ECE-656: Fall 2009

Lecture 26:
Mobility in 3D, 2D, and 1D

Professor Mark Lundstrom
Electrical and Computer Engineering
Purdue University, West Lafayette, IN USA

about this lecture

The goal in this lecture is to examine one scattering mechanism (ADP scattering) in 3D, 2D, and 1D to see how the scattering rate changes with dimensionality. Then we'll compare mobilities in 3D, 2D, and 1D.

This lecture is based on a set of notes prepared by Drs. George Bourianoff and Dmitri Nikonov of Intel Corporation with additional contributions by Dr. Sayed Hasan of Intel.

a question

One frequently hears and sees statements to the fact that quantum confined structures should display higher mobilities because the lower density of states should result in less scattering. Is this statement true?

outline

1) **Review of ADP Scattering in 3D**

2) ADP Scattering in 2D: MCA

3) ADP Scattering in 2D: FGR

4) ADP Scattering in 1D: FGR

5) Mobility in 1D, 2D, and 3D

ADP scattering: 3D in a nutshell

$$S(\vec{p},\vec{p}') = \frac{2\pi}{\hbar}\left|H_{p',p}\right|^2 \delta\left(E'-E\mp\hbar\omega\right)$$

$$H_{p',p} = \frac{1}{\Omega}\int_{-\infty}^{+\infty} e^{-i\vec{p}'\bullet\vec{r}/\hbar}U_S(\vec{r})e^{i\vec{p}\bullet\vec{r}/\hbar}d\vec{r}$$

$$U_S = \sum_\beta K_\beta u_\beta$$

$$\left|K_\beta\right|^2 = \beta^2 D_A^2$$

Note the sum over β

$$u_\beta(\vec{r},t) = A_\beta e^{\pm i\left(\vec{\beta}\bullet\vec{r}-\omega_\beta t\right)}$$

$$\left|A_\beta\right|^2 = \frac{\hbar}{2\rho\Omega\,\omega}\left(N_\omega+\frac{1}{2}\mp\frac{1}{2}\right)$$

$$N_\omega = \frac{1}{e^{\hbar\omega/k_BT}-1}\approx\frac{k_BT}{\hbar\omega}$$

$$N_\omega \approx N_\omega + 1$$

ADP scattering: 3D in a nutshell (ii)

$$H_{p',p} = \frac{1}{\Omega} \int_{-\infty}^{+\infty} e^{-i\vec{p}'\bullet\vec{r}/\hbar} \left(\sum_{\beta} K_\beta u_\beta \right) e^{i\vec{p}\bullet\vec{r}/\hbar} d\vec{r}$$

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} \sum_{\beta} U_{ac} \frac{1}{\Omega} \left| \int_{-\infty}^{+\infty} e^{-i\vec{p}'\bullet\vec{r}/\hbar} \left(e^{\pm i\vec{\beta}\bullet\vec{r}} \right) e^{i\vec{p}\bullet\vec{r}/\hbar} d\vec{r} \right|^2$$

$$U_{ac} = \left|K_\beta\right|^2 \left|A_\beta\right|^2 = \frac{D_A^2 k_B T}{2 c_l}$$

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \delta_{\vec{p}',\vec{p}\pm\hbar\vec{\beta}}$$

$$u_\beta\left(\vec{r}\right) = A_\beta e^{\pm i\vec{\beta}\bullet\vec{r}}$$

$$\left|A_\beta\right|^2 = \frac{\hbar}{2\rho\Omega\,\omega} \left(N_\omega + \frac{1}{2} \mp \frac{1}{2} \right)$$

$$N_\omega = \frac{1}{e^{\hbar\omega/k_B T} - 1} \approx \frac{k_B T}{\hbar\omega}$$

$$N_\omega \approx N_\omega + 1$$

ADP scattering: 3D in a nutshell (iii)

$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\frac{U_{ac}}{\Omega}\delta_{\vec{p}\,',\vec{p}\pm\hbar\vec{\beta}}\,\delta\left(E' - E \mp \hbar\omega\right) \qquad U_{ac} = \frac{D_A^2 k_B T}{2c_l}$$

$$\frac{1}{\tau} = \frac{1}{\tau_m} = \sum_{\vec{p}\,'} S(\vec{p},\vec{p}\,')$$

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} U_{ac} \frac{D_{3D}(E)}{2}$$

$$\tau = \frac{2c_l\hbar^4}{\pi D_A^2 m^* \sqrt{2m^*}} \frac{1}{(k_B T)} \frac{1}{\sqrt{E}}$$

$$\tau = \tau_0 \left(E/k_B T_L\right)^{-1/2}$$

$$s = -1/2$$

$$\tau_0 = \frac{2c_l\hbar^4}{\pi D_A^2 m^* \sqrt{2m^*}} \frac{1}{(k_B T)^{3/2}}$$

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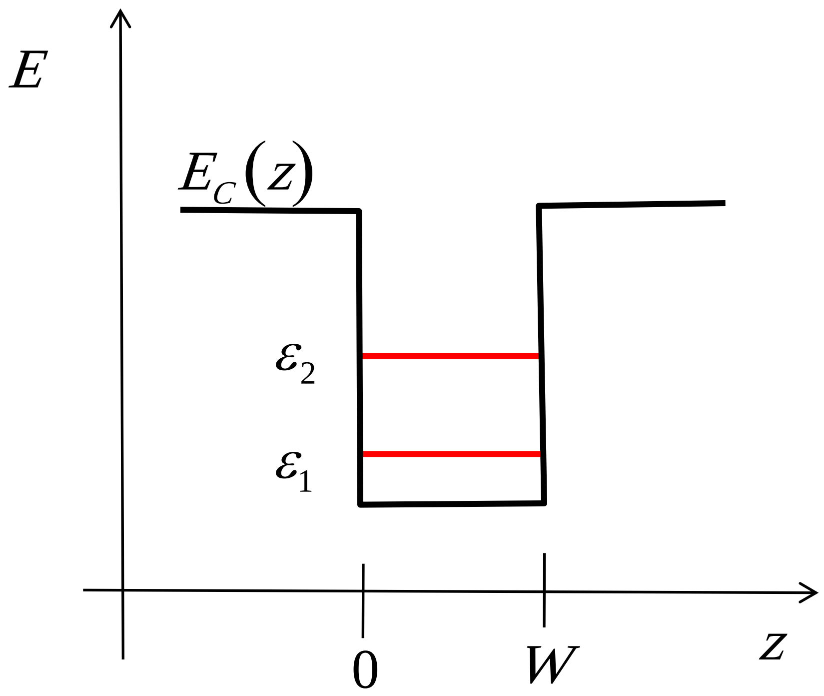
quantum confined carriers



$$\psi_n(x,y,z) = F_n(z)\frac{1}{\sqrt{A}}e^{i\vec{k}_{\parallel}\bullet\vec{\rho}}$$

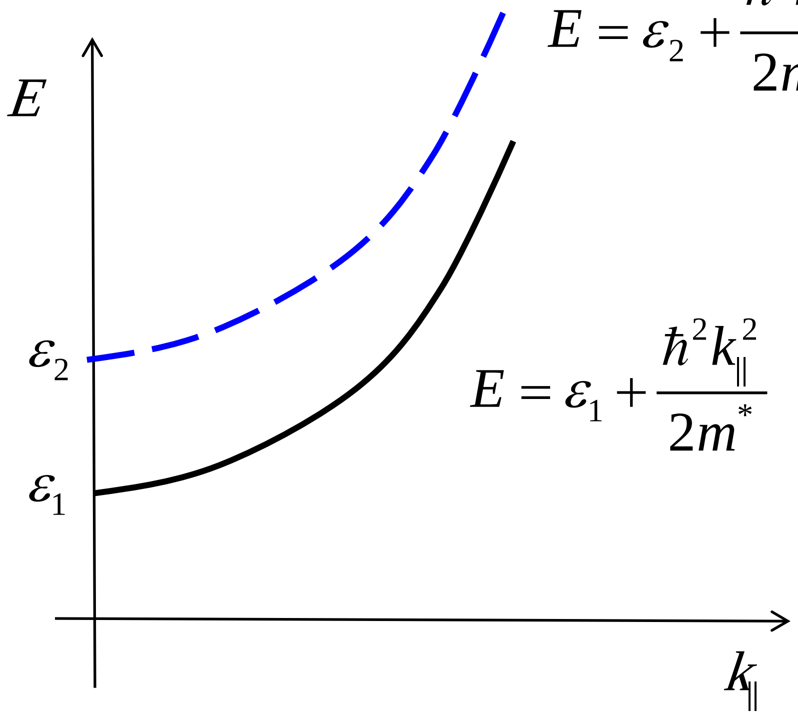
For an infinite well:

$$F_n(z) = \sqrt{\frac{2}{W}}\sin k_n z$$

$$k_n = \frac{n\pi}{W}$$

Electrons are <u>free to move</u> in the x-y plane

quantum confined carriers



$$E = \varepsilon_2 + \frac{\hbar^2 k_{\parallel}^2}{2m^*}$$

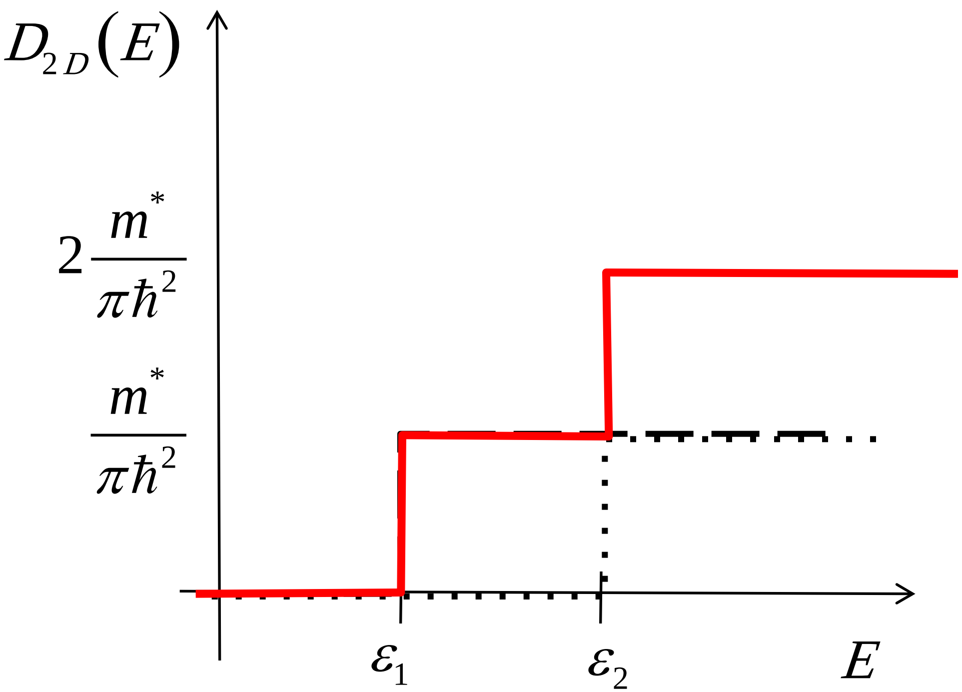
$$E = \varepsilon_1 + \frac{\hbar^2 k_{\parallel}^2}{2m^*}$$

$$E = \varepsilon_n + \frac{\hbar^2 k_{\parallel}^2}{2m^*}$$

$$\varepsilon_n = \frac{\hbar^2 n^{22} \pi^2}{2m^* W^2}$$

Electrons are free to move in the x-y plane

quantum confined carriers

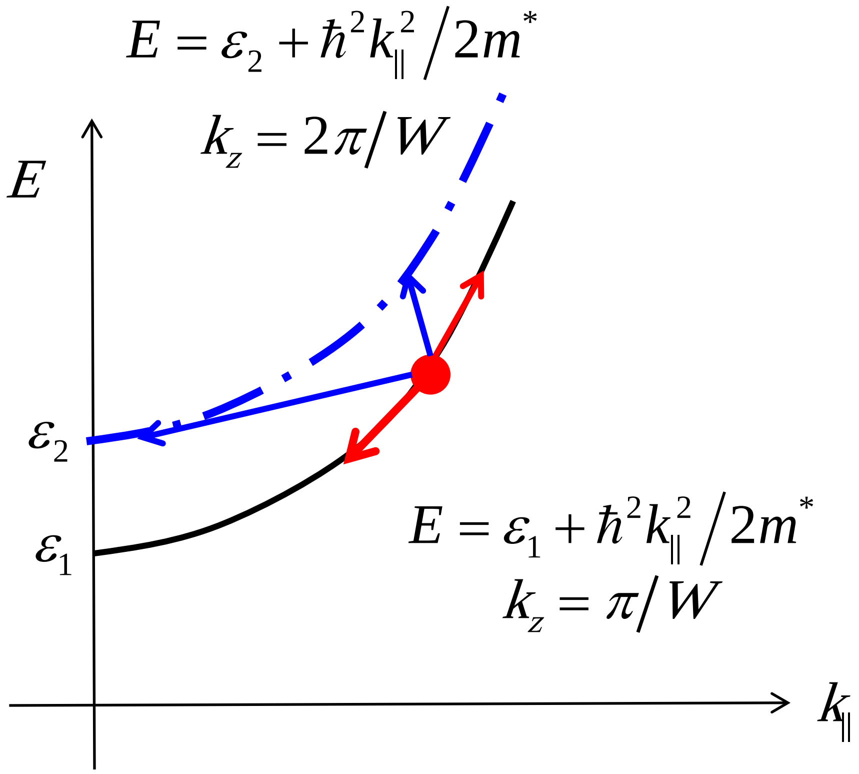


$$E = \varepsilon_n + \frac{\hbar^2 k_\parallel^2}{2m^*}$$

$$\varepsilon_n = \frac{\hbar^2 n^{22} \pi^2}{2m^* W^2}$$

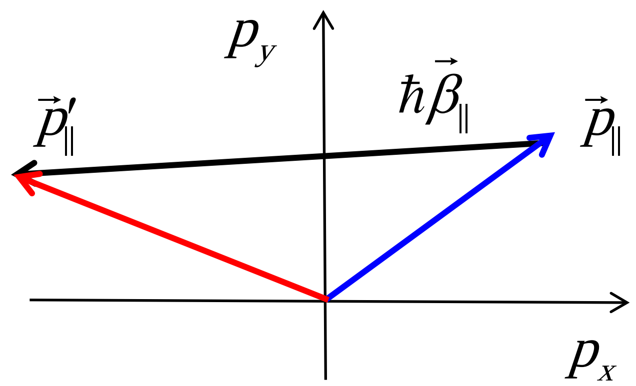
$$D_{2D}(E) = \frac{m^*}{\pi\hbar^2} \sum_{n=1} \Theta(E - \varepsilon_n)$$

momentum conservation approximation



electron: $\vec{p} = \vec{p}_{\parallel} + p_z \hat{z}$

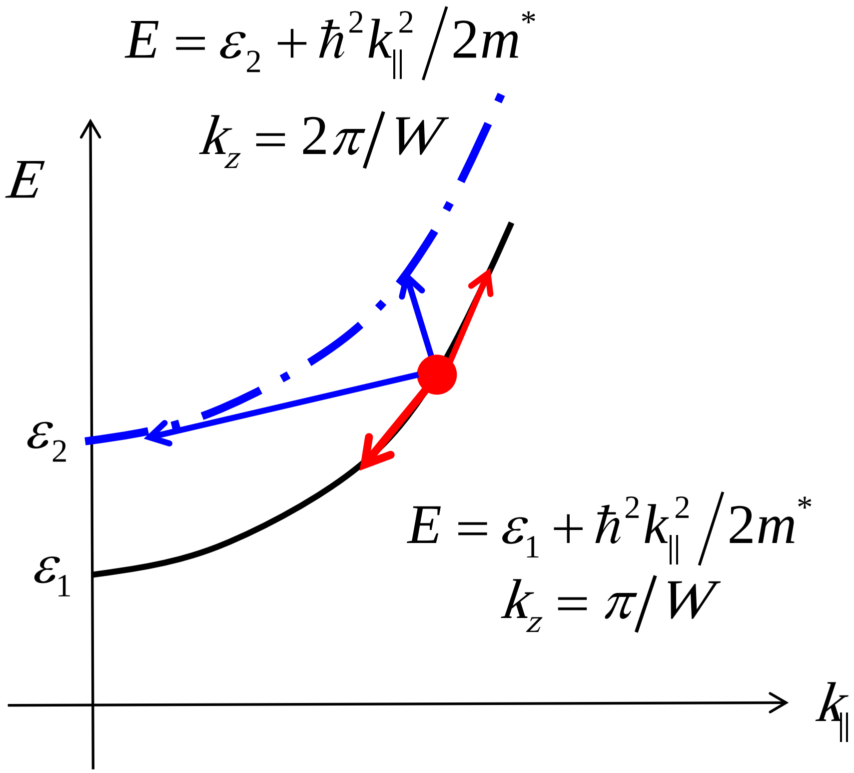
phonon: $\vec{\beta} = \vec{\beta}_{\parallel} + \beta_z \hat{z}$



Intra subband: $\Delta p_z = 0 \quad \beta_z = 0$

Inter subband: $\Delta p_z = p_{zi} - p_{zf} \quad \hbar\beta_z = \Delta p_z$

MCA



$$\Delta p_z \Delta z \geq \hbar$$

Momentum does not need to be strictly conserved!

Recall that for short times, energy is not strictly conserved.

Momentum and energy conservation result from FGR in the appropriate limits.

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what changes from 3D?

$$S\left(\vec{p},\vec{p}'\right)=\frac{2\pi}{\hbar}\left|H_{p',p}\right|^{2}\delta\left(E'-E\mp\hbar\omega\right)$$

$$\left|H_{p',p}\right|^{2}=\frac{1}{\Omega}U_{ac}\sum_{\beta}\frac{1}{\Omega}\left|\int_{-\infty}^{+\infty}\psi_{f}^{*}\left(e^{\pm i\vec{\beta}\bullet\vec{r}}\right)\psi_{i}d\vec{r}\right|^{2}$$

$$\delta_{\vec{p}',\vec{p}\pm\hbar\vec{\beta}}$$

(For 3D electrons and 3D phonons)

$$U_{ac}=\frac{D_{A}^{2}k_{B}T}{2c_{l}}$$

$$\psi_{i}=\frac{1}{\sqrt{\Omega}}e^{i\vec{p}\bullet\vec{r}/\hbar}$$

$$\psi_{f}=\frac{1}{\sqrt{\Omega}}e^{i\vec{p}'\bullet\vec{r}/\hbar}$$

In this section, we will assume 2D electrons and 3D phonons.

2D electrons and 3D phonons

2D electrons:

$$\psi_{i,n}\left(\vec{\rho},z\right)=F_n\left(z\right)\frac{1}{\sqrt{A}}e^{i\vec{k}_{\parallel}\bullet\vec{\rho}} \qquad \psi_{f,n'}\left(\vec{\rho},z\right)=F_{n'}\left(z\right)\frac{1}{\sqrt{A}}e^{i\vec{k}'_{\parallel}\bullet\vec{\rho}}$$

3D phonons:

$$u_{\beta}\left(\vec{r}\right)=A_{\beta}e^{\pm i\vec{\beta}\bullet\vec{r}}=A_{\beta}\left(e^{\pm i\vec{\beta}_{\parallel}\bullet\vec{\rho}}e^{\pm i\beta_z z}\right)$$

$$\left|H_{p',p}\right|^2=\frac{1}{\Omega}U_{ac}\sum_{\vec{\beta}}\left|\int_{-\infty}^{+\infty}\psi_f^*\left(e^{\pm i\vec{\beta}\bullet\vec{r}}\right)\psi_i d\vec{r}\right|^2$$

matrix element for 2D electrons

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \left| \sum_{\beta} \int_{-\infty}^{+\infty} \psi_f^* \left(e^{\pm i \vec{\beta} \bullet \vec{r}} \right) \psi_i d\vec{r} \right|^2$$

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \left| \sum_{\beta} \int_{-\infty}^{+\infty} F_{n'}^*(z) \frac{1}{\sqrt{A}} e^{-i\vec{k}_{\|}' \bullet \vec{\rho}} \left(e^{\pm i \vec{\beta}_{\|} \bullet \vec{\rho}} e^{\pm i \beta_z z} \right) F_n(z) \frac{1}{\sqrt{A}} e^{i \vec{k}_{\|} \bullet \vec{\rho}} d\vec{\rho} d \right|^2$$

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \left| \sum_{\beta} \frac{1}{A} \int e^{-i\left(\vec{k}_{\|}' - \vec{k}_{\|} \mp \vec{\beta}_{\|}\right) \bullet \vec{\rho}} d\vec{\rho} \int_{-\infty}^{+\infty} F_{n'}^*(z) F_n(z) e^{\pm i \beta_z z} dz \right|^2$$

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \delta_{\vec{p}_{\|}', \vec{p}_{\|} \pm \hbar \vec{\beta}_{\|}} \left| \sum_{\beta_z} \int_{-\infty}^{+\infty} F_{n'}^*(z) F_n(z) e^{\pm i \beta_z z} dz \right|^2$$

form factor

$$\left|H_{p',p}\right|^2 = \frac{1}{A} U_{ac} \delta_{\vec{p}'_{\|}, \vec{p}_{\|} \pm \hbar \vec{\beta}_{\|}} \left|F_{n',n}\right|^2$$

$$\left|F_{n',n}\right|^2 = \left| \frac{1}{L} \sum_{\beta_z} \int_{-\infty}^{+\infty} F_{n'}^*(z) F_n(z) e^{\pm i\beta_z z} dz \right|^2$$

3D → 2D

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \delta_{\vec{p}', \vec{p} \pm \hbar \vec{\beta}} \rightarrow \frac{1}{A} U_{ac} \delta_{\vec{p}'_{\|}, \vec{p}_{\|} \pm \hbar \vec{\beta}_{\|}} \left|F_{n',n}\right|^2$$

Momentum conservation is replaced by momentum conservation in the plane times a “form factor.”

evaluation of the form factor

$$\left|F_{n',n}\right|^2 = \left|\frac{1}{L}\sum_{\beta_z}\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)e^{\pm i\beta_z z}dz\right|^2$$

$$\left|F_{n',n}\right|^2 = \left|\frac{1}{L}\frac{L}{2\pi}\int d\beta_z\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)e^{\pm i\beta_z z}dz\right|^2$$

$$\left|F_{n',n}\right|^2 = \frac{1}{2\pi}\int d\beta_z\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)e^{i\beta_z z}dz\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)e^{-i\beta_z z}dz$$

$$\left|F_{n',n}\right|^2 = \frac{1}{2\pi}\int d\beta_z\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)e^{i\beta_z z}dz\int_{-\infty}^{+\infty} F_{n'}^*(z')F_n(z')e^{-i\beta_z z'}dz'$$

$$\left|F_{n',n}\right|^2 = \frac{1}{2\pi}\int e^{i\beta_z(z-z')}d\beta_z\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)dz\int_{-\infty}^{+\infty} F_{n'}^*(z')F_n(z')dz'$$

evaluation of the form factor (ii)

$$\left|F_{n',n}\right|^2 = \frac{1}{2\pi}\int e^{i\beta_z(z-z')}\,d\beta_z \int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)dz \int_{-\infty}^{+\infty} F_{n'}^*(z')F_n(z')dz'$$

Do the integral over β_z first and use: $\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i\beta_z(z-z')}\,d\beta_z = \delta(z-z')$

$$\left|F_{n',n}\right|^2 = \int_{-\infty}^{+\infty} \left|F_{n'}(z)\right|^2 dz \int_{-\infty}^{+\infty} \left|F_n(z)\right|^2 dz$$

Assume an infinite barrier quantum well: $F_n(z) = \sqrt{\frac{2}{W}}\sin(n\pi z/W)$

$$\left|F_{n',n}\right|^2 = \frac{1}{2W}\left(2+\delta_{n,n'}\right)$$

= 3/2*W* for intra-subband scattering
= 1/*W* for inter-subband scattering

3D to 2D re-cap

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \delta_{\vec{p}',\vec{p}\pm\hbar\vec{\beta}} \rightarrow \frac{1}{A} U_{ac} \delta_{\vec{p}'_{\parallel},\vec{p}_{\parallel}\pm\hbar\vec{\beta}_{\parallel}} \left|F_{n',n}\right|^2$$

$$\left|F_{n',n}\right|^2 = \int_{-\infty}^{+\infty} \left|F_{n'}(z)\right|^2 dz \int_{-\infty}^{+\infty} \left|F_n(z)\right|^2 dz$$

$$\left|F_{n',n}\right|^2 = \frac{1}{2W}\left(2 + \delta_{n,n'}\right)$$ (infinite barrier well)

$$U_{ac} = \frac{D_A^2 k_B T}{2 c_l}$$

For intraband scattering, the scattering rate will be 50% greater than for the MCA.)

2D scattering rate

$$S\left(\vec{p},\vec{p}'\right)=\frac{2\pi}{\hbar}\frac{U_{ac}}{A}\delta_{\vec{p}_{\parallel}',\vec{p}_{\parallel}\pm\hbar\vec{\beta}_{\parallel}}\left|F_{n',n}\right|^2\delta\left(E'-E\mp\hbar\omega\right) \qquad U_{ac}=\frac{D_A^2 k_B T}{2c_l}$$

$$\frac{1}{\tau_{n,n'}}=\frac{1}{\tau_m}=\sum_{\vec{p}_{\parallel}'}S\left(\vec{p}_{\parallel},\vec{p}_{\parallel}'\right)$$

$$\left|F_{n',n}\right|^2=\frac{1}{2}\left(2+\delta_{n,n'}\right)$$

(infinite barrier well)

$$\frac{1}{\tau_{n,n'}}=\frac{2\pi}{\hbar}U_{ac}\frac{D_{2D}\left(E\right)}{2}\frac{\left(2+\delta_{n,n'}\right)}{2}$$

$$\tau=\frac{2c_l\hbar^3}{D_A^2 m^*}\frac{1}{k_B T}\left(\frac{E}{k_B T_L}\right)^0$$

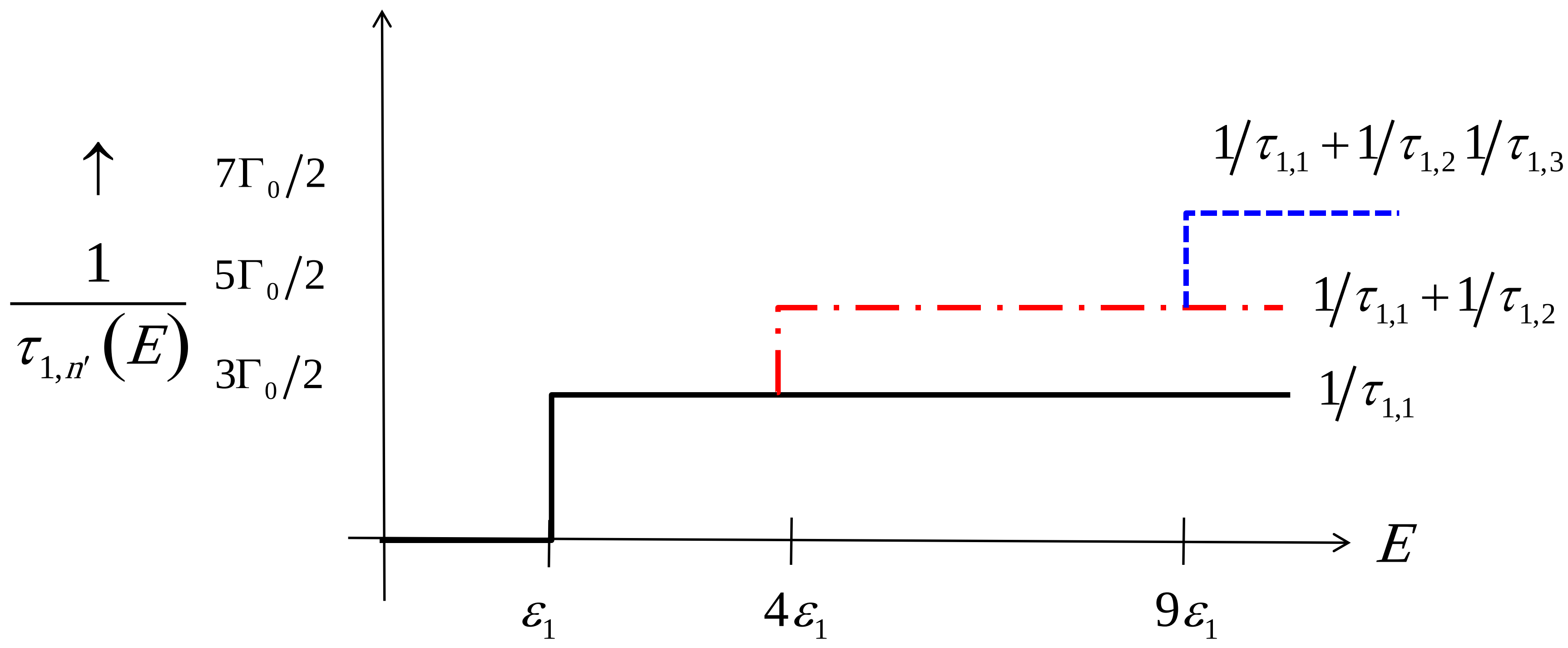
$$\tau=\tau_0\left(E/k_B T_L\right)^0$$

$$s=0$$

$$\tau_0=\frac{2c_l\hbar^3}{D_A^2 m^*}\frac{1}{k_B T}$$

2D scattering rate vs. energy

$$\frac{1}{\tau_{n,n'}} = \frac{2\pi}{\hbar} U_{ac} \frac{D_{2D}(E)}{2} \frac{\left(2+\delta_{n,n'}\right)}{2} = \Gamma_0 \frac{\left(2+\delta_{n,n'}\right)}{2}$$



outline

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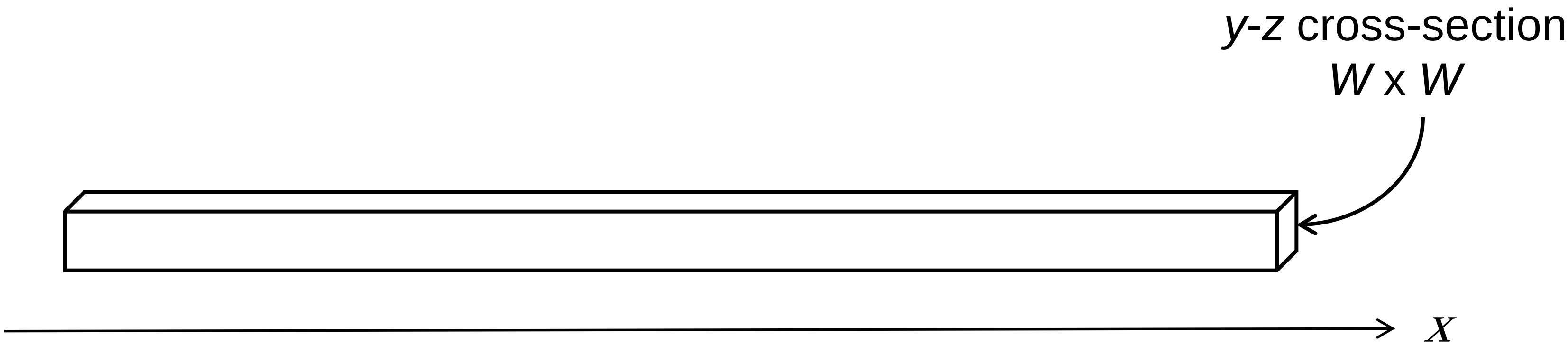
3D → 1D

Expect: $\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \delta_{\vec{p}',\vec{p}\pm\hbar\vec{\beta}} \rightarrow \frac{1}{L} U_{ac} \delta_{p_x', p_x \pm \hbar\beta_x} \left|F_{l',l}\right|^2$



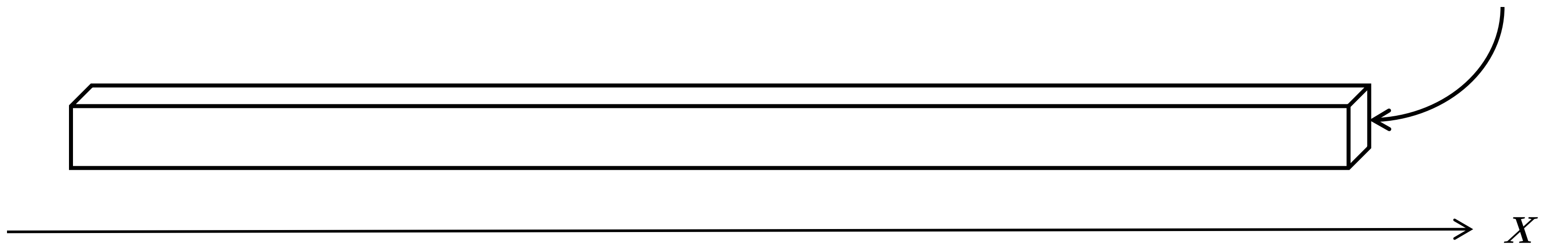
1D electrons:

$$\psi_l\left(x,y,z\right) = F_l\left(y,z\right)\frac{1}{\sqrt{L}} e^{i\vec{k}_x \bullet x\hat{x}}$$

form factor in 1D

$$\left|H_{p',p}\right|^2 = \frac{1}{L} U_{ac} \delta_{p_x, p_x \pm \hbar\beta_x} \left|F_{l,l'}\right|^2 \qquad U_{ac} = \frac{D_A^2 k_B T}{2 c_l}$$

y-*z* cross-section
W x *W*



(infinite barrier well)

$$F_m(y) = \sqrt{2/W}\sin\left(m\pi y/W\right) \qquad F_n(z) = \sqrt{2/W}\sin\left(n\pi z/W\right)$$

$$F_{m'}(y) = \sqrt{2/W}\sin\left(m\pi y/W\right) \qquad F_{n'}(z) = \sqrt{2/W}\sin\left(n\pi z/W\right)$$

$$\left|F_{l',l}\right|^2 = \int_{-\infty}^{+\infty} F_{n'}^2(y)\,dy \int_{-\infty}^{+\infty} F_{n'}^2(z)\,dz \int_{-\infty}^{+\infty} F_n^2(y)\,dy \int_{-\infty}^{+\infty} F_n^2(z)\,dz$$

$$\left|F_{l',l}\right|^2 = \left[\frac{1}{2W}\left(2 + \delta_{l,l'}\right)\right]^2$$

1D scattering rate

$$S\left(p_x, p_x'\right) = \frac{2\pi}{\hbar}\frac{U_{ac}}{L}\delta_{p_x', p_x \pm \hbar\beta_x}\left|F_{l',l'}\right|^2\delta\left(E' - E \mp \hbar\omega\right) \quad U_{ac} = \frac{D_A^2 k_B T}{2c_l}$$

$$\frac{1}{\tau_{l,l'}} = \frac{1}{\tau_m} = \sum_{p_x'} S\left(p_x, p_x'\right)$$

$$\left|F_{l',l}\right|^2 = \left[\frac{1}{2}\left(2 + \delta_{l,l'}\right)\right]^2$$

(infinite barrier well)

$$\frac{1}{\tau_{l,l'}} = \frac{2\pi}{\hbar}U_{ac}\frac{D_{1D}(E)}{2}\left[\frac{\left(2 + \delta_{l,l'}\right)}{2}\right]^2$$

$$\tau_{1,1} = \frac{8c_l\hbar^2}{9D_A^2\sqrt{2m^*}}\frac{1}{\left(k_BT\right)^{1/2}}\left(\frac{E}{k_BT_L}\right)^{1/2}$$

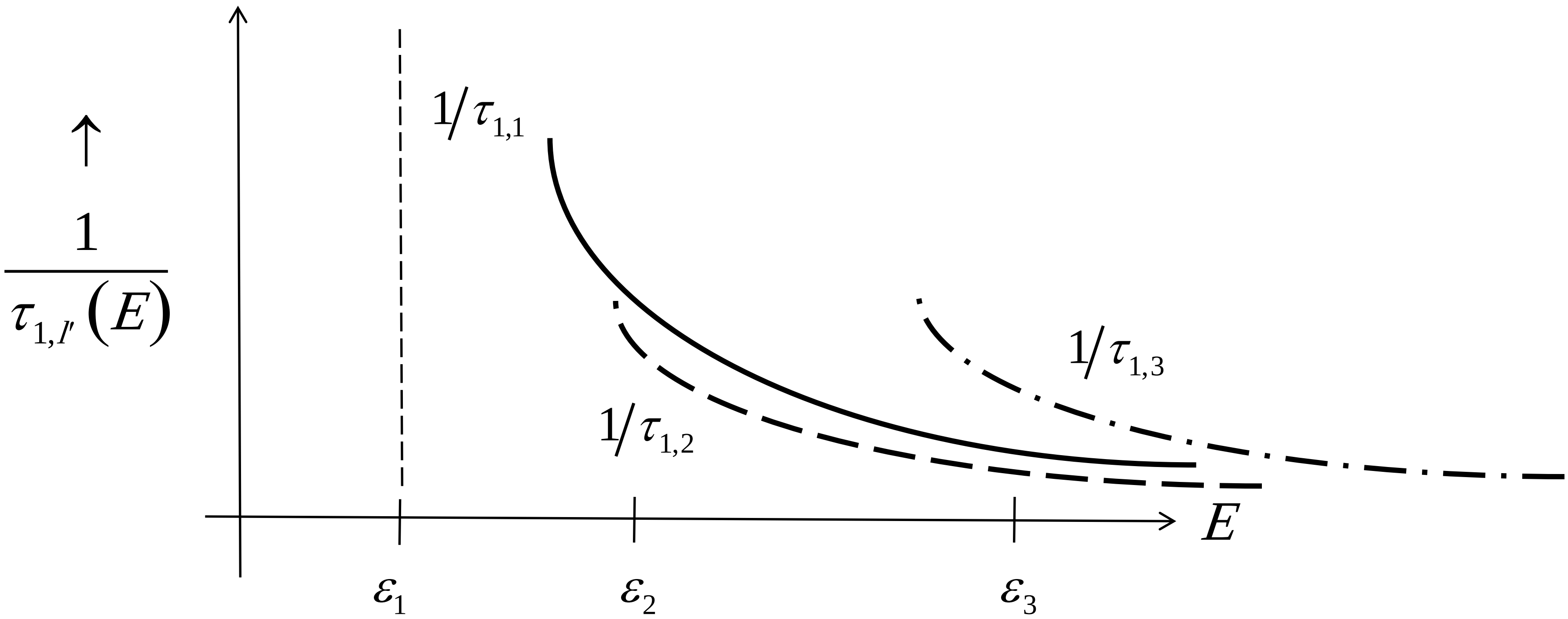
$$\tau_{11} = \tau_0\left(E/k_BT_L\right)^{1/2}$$

$$s = +1/2$$

$$\tau_0 = \frac{8c_l\hbar^2}{9D_A^2\sqrt{2m^*}}\frac{1}{\left(k_BT\right)^{1/2}}$$

2D scattering rate vs. energy

$$\frac{1}{\tau_{l,l'}} = \frac{2\pi}{\hbar} U_{ac} \frac{D_{1D}(E)}{2} \left[\frac{(2+\delta_{l,l'})}{2} \right]^2$$



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ADP scattering in 1D, 2D, and 3D

$$S(p,p') = \frac{2\pi}{\hbar}\frac{U_{ac}}{L^d}\delta_{p',p\pm\hbar\beta}\left|F_{l',l'}\right|^2\delta(E'-E\mp\hbar\omega) \qquad U_{ac} = \frac{D_A^2 k_B T}{2c_l}$$

$$d = 1,2,3$$

The vectors, *p*, *p'*, and *β* refer to the unconfined dimensions.

$$\frac{1}{\tau_{l,l'}} = \frac{2\pi}{\hbar}U_{ac}\frac{D_d(E)}{2}\left|F_{l',l}\right|^2 \qquad d = 1,2,3$$

$$\left|F_{l',l}\right|^2 = \left[\frac{1}{2}\left(2+\delta_{l,l'}\right)\right]^{3-d}$$

(Form factor for infinite barrier well)

ADP mobilities due to intravalley scattering

$$\frac{1}{\tau_{1,1}} = \frac{2\pi}{\hbar} U_{ac} \frac{D_d(E)}{2} \left| F_{1,1} \right|^2$$

d = 1, 2, or 3

$$\left| F_{l',l} \right|^2 = (3/2)^{3-d}$$

$$\mu_n = \frac{q\langle\langle \tau_f \rangle\rangle}{m^*} \qquad \tau_f(E) = \tau_0 (E/k_B T)^s \qquad \langle\langle \tau_f \rangle\rangle = \tau_0 \frac{\Gamma(s + d/2 + 1)}{\Gamma(d/2 + 1)}$$

(non-degenerate semiconductor)

With these expression and the previously-derived expressions for scattering rates, we can readily compare the mobility due to intra-valley ADP scattering in 1D, 2D, and 3D

ADP scattering times in 1D, 2D, and 3D

$$\mu_n = \frac{q\langle\langle \tau_f \rangle\rangle}{m^*} \qquad \tau_f(E) = \tau_0 \left(E/k_B T\right)^s$$

1D

$$\tau_{11} = \tau_0 \left(E/k_B T_L\right)^{1/2}$$

$$\tau_0 = \frac{8 c_l \hbar^2}{9 D_A^2 \sqrt{2m^*}} \frac{1}{\left(k_B T\right)^{1/2}}$$

2D

$$\tau_{11} = \tau_0 \left(E/k_B T_L\right)^{0}$$

$$\tau_0 = \frac{2 c_l \hbar^3}{D_A^2 m^*} \frac{1}{k_B T}$$

3D

$$\tau_{11} = \tau_0 \left(E/k_B T_L\right)^{-1/2}$$

$$\tau_0 = \frac{2 c_l \hbar^4}{\pi D_A^2 m^* \sqrt{2m^*}} \frac{1}{\left(k_B T\right)^{3/2}}$$

$$\tau_0 \propto \frac{1}{\left(m^*\right)^{d/2}} \qquad \tau_0 \propto \frac{1}{\left(k_B T\right)^{d/2}}$$

ADP mobilities in 1D, 2D, and 3D

$$\mu_n = \frac{q\langle\langle \tau_f \rangle\rangle}{m^*}$$

1D	**2D**	**3D**
$\langle\langle \tau \rangle\rangle = \frac{16 c_l \hbar^2}{9 D_A^2 \sqrt{2\pi m^*}} \frac{1}{(k_B T)^{1/2}}$	$\frac{2 c_l \hbar^3}{D_A^2 m^*} \frac{1}{k_B T}$	$\frac{8 c_l \hbar^4}{3\pi D_A^2 m^* \sqrt{2\pi m^*}} \frac{1}{(k_B T)^{3/2}}$
$\mu_n = \frac{16 c_l q \hbar^2}{9 D_A^2 m^* \sqrt{2\pi m^*}} \frac{1}{(k_B T)^{1/2}}$	$\frac{2 c_l q \hbar^3}{D_A^2 (m^*)^2} \frac{1}{k_B T}$	$\frac{8 c_l q \hbar^4}{3\pi D_A^2 (m^*)^2 \sqrt{2\pi m^*}} \frac{1}{(k_B T)^{3/2}}$

1D vs. 2D vs. 3D

1D:

$$\mu_n = \frac{16 c_l q \hbar^2}{9 D_A^2 m^* \sqrt{2\pi m^*}} \frac{1}{\left(k_B T\right)^{1/2}}$$

2D:

$$\mu_n = \frac{2 c_l q \hbar^3}{D_A^2 \left(m^*\right)^2} \frac{1}{k_B T}$$

3D:

$$\mu_n = \frac{8 c_l q \hbar^4}{3\pi D_A^2 \left(m^*\right)^2 \sqrt{2\pi m^*}} \frac{1}{\left(k_B T\right)^{3/2}}$$

Nikonov and Bourianoff show that:

$$\frac{\mu_{2D}}{\mu_{3D}} \le \frac{\sqrt{\pi}}{2} \qquad \frac{\mu_{1D}}{\mu_{2D}} \le \frac{2\sqrt{\pi}}{3}$$

For one suband, with non-degenerate statistics and phonon scattering, the mobility of a quantum confined device is slightly less than or approximately equal to the corresponding bulk mobility.

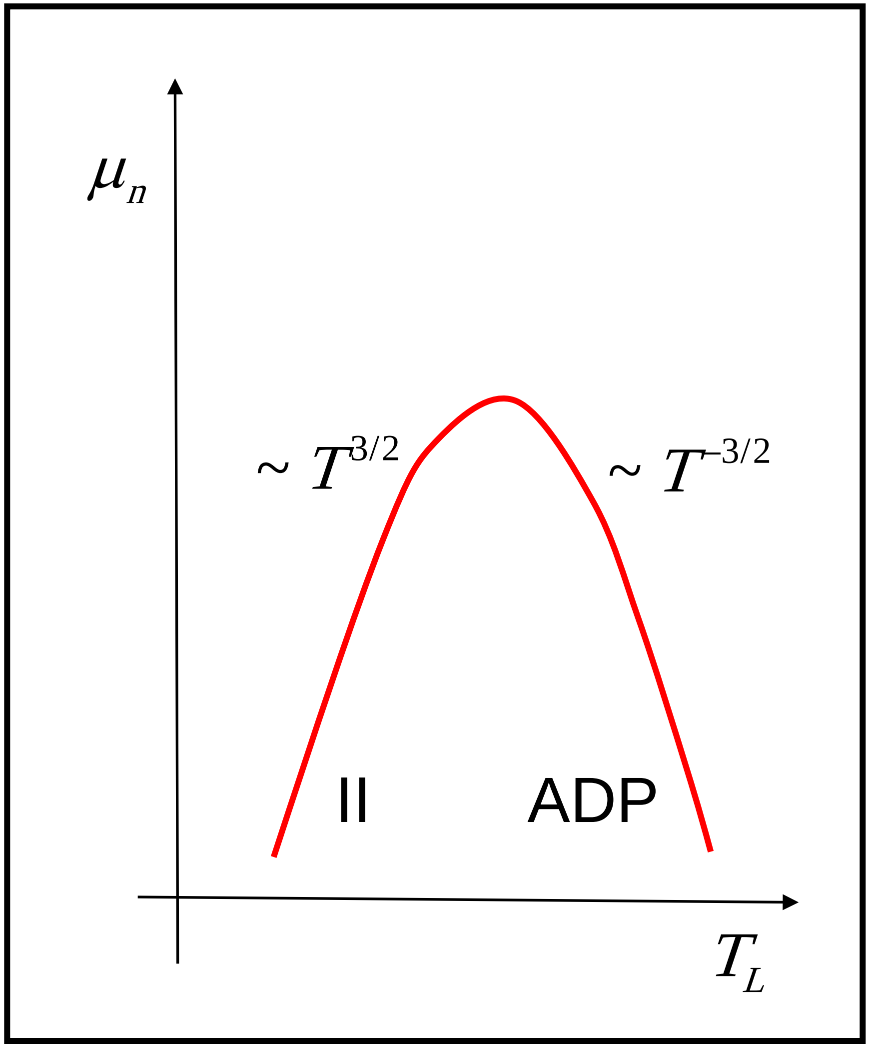
temperature dependent ADP mobility in 3D

$$\mu_n = \frac{8 c_l q \hbar^4}{3\pi D_A^2 \left(m^*\right)^2 \sqrt{2\pi m^*}} \frac{1}{\left(k_B T\right)^{3/2}}$$

$T^{-3/2}$ power is taken as the "signature" of acoustic phonon scattering.

In 2D, we expect, T^{-1} behavior.

In 1D, we expect, $T^{-1/2}$ behavior.



more to the story….

1) Confined phonons:

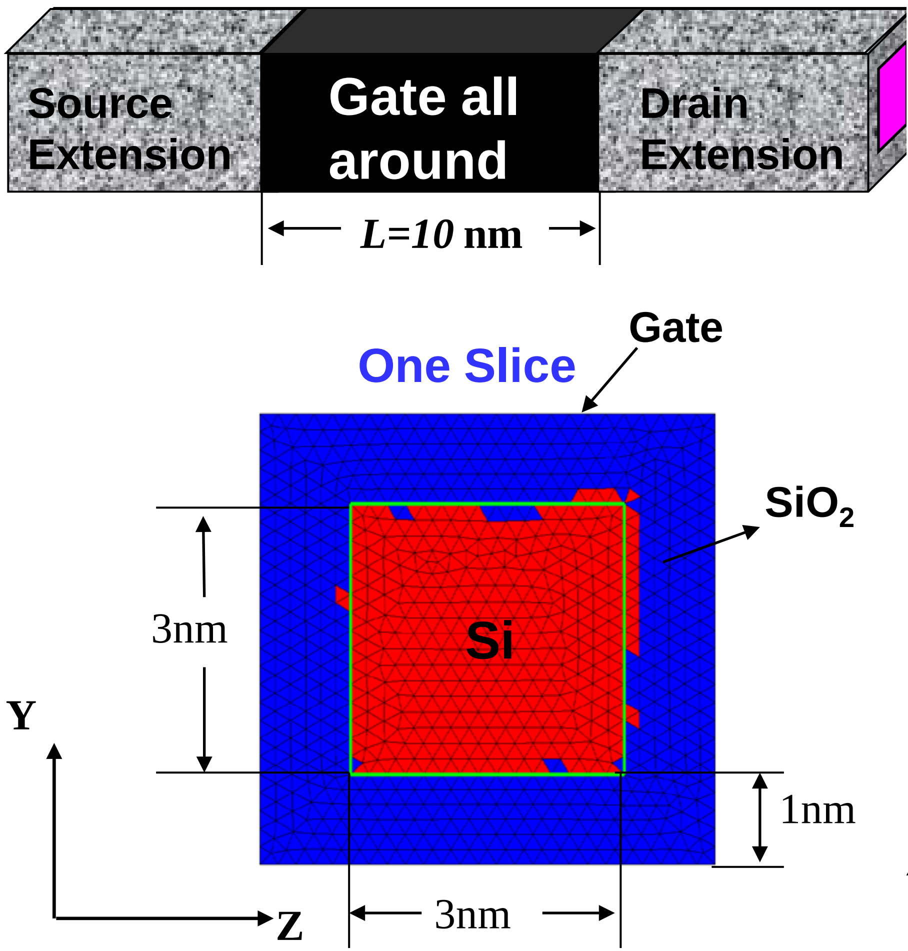
See:

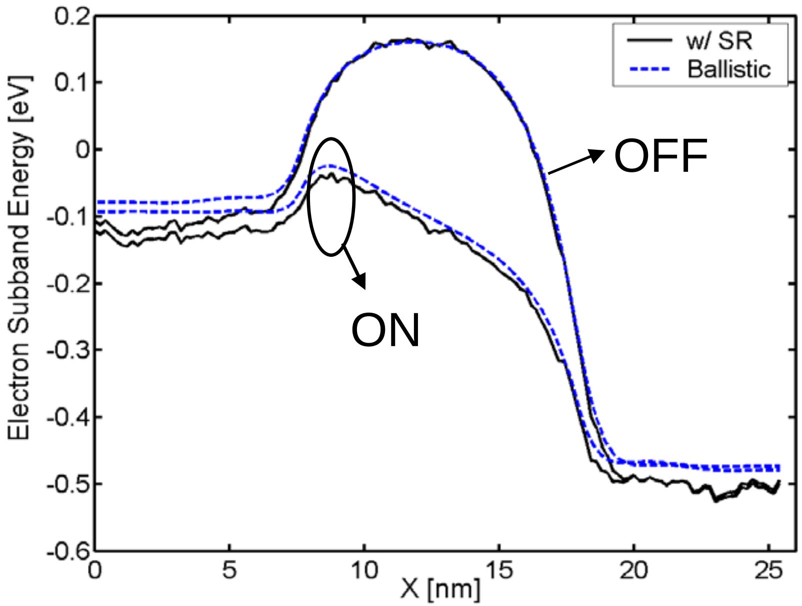
B.K. Ridley, *Electrons and Phonons in Semiconductor Multilayers*, Cambridge Univ. Press, Cambridge, UK, 1997.

M. A. Stroscio and M. Dutta, Phonons in Nanostructures, Cambridge Univ. Press, Cambridge, UK, 2001.

more to the story…

2) Surface roughness:





Jing Wang, E. Polizzi, A. Ghosh, S. Datta, and M. Lundstrom, “A Theoretical Investigation of Surface Roughness Scattering in Silicon Nanowire Transistors,” *Appl. Phys. Lett*., **87**, 043101, 2005.

a question

One frequently hears and sees statements to the fact that quantum confined structures should display higher mobilities because the lower density of states should result in less scattering. Is this statement true?

$$\frac{1}{\tau_{n',n}} = \frac{2\pi}{\hbar} U_{ac} \frac{D_d\left(E\right)}{2} \left|F_{n',n}\right|_d^2$$

Reducing the dimensionality decreases the density of states, but it also increases the form factor. The two effects roughly compensate, so for our toy problem, the mobility is very similar in all dimensions.

questions?

1) Review of ADP Scattering in 3D

2) ADP Scattering in 2D: MCA

3) ADP Scattering in 2D: FGR

4) ADP Scattering in 1D: FGR

5) Mobility in 1D, 2D, and 3D