ECE 656: Fall 2009
Lecture 26 Homework SOLUTION

1) In Lecture 26, we worked out the scattering rate due to acoustic phonon deformation potential scattering in 2D. Repeat that calculation, carefully checking all of the algebra, but this time assume deformation potential scattering from optical phonons.

ODP Scattering in 2D

Recall from 3D:

$$S(\vec{p},\vec{p}') = \frac{2\pi}{\hbar}\left|H_{p',p}\right|^2 \delta\left(E' - E \mp \hbar\omega\right)$$

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega}U_{op}^{a,e}\sum_{\beta}\left|\int_{-\infty}^{+\infty}\psi_f^*\left(e^{\pm i\vec{\beta}\bullet\vec{r}}\right)\psi_i d\vec{r}\right|^2$$

$$\psi_i = \frac{1}{\sqrt{\Omega}}e^{i\vec{p}\bullet\vec{r}/\hbar}$$

$$\psi_f = \frac{1}{\sqrt{\Omega}}e^{i\vec{p}'\bullet\vec{r}/\hbar}$$

According to Lecture 23: $\left|K_\beta\right|^2 = D_0^2$

According to Lecture 24: $\left|A_\beta\right|^2 \rightarrow \dfrac{\hbar}{2\rho\Omega\omega}N_\omega$

$$U_{op}^{a,e} = \Omega\left|K_\beta\right|^2\left|A_\beta\right|^2 = \frac{D_0^2\hbar}{2\rho\omega_0}\left(N_0 + \frac{1}{2} \mp \frac{1}{2}\right)$$ (note typo in Lecture 24)

$$\frac{1}{\tau} = \frac{2\pi}{\hbar}U_{op}^{a,e}\frac{D_{3D}\left(E \pm \hbar\omega_0\right)}{2}$$

What changes in 2D?

Wavefunctions:

$$\psi_{i,n}(\vec{\rho},z) = F_n(z)\frac{1}{\sqrt{A}}e^{i\vec{k}_\parallel \bullet \vec{\rho}}$$
$$\psi_{f,n'}(\vec{\rho},z) = F_{n'}(z)\frac{1}{\sqrt{A}}e^{i\vec{k}'_\parallel \bullet \vec{\rho}}$$

But keep the 3D phonons:

$$u_\beta(\vec{r}) = A_\beta e^{\pm i\vec{\beta}\bullet\vec{r}} = A_\beta\left(e^{\pm i\vec{\beta}_\parallel \bullet \vec{\rho}}e^{\pm i\beta_z z}\right)$$

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega}U_{op}^{a,e}\sum_{\vec{\beta}}\left|\int_{-\infty}^{+\infty}\psi_f^*\left(e^{\pm i\vec{\beta}\bullet\vec{r}}\right)\psi_i d\vec{r}\right|^2 \rightarrow \frac{1}{A}U_{op}^{a,e}\frac{1}{L}\sum_{\vec{\beta}}\left|\int_{-\infty}^{+\infty}\psi_f^*\left(e^{\pm i\vec{\beta}\bullet\vec{r}}\right)\psi_i d\vec{r}\right|^2$$

Note that the sum over beta goes outside of the magnitude squared. There are several typos related to this in the L26 slides.

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega}U_{op}^{a,e}\delta_{\vec{p}',\vec{p}\pm\hbar\vec{\beta}} \rightarrow \frac{1}{A}U_{op}^{a,e}\delta_{\vec{p}'_\parallel,\vec{p}_\parallel\pm\hbar\vec{\beta}_\parallel}\left|F_{n',n}\right|^2$$

for an infinite barrier quantum well:

$$\left|F_{n',n}\right|^2 = \frac{1}{2W}\left(2+\delta_{n,n'}\right)$$

$$S(\vec{p},\vec{p}') = \frac{2\pi}{\hbar}\frac{U_{op}^{a,e}}{A}\delta_{\vec{p}'_\parallel,\vec{p}_\parallel\pm\hbar\vec{\beta}_\parallel}\left|F_{n',n}\right|^2\delta\left(E'-E\mp\hbar\omega_0\right)$$

$$\frac{1}{\tau_{n,n'}} = \frac{1}{\tau_m} = \sum_{\vec{p}'_\parallel}S\left(\vec{p}_\parallel,\vec{p}'_\parallel\right)$$

$$\left.\frac{1}{\tau_{n,n'}}\right)^{a,e} = \frac{2\pi}{\hbar}U_{op}^{a,e}\frac{D_{2D}\left(E\pm\hbar\omega_0\right)}{2}\frac{\left(2+\delta_{n,n'}\right)}{2}$$

$$\left.\frac{1}{\tau_{n,n'}}\right)^{a,e} = \frac{\pi}{\hbar}\frac{D_0^2\hbar}{\rho\omega_0}\left(N_0+\frac{1}{2}\mp\frac{1}{2}\right)\frac{D_{2D}\left(E\pm\hbar\omega_0\right)}{2}\frac{\left(2+\delta_{n,n'}\right)}{2}$$

To see what this looks like, you should let $\hbar\omega_0 = k_B T$ and plot the absorption and emission scattering rates for an electron in subband 1 (include 2 subbands)