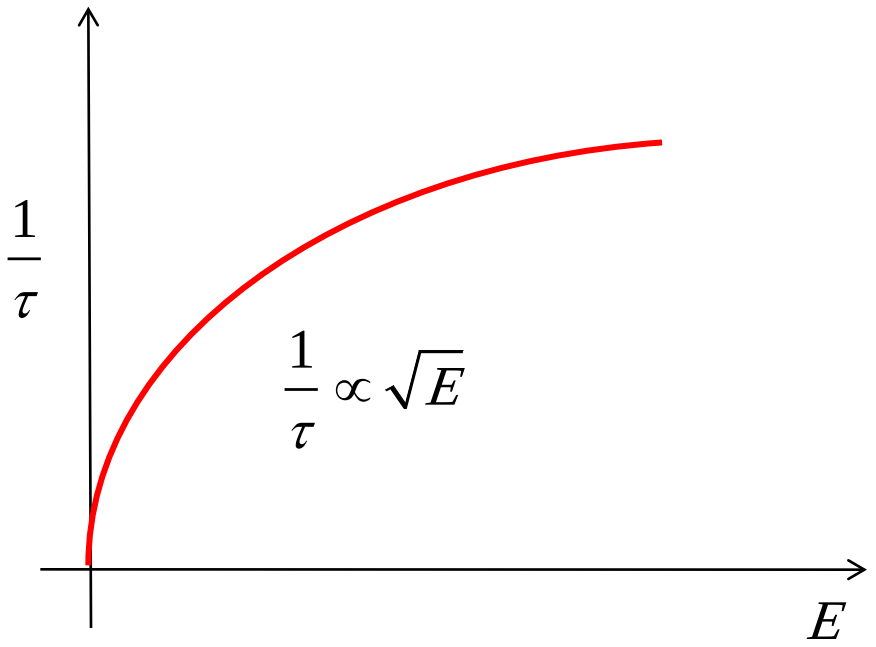
$$\left|H_{p',p}\right|^2=\frac{1}{\Omega}\left|K_\beta\right|^2\frac{\hbar}{2\rho\,\omega}\left(N_\omega+\frac{1}{2}\mp\frac{1}{2}\right)\delta_{\vec{p}',\vec{p}\pm\vec{\beta}}\qquad N_\omega=\frac{1}{e^{\hbar\omega/k_BT}-1}\approx\frac{k_BT}{\hbar\omega}\approx N_\omega+1$$

$$\left|K_\beta\right|^2=\beta^2D_A^2$$

$$\frac{1}{\tau}=\frac{2\pi}{\hbar}\left(\frac{D_A^2k_BT}{c_l}\right)\frac{D_{3D}\left(E\right)}{2}$$

(ABS + EMS)

$$\frac{1}{\tau}=\frac{1}{\tau_m}$$ (isotropic)



ODP scattering

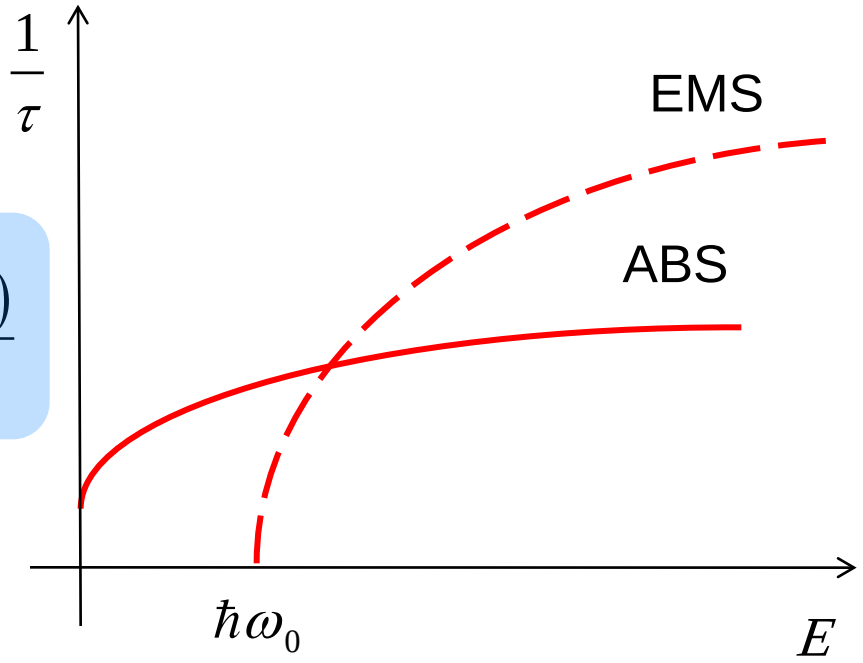
$$S(\vec{p},\vec{p}') \approx \frac{2\pi}{\hbar}\left|H_{p,p'}\right|^2 \delta\left(E' - E \pm \hbar\omega_0\right)$$ (inelastic at RT and below)

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega}\left|K_\beta\right|^2 \frac{\hbar}{2\rho\ \omega_0}\left(N_0 + \frac{1}{2} \mp \frac{1}{2}\right)\delta_{\vec{p}',\vec{p}\pm\vec{\beta}}$$

$$N_0 = \frac{1}{e^{\hbar\omega_0/k_B T} - 1} < N_0 + 1$$

$$\left|K_\beta\right|^2 = D_0^2$$

$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\left(\frac{\hbar D_O^2}{2\rho\omega_0}\right)\left(N_0 + \frac{1}{2} \mp \frac{1}{2}\right)\frac{D_{3D}\left(E \pm \hbar\omega_0\right)}{2}$$



$$\frac{1}{\tau_{abs}} \neq \frac{1}{\tau_{ems}} \qquad \frac{1}{\tau} = \frac{1}{\tau_m}$$ (isotropic)

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POP scattering

$$S(\vec{p},\vec{p}') \approx \frac{2\pi}{\hbar}\left|H_{p,p'}\right|^2 \delta\left(E' - E \pm \hbar\omega_0\right)$$ (inelastic at RT and below)

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega}\left|K_\beta\right|^2 \frac{\hbar}{2\rho\ \omega_0}\left(N_0 + \frac{1}{2} \mp \frac{1}{2}\right)\delta_{\vec{p}',\vec{p}\pm\vec{\beta}} \qquad N_0 = \frac{1}{e^{\hbar\omega_0/k_B T} - 1} < N_0 + 1$$

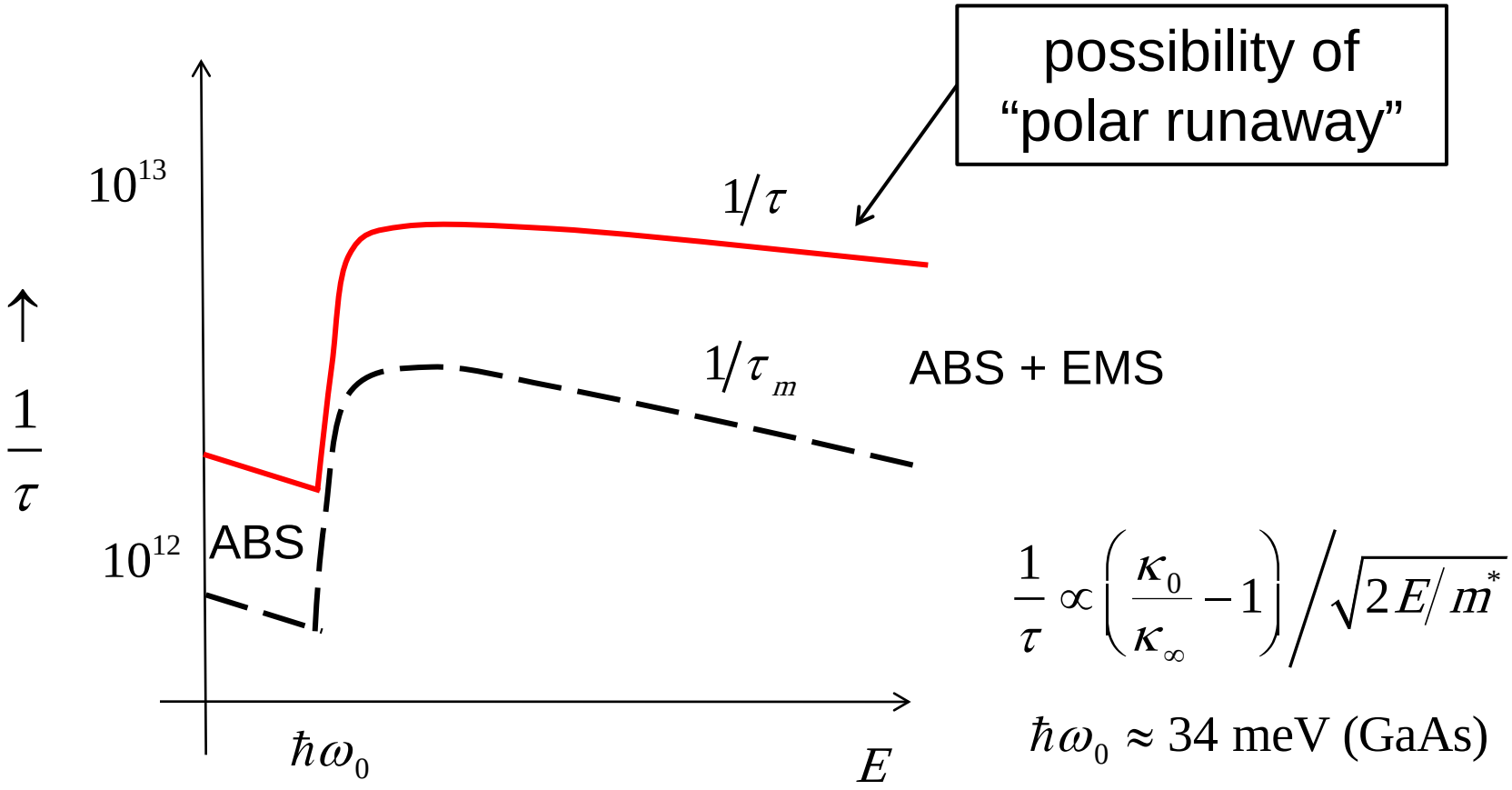
$$\left|K_\beta\right|^2 = \frac{\rho q^2 \omega_0^2}{\beta^2 \kappa_0 \varepsilon_0}\left(\frac{\kappa_0}{\kappa_\infty} - 1\right)$$ (favors small angle scattering)

$$\frac{1}{\tau} = \frac{1}{\tau_{abs}} + \frac{1}{\tau_{ems}} = \frac{q^2\omega_0^2\left(\frac{\kappa_0}{\kappa_\infty} - 1\right)}{2\pi\kappa_0\varepsilon_0\hbar\sqrt{2E/m^*}}\left[N_0 \sinh^{-1}\left(\frac{E}{\hbar\omega_0}\right) + \left(N_0 + 1\right)\sinh^{-1}\left(\frac{E}{\hbar\omega_0} - 1\right)\right]$$

$$\frac{1}{\tau} > \frac{1}{\tau_m} > \frac{1}{\tau_E}$$

Lundstrom, pp. 84 – 86 for momentum and energy relaxation rates.

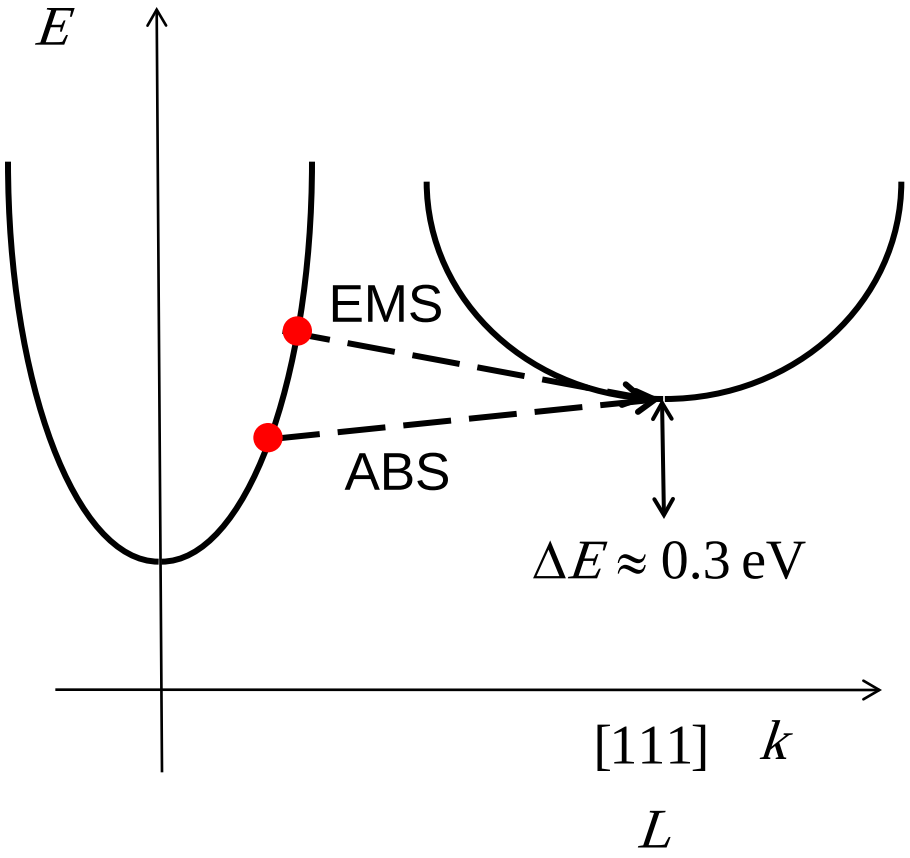
POP scattering



$$\frac{1}{\tau} \propto \left(\frac{\kappa_0}{\kappa_\infty} - 1 \right) \Big/ \sqrt{2E/m^*}$$

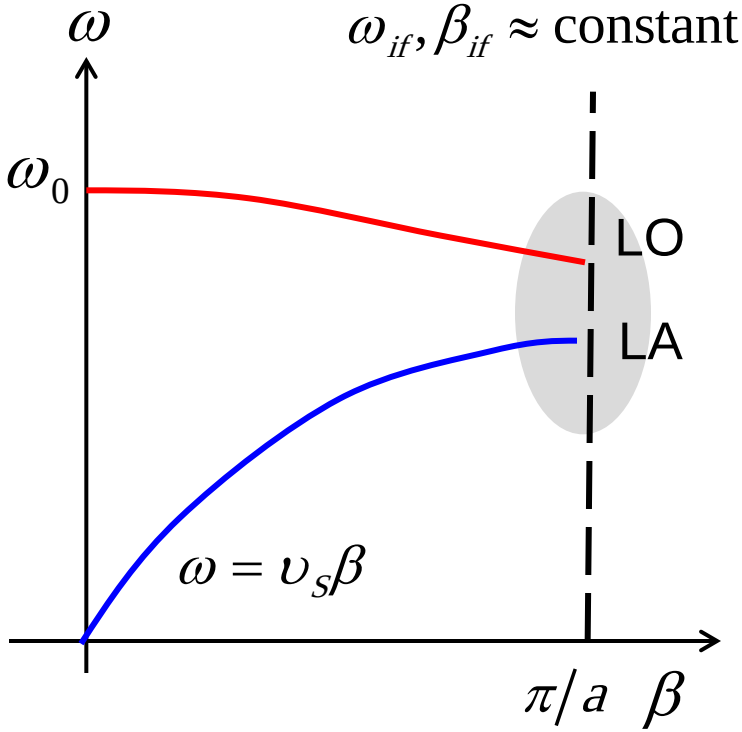
$\hbar\omega_0 \approx 34$ meV (GaAs)

IV scattering (GaAs)



"intervalley phonons"

$\omega_{if}, \beta_{if} \approx$ constant



Requires phonons with momentum near the zone boundary.

IV scattering (GaAs)

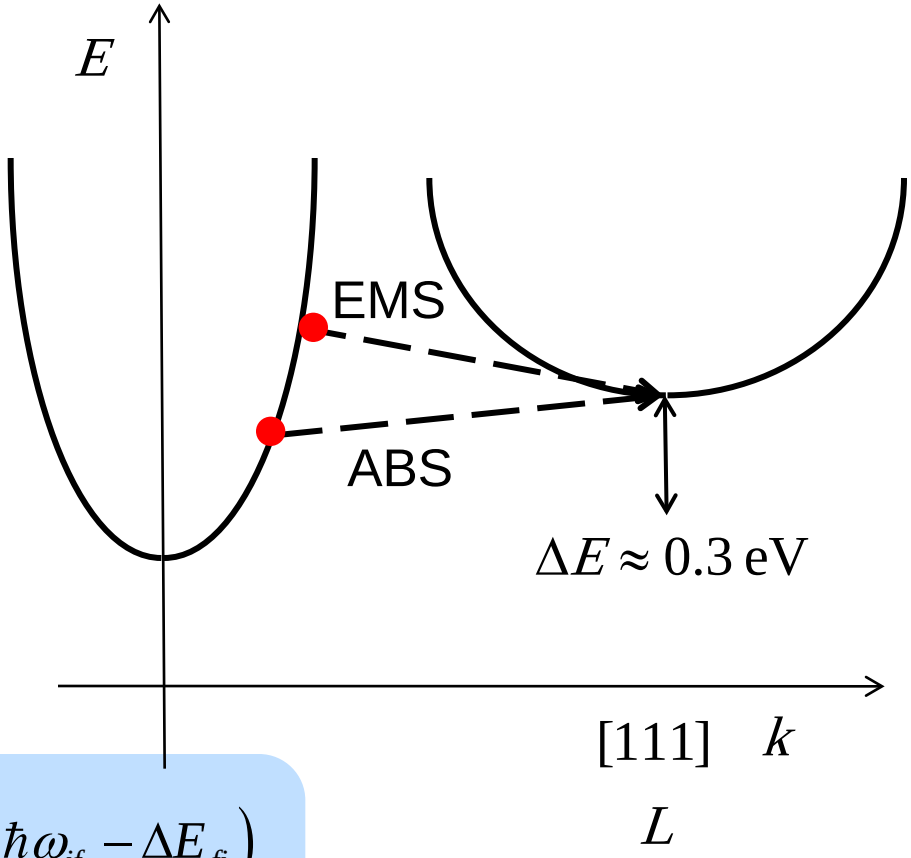
Postulate:

$$U_S = D_{if} u_\beta$$

Then IV scattering looks like ODP scattering

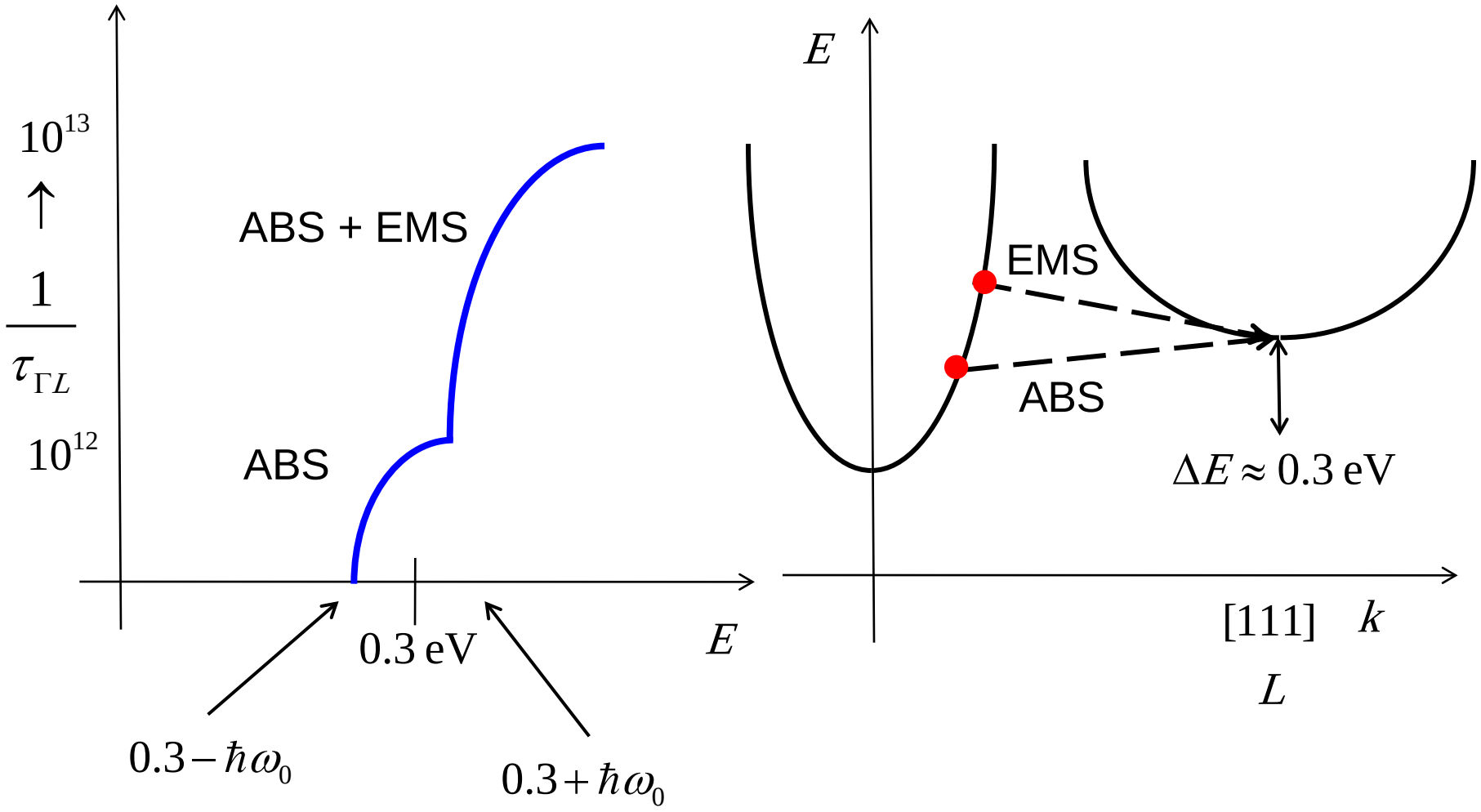
Isotropic: $\frac{1}{\tau} = \frac{1}{\tau_m}$

Number of final valleys: Z_f

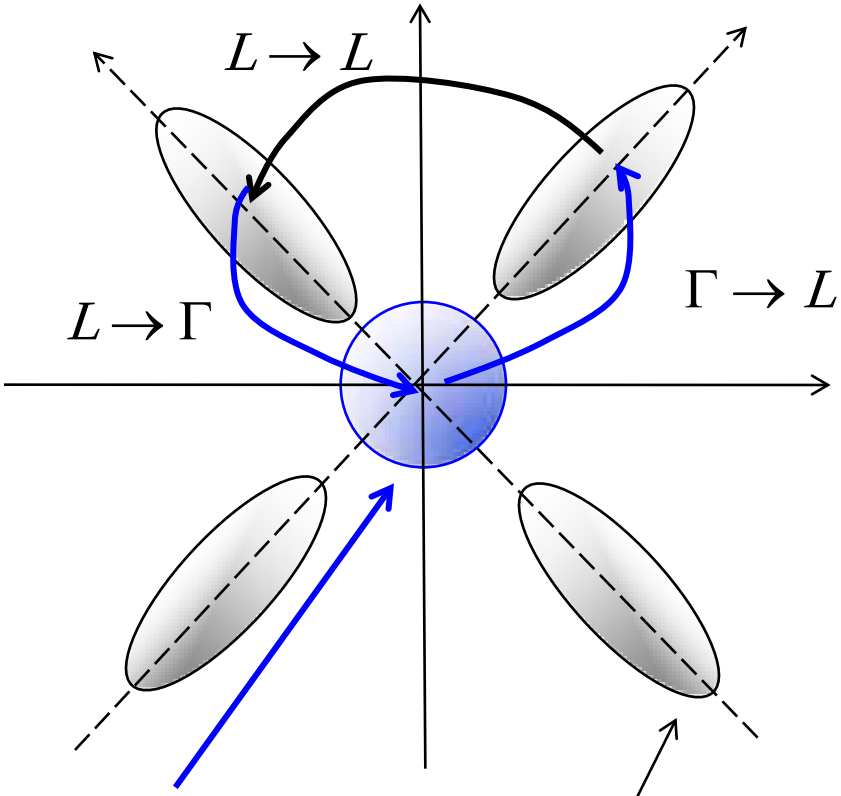


$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\left(\frac{\hbar D_{if}^2 Z_f}{2\rho\omega_{if}}\right)\left(N_{if} + \frac{1}{2} \mp \frac{1}{2}\right)\frac{D_f\left(E \pm \hbar\omega_{if} - \Delta E_{fi}\right)}{2}$$

IV scattering (GaAs)



L-L and L-Γ IV scattering (GaAs)



Γ valley constant energy surface.

L valley constant energy surfaces.

$$\frac{1}{\tau} \propto D_{if}^2 Z_f \frac{D_f\left(E \pm \hbar\omega_{if} - \Delta E_{fi}\right)}{2}$$

$\Gamma \rightarrow L$: $Z_f = 4$ $\Delta E_{fi} = 0.3$ eV

$L \rightarrow L$: $Z_f = 3$ $\Delta E_{fi} = 0$ eV

$L \rightarrow \Gamma$: $Z_f = 1$ $\Delta E_{fi} = -0.3$ eV

Compare rates:

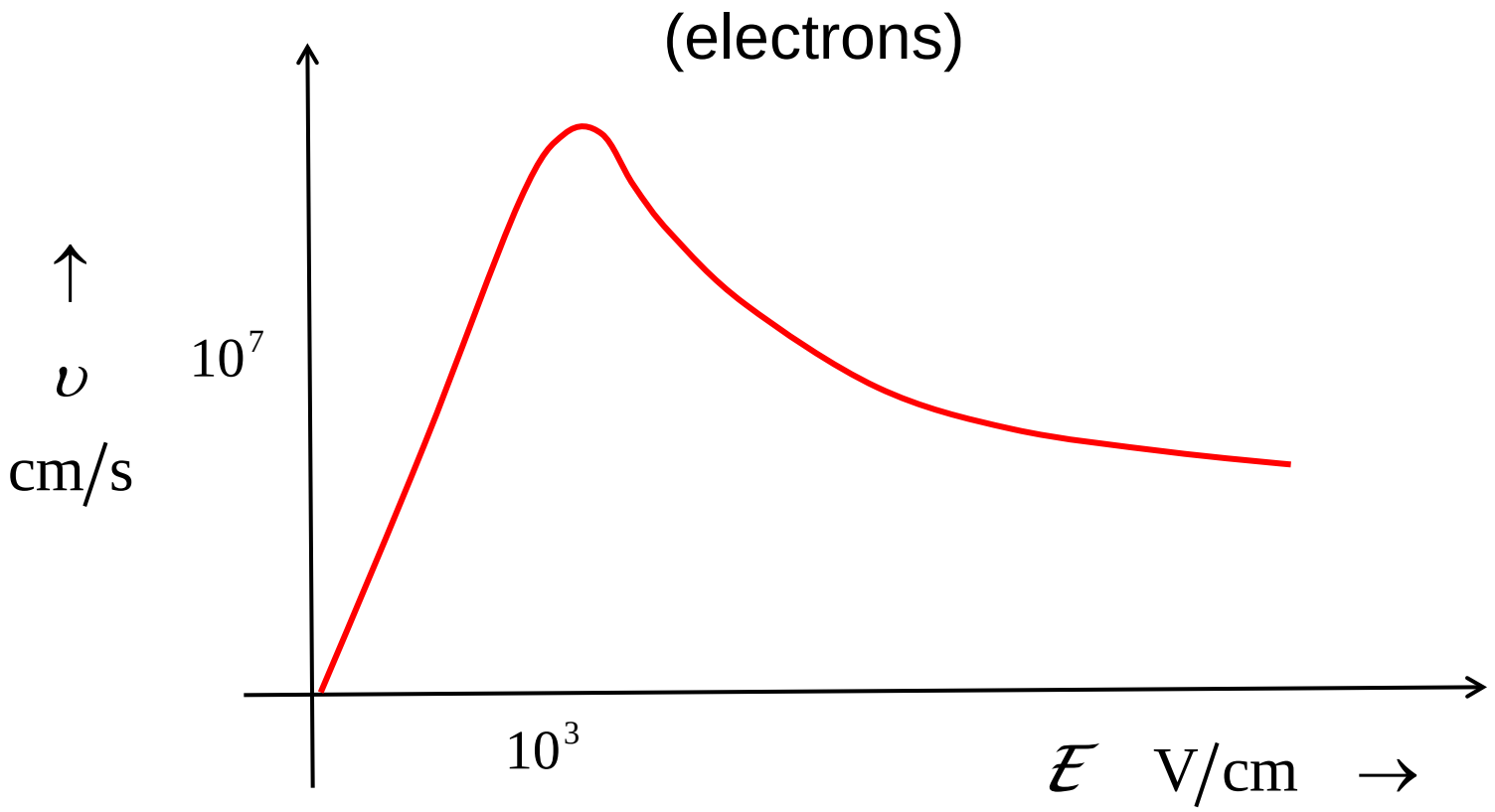
$1/\tau_{\Gamma\rightarrow\Gamma}$ (POP)

$1/\tau_{\Gamma\rightarrow L}$

$1/\tau_{L\rightarrow L}$

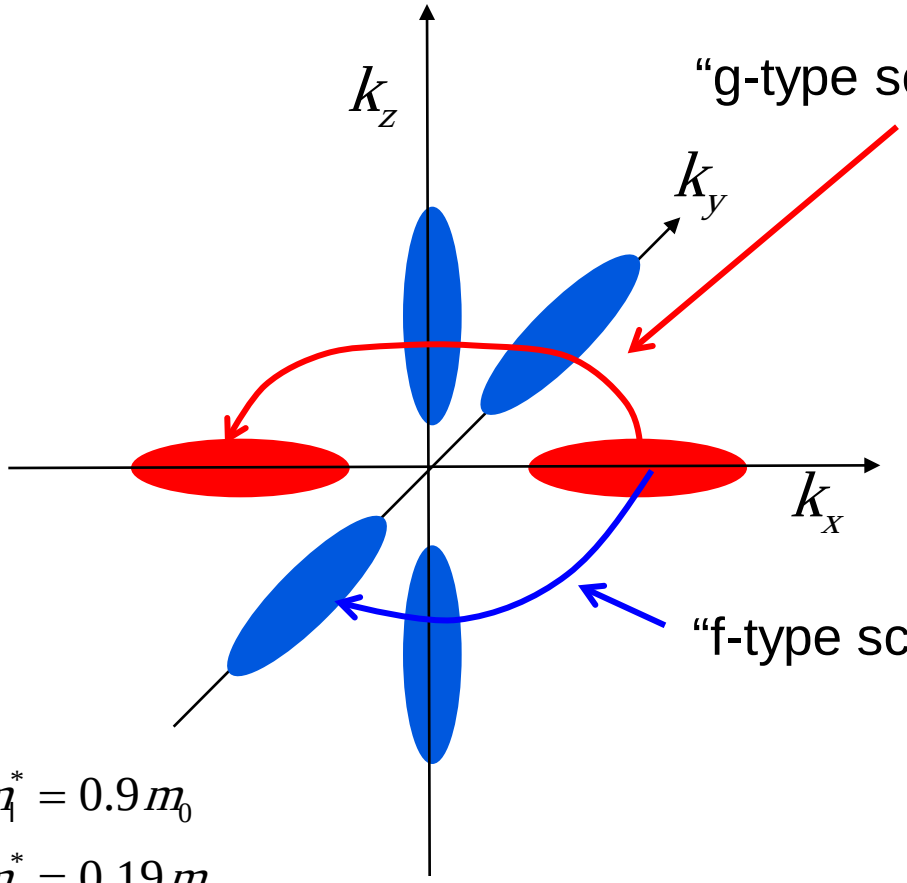
$1/\tau_{L\rightarrow\Gamma}$

velocity vs. electric field: GaAs



equivalent IV scattering (Si)

Si conduction band



$$\text{g-type}: \quad Z_f = 1 \quad \Delta E_{fi} = 0 \text{ eV}$$

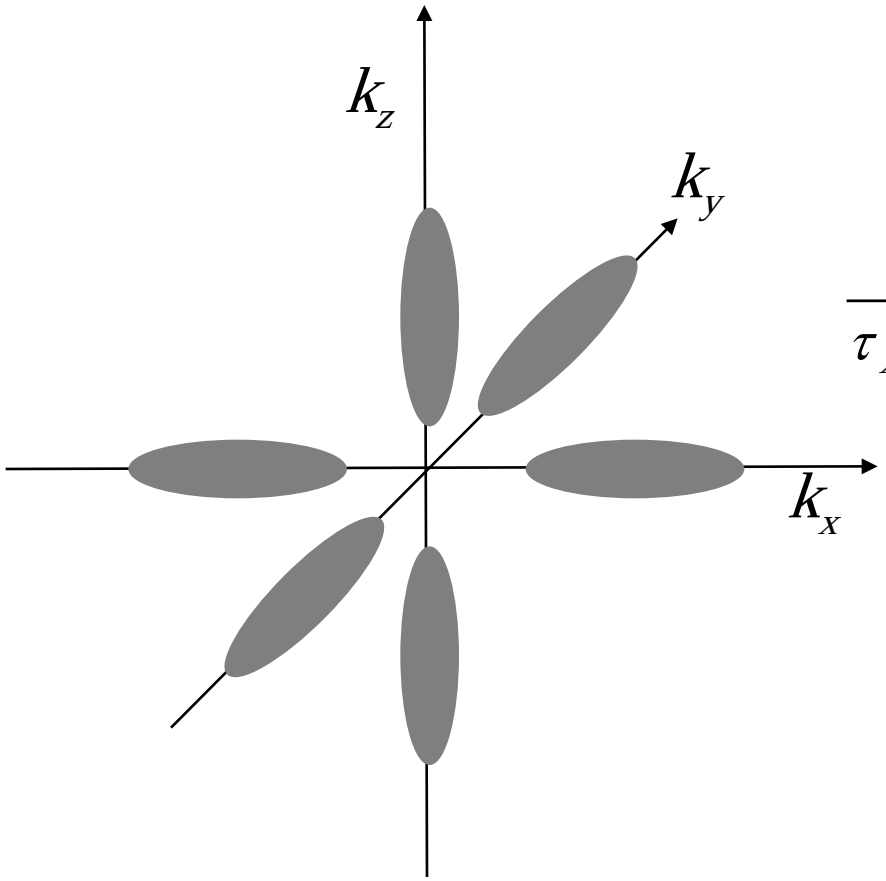
$$f\text{-type}: \quad Z_f = 4 \quad \Delta E_{fi} = 0 \text{ eV}$$

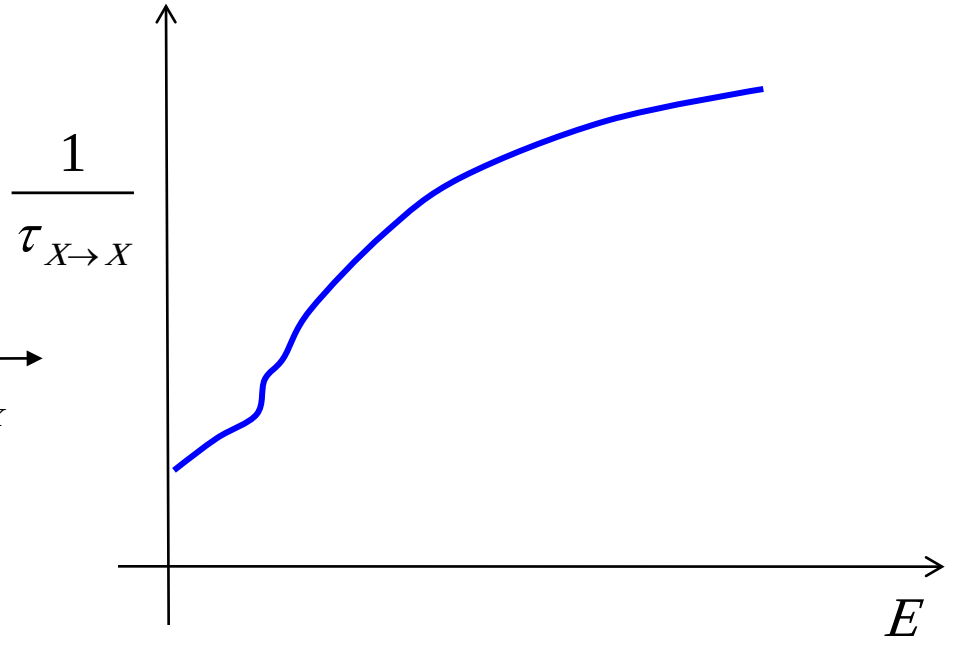
$$m_l^* = 0.9 m_0$$

$$m_t^* = 0.19 m_0$$

equivalent IV scattering (Si)

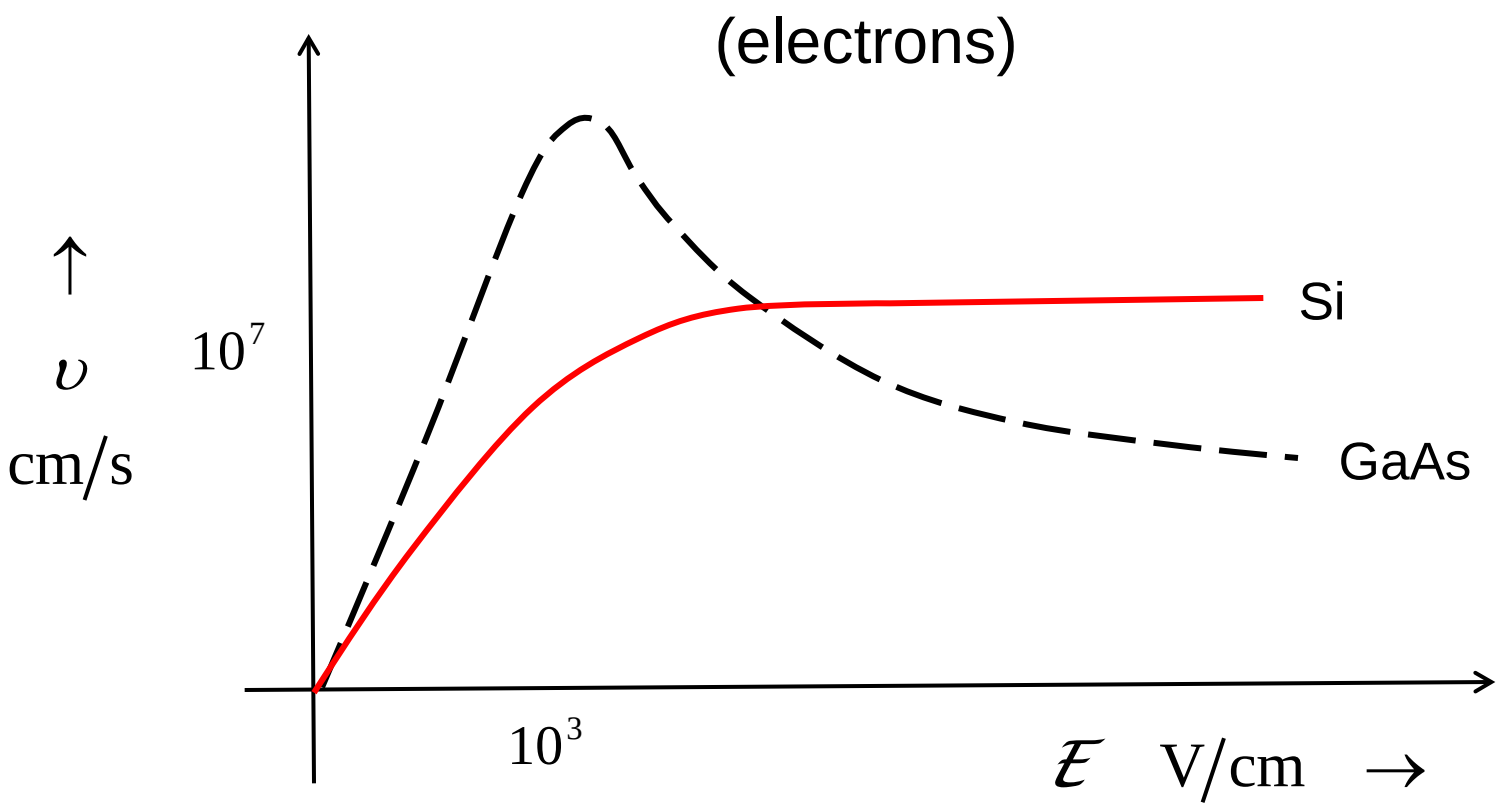
Si conduction band



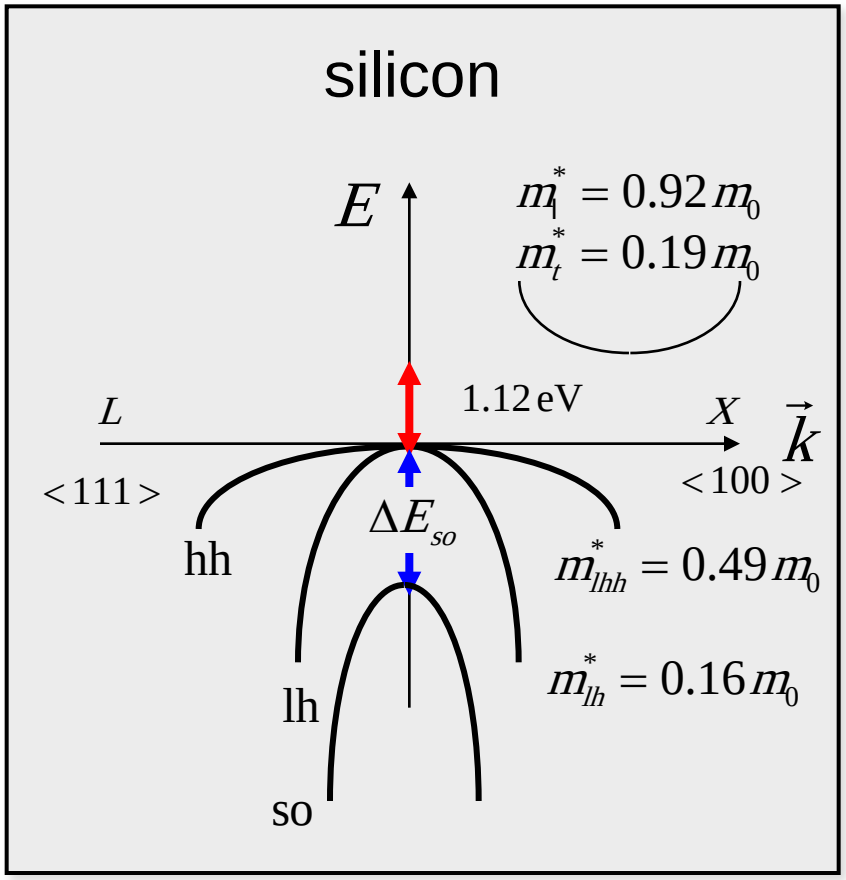


Important at room temperature in Si

velocity vs. electric field: Si



about holes



$$\Delta E_{so} = 44 \text{ meV (Si)}$$

$$= 290 \text{ meV (Ge)}$$

Intravalley:

$$\frac{1}{\tau_{hh\to hh}} \quad \frac{1}{\tau_{lh\to lh}} \quad \frac{1}{\tau_{so\to so}}$$

Intervalley:

$$\frac{1}{\tau_{hh\to lh}} \quad \frac{1}{\tau_{lh\to hh}}$$

$$\frac{1}{\tau_{hh\to so}} \quad \frac{1}{\tau_{so\to hh}} \quad \frac{1}{\tau_{lh\to so}} \quad \frac{1}{\tau_{so\to lh}}$$

Valence band is complex (warped) and can be engineered by strain.

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key questions

1) What is the total scattering rate vs. energy for common semiconductors?

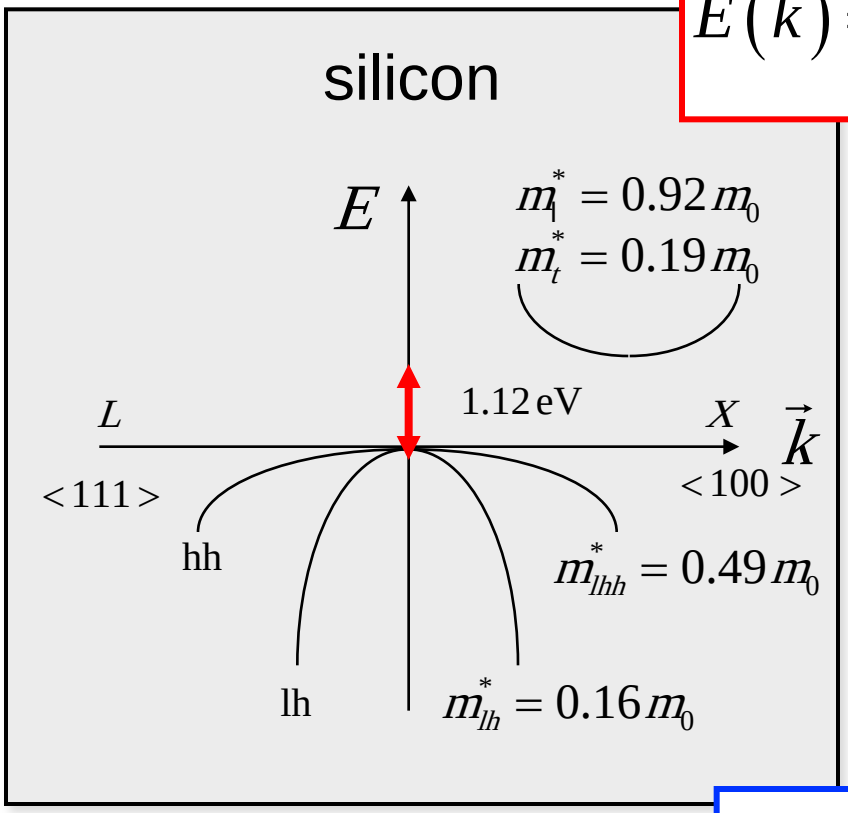
$$\Gamma = \sum_i \frac{1}{\tau_i}$$

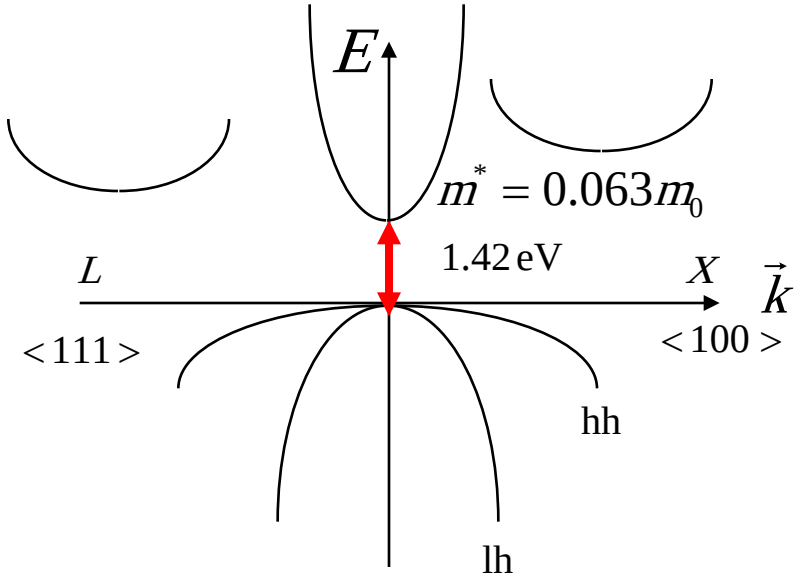
2) How do covalent semicondictors (e.g. Si, Ge) differ from polar semiconductors (e.g. GaAs, InP, InGaAs, ZnSe)?

model bandstructure (for analytical calculations)

Si, covalent, indirect BG

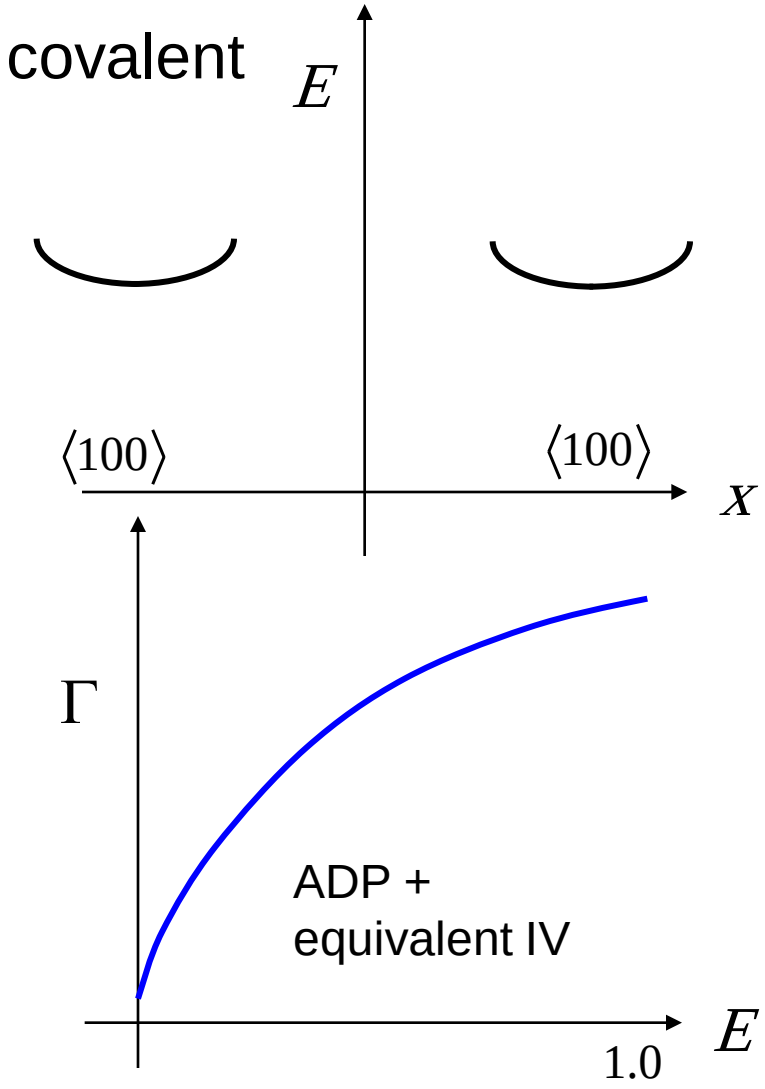
$$E(k) = E_C + \frac{\hbar^2 k^2}{2m_n^*}$$

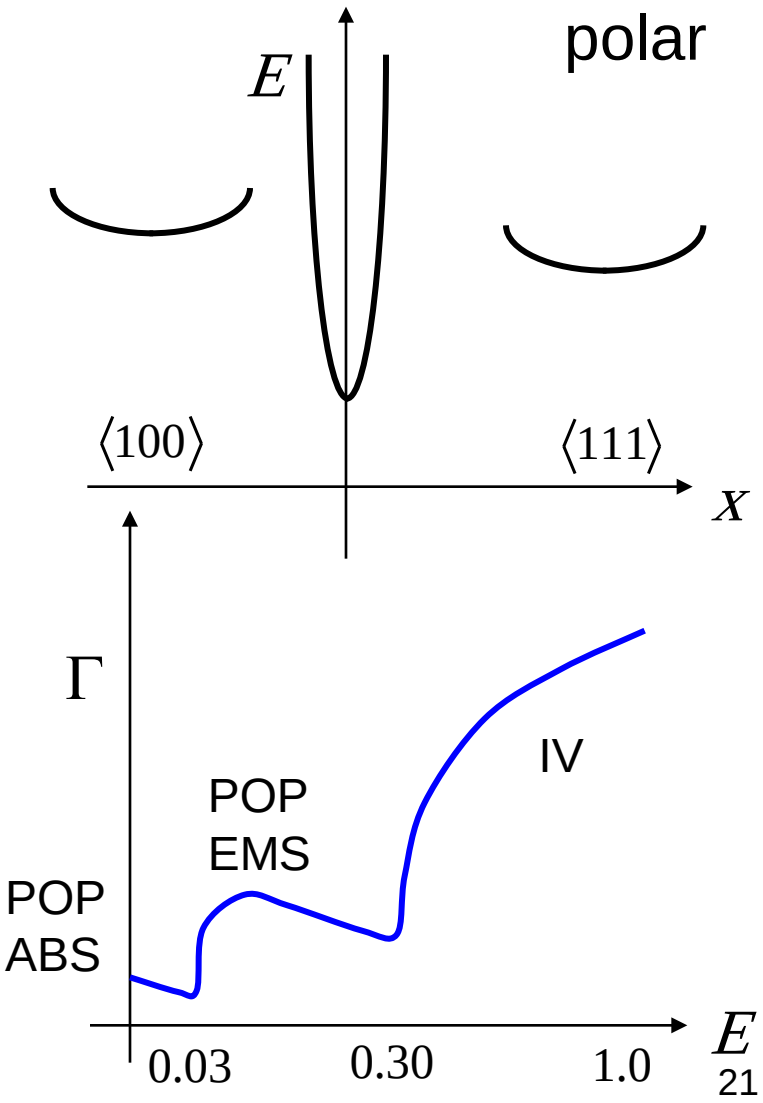




$$E(k) = E_V - \frac{\hbar^2 k^2}{2m_p^*}$$

covalent vs. polar semiconductors

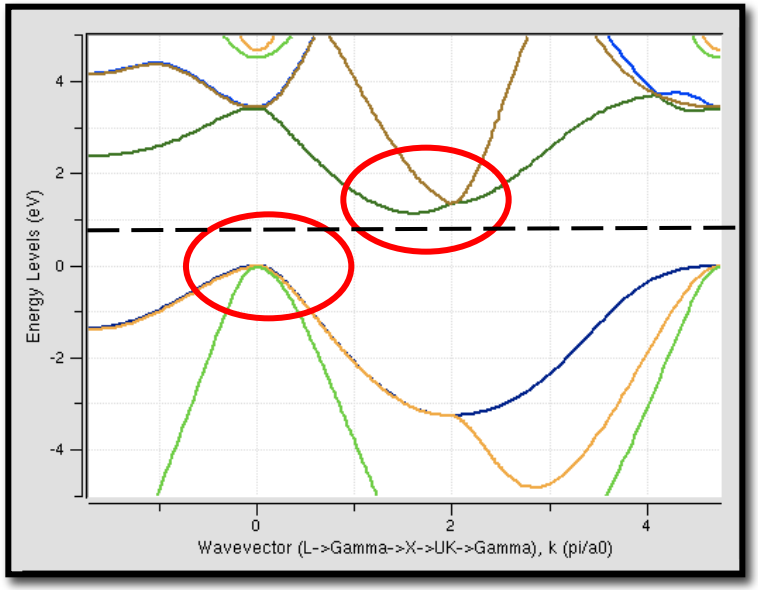


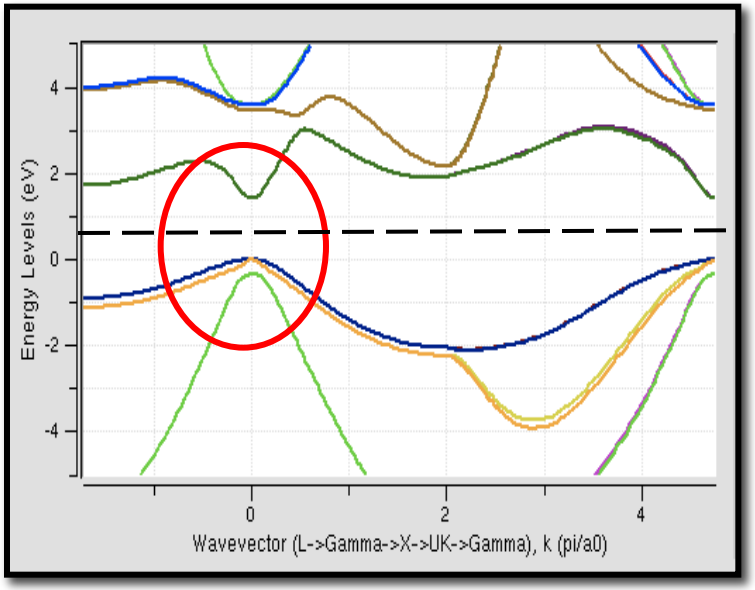


energy bands

For any wavevector, *k*, there is an infinite set of eigenenergies, $E_n\left(\vec{k}\right)$

Si, covalent, indirect bandgap





('Bandstructure Lab' at www.nanoHUB.org)

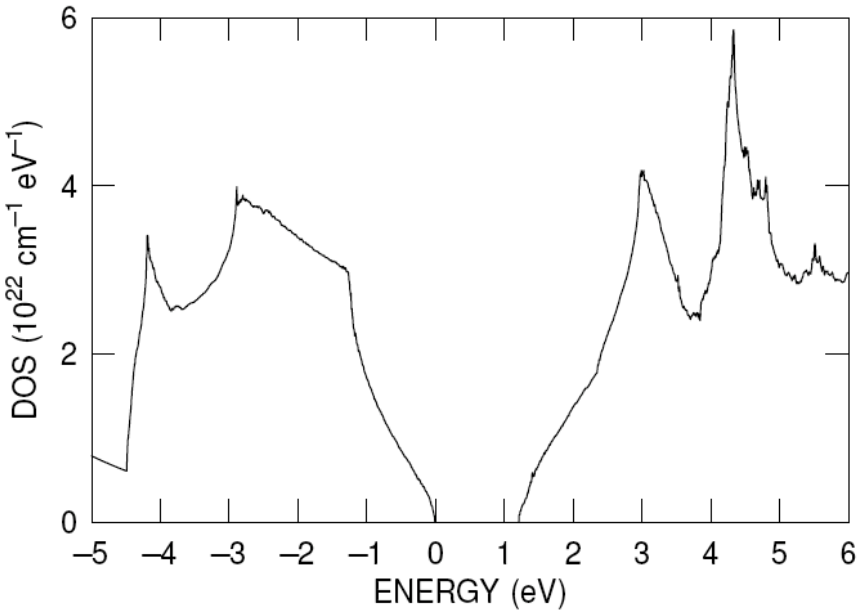
“full band” scattering rates

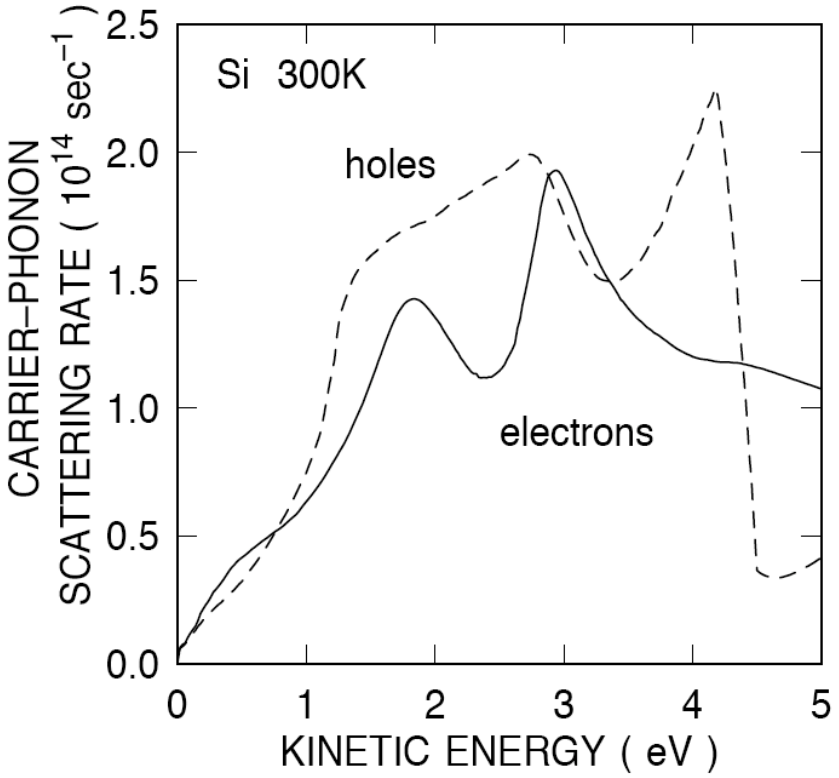
For a good, general reference on the numerical evaluation of scattering rates in common semiconductors, see:

[1] Massimo V. Fischetti, Monte Carlo Simulation of Transport in Technologically Significant Semiconductors of the Diamond and Zinc-Blende Structures-Part I: Homogeneous Transport,” *IEEE Trans. Electron Dev*., **38**, pp. 634-649, 1991

[2] M. V. Fischetti, N. Sano, S. E. Laux, and K. Natori, “Full-bandstructure theory of high-field transport and impact ionization of electrons and holes in Ge, Si, GaAs, InAs, and $In_{0.53}Ga_{0.47}As$,” *unpublished manuscript*., January 3, 2001.

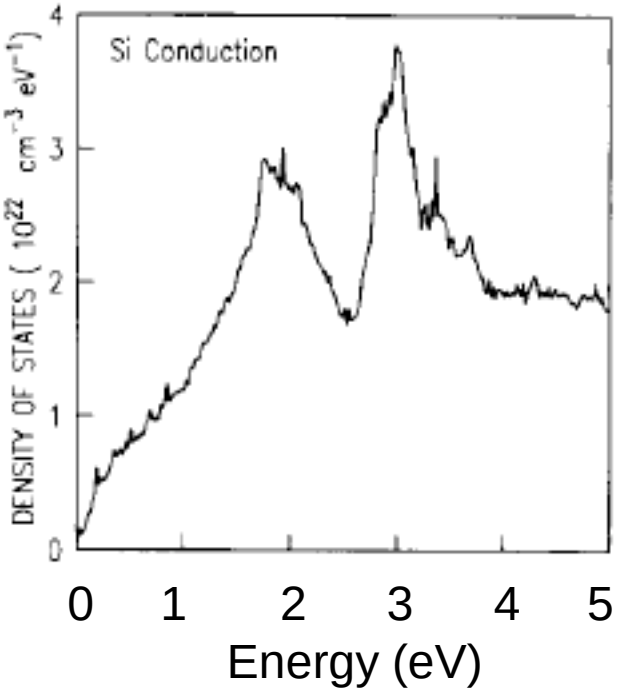
electrons and holes in Si

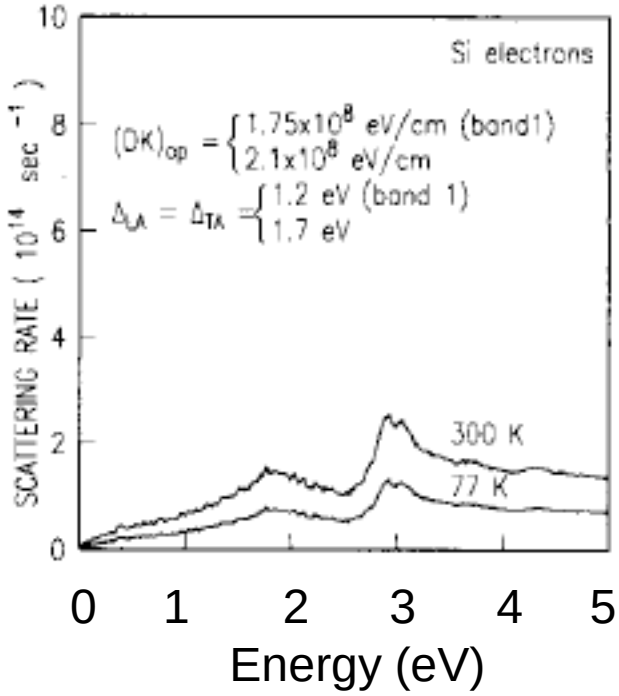




[2] Figures provided by Massimo V. Fischetti, October, 2009.

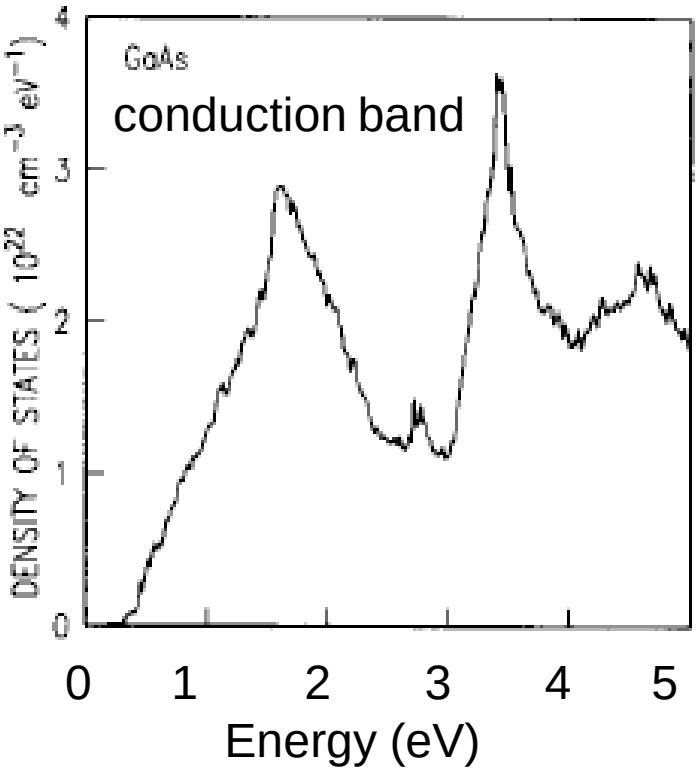
electrons in Si

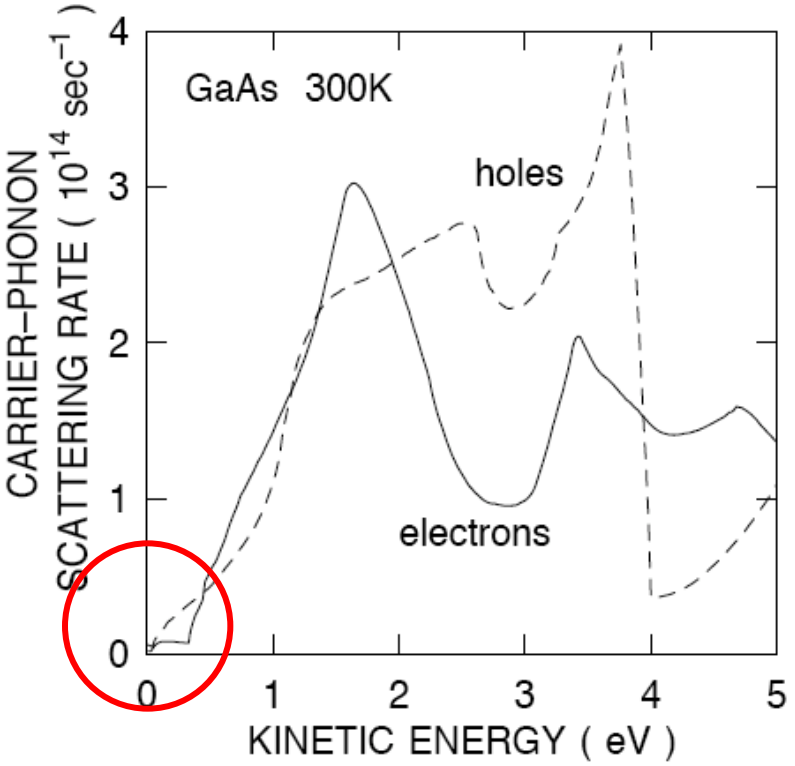




[1] Massimo V. Fischetti," *IEEE Trans. Electron Dev.*, **38**, pp. 634-649, 1991

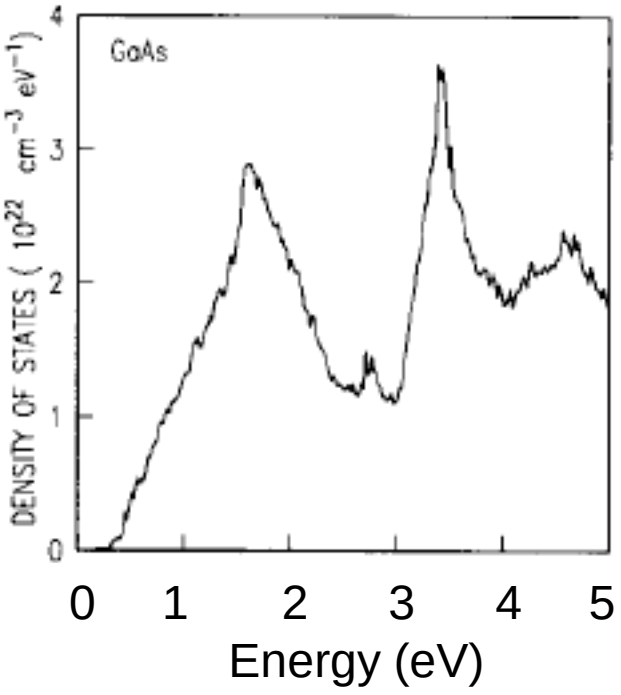
electrons and holes in GaAs

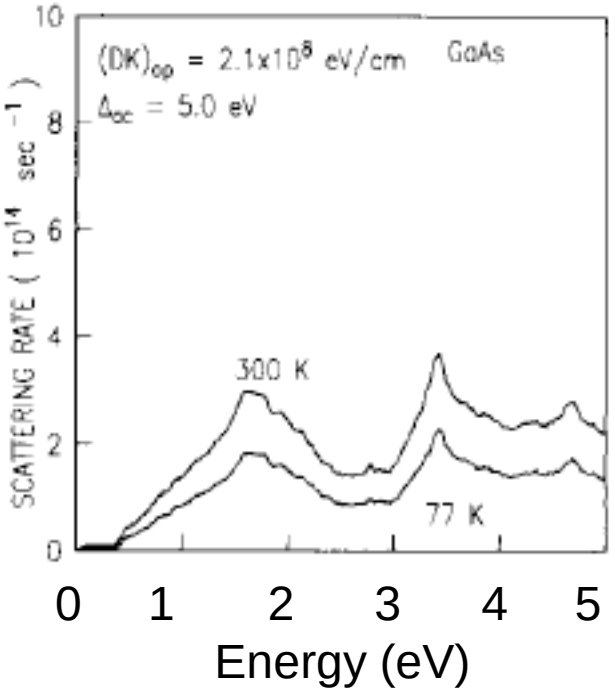




DOS: [1] M. V. Fischetti," *IEEE Trans. Electron Dev.*, **38**, pp. 634-649, 1991
Scattering rate: [2] Provided by M. V. Fischetti, October, 2009)

electrons in GaAs





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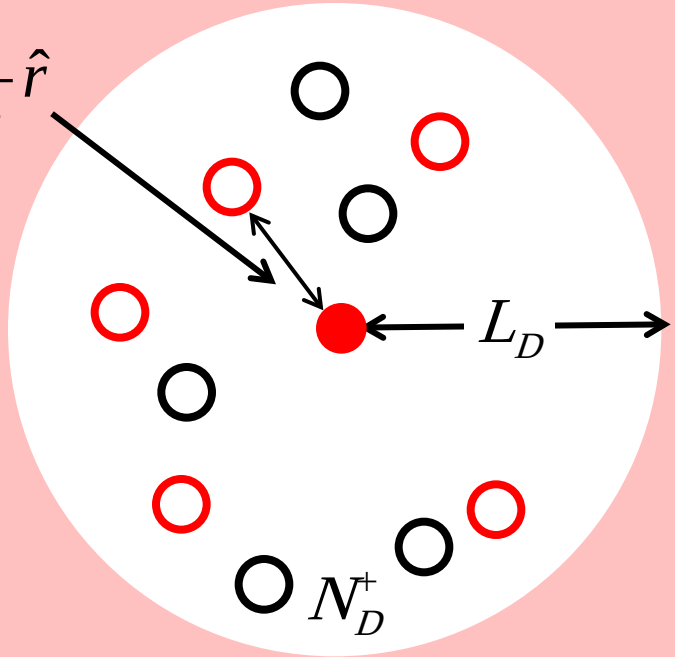
long and short range Coulomb effects

$$\vec{F}_e(\vec{r},t) = -q\vec{\mathcal{E}}(\vec{r},t)$$

$$\nabla \bullet \left[\kappa_S \varepsilon_0 \vec{\mathcal{E}}(\vec{r},t)\right] = \rho(\vec{r},t)$$

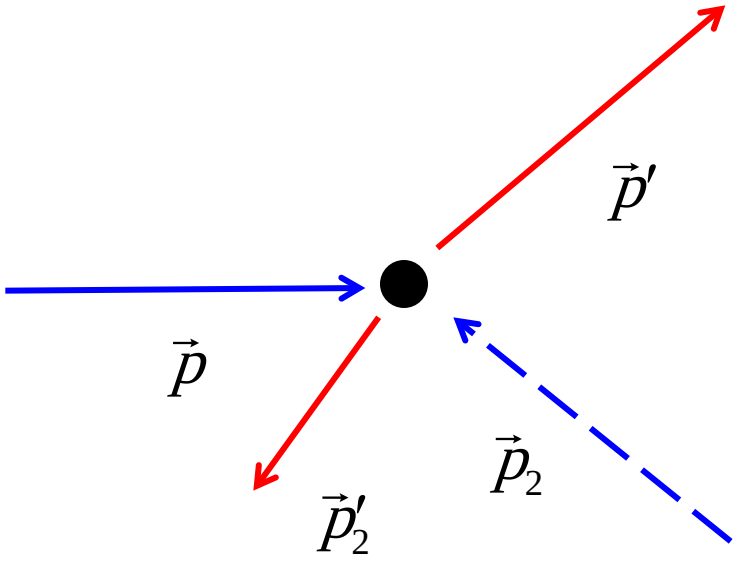
$$\vec{F}_e(\vec{r},t) = \frac{q_1 q_2}{4\pi\kappa_S\varepsilon_0 r^2}\hat{r}$$



n_0 cm^{-3}

$$\rho = q\left(N_D^+ - n_0\right) \text{ C/cm}^{-3}$$

binary e-e scattering



$$\vec{p} + \vec{p}_2 = \vec{p}' + \vec{p}_2'$$

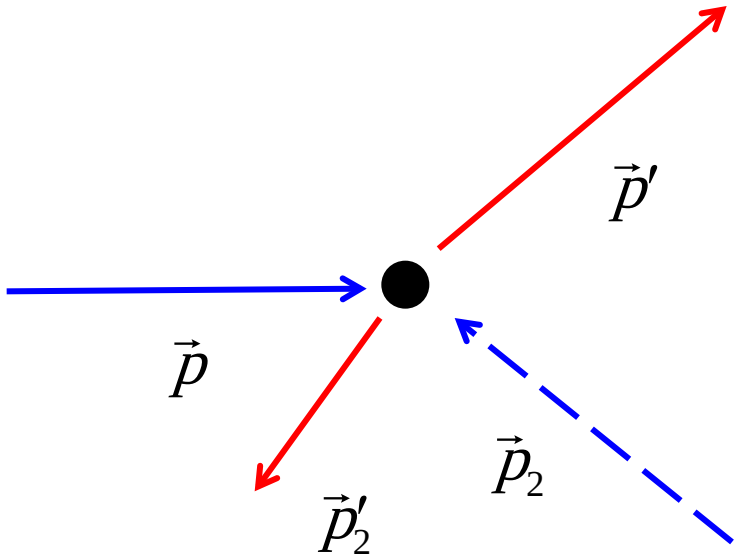
$$E + E_2 = E' + E_2'$$

Important when n > ~ 10^{17} cm^{-3}

$$S\left(\vec{p}, \vec{p}'\right) \rightarrow S\left(\vec{p}, \vec{p}_2; \vec{p}', \vec{p}_2'\right)$$

$$S\left(\vec{p}, \vec{p}_2; \vec{p}', \vec{p}_2'\right) \propto \delta_{\vec{p}+\vec{p}_2, \vec{p}'+\vec{p}_2'} \delta\left(E' + E_2' - E - E_2\right)$$

binary e-e scattering rate



$$\frac{1}{\tau} = \sum_{\vec{p}'} S\left(\vec{p}, \vec{p}'\right) \rightarrow$$

Important when n > ~ 10^{17} cm^{-3}

$$\frac{1}{\tau_{e-e}} = \sum_{\vec{p}'} S\left(\vec{p}, \vec{p}_2; \vec{p}', \vec{p}_2'\right) f\left(\vec{p}_2\right)\left[1 - f\left(\vec{p}\right)\right]\left[1 - f\left(\vec{p}_2'\right)\right]$$

(a very difficult problem to solve)

collision integral

Assume a non-degenerate semiconductor:

$$\left.\frac{\partial f}{\partial t}\right|_{coll} = \sum_{\vec{p}',\vec{p}_2'} S\left(\vec{p}',\vec{p}_2';\vec{p},\vec{p}_2\right) f\left(\vec{p}'\right) f\left(\vec{p}_2'\right) - S\left(\vec{p},\vec{p}_2;\vec{p}',\vec{p}_2'\right) f\left(\vec{p}\right) f\left(\vec{p}_2\right)$$

The probability for a transition and its inverse are equal (see. eqn. (2.100) p. 90 of Lundstrom)

$$S\left(\vec{p},\vec{p}_2;\vec{p}',\vec{p}_2'\right) = S\left(\vec{p}',\vec{p}_2';\vec{p},\vec{p}_2\right)$$

$$\left.\frac{\partial f}{\partial t}\right|_{coll} = \sum_{\vec{p}',\vec{p}_2'} S\left(\vec{p},\vec{p}_2;\vec{p}',\vec{p}_2'\right)\left[f\left(\vec{p}\right) f\left(\vec{p}_2\right) - f\left(\vec{p}'\right) f\left(\vec{p}_2'\right)\right]$$

Eventually the electron system comes into equilibrium – every in-scattering is balanced by a corresponding out-scattering and…

$$f\left(\vec{p}\right) f\left(\vec{p}_2\right) = f\left(\vec{p}'\right) f\left(\vec{p}_2'\right)$$

“equilibrium” solution”

$$f\left(\vec{p}\right) f\left(\vec{p}_2\right) = f\left(\vec{p}'\right) f\left(\vec{p}_2'\right)$$

For a solution, try: $f\left(\vec{p}\right) = e^{Kp^2}$

$$f\left(\vec{p}\right) f\left(\vec{p}_2\right) = f\left(\vec{p}'\right) f\left(\vec{p}_2'\right) \rightarrow e^{K\left(p^2+p_2^2\right)} = e^{K\left(p'^2+p_2'^2\right)}$$

True because energy is conserved.

The average energy per carrier for an equilibrium Maxwellian is $3k_BT_L/2$

When the electron system is out of equilibrium with the lattice, then the average energy per carrier can be written as: $3k_BT_e/2$

$$T_e > T_L$$

e-e scattering

When e-e- scattering dominates, electrons exchange energy among themselves and an equilibrium shape (Maxwellian or Fermi-Dirac) distribution is established.

$$f\left(\vec{p}\right) \propto e^{p^2/2mk_BT_e}$$

The electron temperature, however, may be quite different from the lattice temperature.

To first order, electron-electron scattering does not lower the mobility, because the total momentum of the electron system is conserved, but there can be an indirect effect because the shape of the distribution affects the average scattering rate for other scattering processes.

outline

1) Review

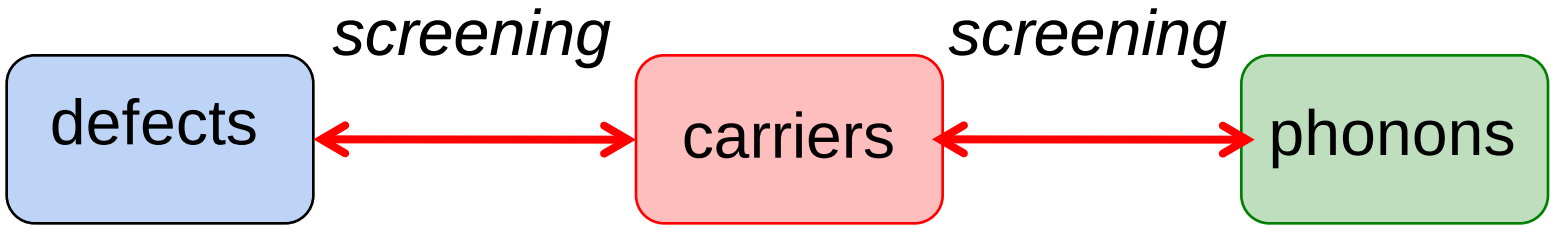
2) POP and IV scattering

3) Scattering in common semiconductors

4) Electron-electron scattering

5) Summary

scattering in semiconductors



- ionized impurities
- neutral impurities
- dislocations
- surface roughness
- alloy

- electron-electron
- electron-plasmon
- electron-hole

- intravalley
  -ADP
  -ODP
  -POP
  -PZ

- intervalley
  -acoustic
  -optical

questions

1) Review

2) POP and IV scattering

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