ECE 656: Fall 2009
Lecture 28 Homework SOLUTION

1) Repeat the derivation of the current equation presented in L28, but this time, do it in 3D.

$$\phi(p) = p_z$$

$$n_\phi(\vec{r},t) = \frac{1}{\Omega}\sum_{\vec{p}} \phi(p) f\left(\vec{r},\vec{p},t\right)$$

$$n_\phi(\vec{r},t) = P_z(\vec{r},t) = n\left\langle p_z \right\rangle$$

$$F_{\phi i} = \frac{1}{\Omega}\sum_{\vec{p}} p_z \upsilon_i f\left(\vec{r},\vec{p},t\right) \equiv 2W_{zi} = n\left\langle p_z \upsilon_i \right\rangle$$

$$\nabla \bullet \vec{F}_\phi = \frac{\partial}{\partial x_i}\left(2W_{zi}\right)$$

$$R_\phi \equiv \frac{n_\phi - n_\phi^0}{\left\langle \tau_\phi \right\rangle} = \frac{P_z - P_z^0}{\left\langle \tau_m \right\rangle} = \frac{P_z}{\left\langle \tau_m \right\rangle}$$

$$G_\phi = -q\vec{\mathcal{E}} \bullet \left\{ \frac{1}{L}\sum_{\vec{p}} \nabla_p \phi f \right\} = (-q) n \mathcal{E}_z$$

$$\frac{\partial n_\phi}{\partial t} = -\frac{dF_{\phi x}}{dx} + G_\phi - R_\phi \quad \rightarrow \frac{\partial P_z\left(\vec{r},t\right)}{\partial t} = -\frac{\partial}{\partial x_i}\left(2W_{iz}\right) - nq\mathcal{E}_z - \frac{P_z}{\left\langle \tau_m \right\rangle}$$

Now summarize:

The z-component of the total momentum density:

$$\frac{\partial P_z\left(\vec{r},t\right)}{\partial t} = -\frac{\partial}{\partial x_i}\left(2W_{iz}\right) - nq\mathcal{E}_z - \frac{P_z}{\left\langle \tau_m \right\rangle}$$

The jth component of the total momentum density:

$$\frac{\partial P_j(\vec{r},t)}{\partial t} = -\frac{\partial}{\partial x_i}\left(2W_{ij}\right) - nq\mathcal{E}_j - \frac{P_j}{\langle \tau_m \rangle}$$

Or, in vectorial notation:

$$\frac{\partial \vec{P}(\vec{r},t)}{\partial t} = -\nabla \bullet \left(2\vec{W}\right) - nq\vec{\mathcal{E}} - \frac{\vec{P}}{\langle \tau_m \rangle}$$

Now convert this to a current equation:

$$P_j = n\langle p_j \rangle = nm^* \langle \upsilon_j \rangle$$

$$J_{nj} = (-q)n\langle \upsilon_j \rangle$$

so:

$$J_{nj} = \frac{(-q)}{m^*} P_j$$

With this, the momentum balance equation becomes:

$$\frac{\partial J_{nj}(\vec{r},t)}{\partial t} = \frac{2q}{m^*}\frac{\partial W_{ij}}{\partial x_i} + \frac{nq^2}{m^*}\mathcal{E}_j - \frac{J_{nj}}{\langle \tau_m \rangle}$$

Now solve for the current:

$$J_{nj} + \langle \tau_m \rangle \frac{\partial J_{nj}(\vec{r},t)}{\partial t} = \frac{n q^2 \langle \tau_m \rangle}{m^*}\mathcal{E}_j + \frac{2q\langle \tau_m \rangle}{m^*}\frac{\partial W_{ij}}{\partial x_i}$$

Define:

$$\mu_n \equiv \frac{q\langle \tau_m \rangle}{m^*}$$

Simplifying the current equation:

Assume:

$$\langle \tau_m \rangle \frac{\partial J_{nj}(\vec{r},t)}{\partial t} << J_{nj}$$

Then look at W:

$$W_{ij} = \begin{bmatrix} W_{xx} & W_{xy} & W_{xz} \\ W_{yx} & W_{yy} & W_{yz} \\ W_{zx} & W_{zy} & W_{zz} \end{bmatrix}$$

$$W_{ij} = \frac{W}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$W_{ij} = \frac{W}{3}\delta_{ij}$$

$$W = \frac{1}{2}m\left(\langle \upsilon_x^2 \rangle + \langle \upsilon_y^2 \rangle + \langle \upsilon_z^2 \rangle\right)$$

With this assumptions, our current equation becomes:

$$J_{nj} = nq\mu_n \mathcal{E}_j + \frac{2}{3}\mu_n \frac{\partial W}{\partial x_j}$$

If we further assume:

$$W = nu$$

where

$$u = \frac{3}{2}k_B T$$

(which ignores the drift energy)

then

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + D_n \vec{\nabla} n$$

where

$$D_n = \frac{k_B T}{q}\mu_n$$