ECE 656: Fall 2009
Lecture 27 Homework SOLUTION

1) Provide the missing steps and show that

$$\frac{1}{\tau_m(E)} = \sum_{p'_{\parallel}} (1-\cos\theta) S(p_{\parallel}, p'_{\parallel}) = \frac{D_A^2 k_B T}{4\hbar^3 \rho_m (\upsilon_F \upsilon_S)^2} E$$

for $E > 0$.

When we ignored the two-component wavefunction, we found the momentum relaxation rate from:

$$\frac{1}{\tau} = \frac{1}{\tau_m} = \sum_{\vec{p}'_{\parallel}} S(\vec{p}_{\parallel}, \vec{p}'_{\parallel})$$

where

$$S(\vec{p}_{\parallel}, \vec{p}_{\parallel}) = \frac{2\pi}{\hbar} \frac{U_{ac}}{A} \delta_{\vec{p}'_{\parallel}, \vec{p}_{\parallel} \pm \hbar \vec{\beta}_{\parallel}} \delta(E' - E)$$

with

$$U_{ac} = \frac{D_A^2 k_B T}{2\rho_m \upsilon_s^2}$$

In this case, the sum over final states involved an integral over angle that was simply

$$\int_0^{2\pi} d\theta = 2\pi$$

With the two-component wavefunction, the transition rate became:

$$S(p_{\parallel}, p'_{\parallel}) = \frac{2\pi}{\hbar} \left[\frac{1}{2}(1+\cos\theta) \right] U_{ac} \delta_{\vec{p}'_{\parallel}, \vec{p}_{\parallel} \pm \hbar \vec{\beta}_{\parallel}} \delta(E' - E)$$

Computing the momentum relaxation rate brings in another factor of $\left(1-\cos\theta\right)$, so the extra factor in this case is

$$\left[\frac{1}{2}\left(1+\cos\theta\right)\left(1-\cos\theta\right)\right]=\frac{1}{2}\left(1-\cos^2\theta\right)$$

The integral over angle becomes

$$\int_0^{2\pi}\frac{1}{2}\left(1-\cos^2\theta\right)d\theta=\frac{1}{2}\int_0^{2\pi}\sin^2\theta d\theta=\frac{1}{4}\int_0^{2\pi}\left(1-\cos2\theta\right)d\theta=\frac{2\pi}{4}$$

So the integral is ¼ if what we got by ignoring the two-component wavefunction, so the scattering is reduced by a factor of 4.

2) Assume $n_S=1.0\times10^{12}\text{ cm}^{-2}$ and compute the room temperature, ADP scattering-limited mobility for electrons in graphene. Also compute the average mean-free-path for backscattering. You will need to hunt down the appropriate materials parameters.

First we need to hunt down some material parameters.

According to T. Stauber, et al., Phys. Rev. B, 76, 205423 (2007)

$\upsilon_s=7.33\times10^3\text{ cm/s}$ (for the LA phonon velocity – ref. 41).

According to T. Fang, et al., Phys. Rev. B, 78, 295403 (2008)

$\upsilon_s\approx2\times10^6\text{ cm/s}$, which is more than double the value quoted above.

$\rho_m\approx7.6\times10^{-8}\text{ gm/cm}^2$

$D_A\approx16\text{ eV}$ (ref. 23)

According to E. H. Hwang and das Sarma, Phys. Rev. B, 77, 115449 (2008)

$\upsilon_s\approx2\times10^6\text{ cm/s}$

$\rho_m\approx7.6\times10^{-8}\text{ gm/cm}^2$

$D_A \approx 19 \text{ eV}$ (but they point out that values of 10-30 are quorted in the literature, refs. 19, 21, 24)

According to R. Shishir and D.K. Ferry, J. Phys.: Cond. Matter, 21, 232204 (2009)

$D_A \approx 14 \text{ eV}$ (but they note that Chen has used 18, ref. 3)

According to S.S. Kubakaddi, Phys. Rev. B, 79, 075417 (2009)

$\rho_m = 7.6 \times 10^{-8} \text{ gm/cm}^2$

$\upsilon_s = 2 \times 10^6 \text{ cm/s}$

$\upsilon_F = 9.874 \times 10^7 \text{ cm/s}$

$D_A = 19 \text{ eV}$

According to Chen, et al., Nature Nanotechnology, 3, 206, (2008)

$\upsilon_s = 2.1 \times 10^4 \text{ m/s}$

$\rho_m = 7.6 \times 10^{-7} \text{ kg/m}^2$

$\upsilon_F = 1 \times 10^6 \text{ m/s}$

$D_A = 18 \pm 1 \text{ eV}$

According to Shishir et al., J. Comp. Electronics, 8, 43, (2009)

$\upsilon_s = 2 \times 10^6 \text{ cm/s}$

$\upsilon_F = 8.15 \times 10^7 \text{ cm/s}$

$\rho_m = 7.6 \times 10^{-8} \text{ gm/cm}^2$

$D_A = 4.75 \text{ eV}$ (possibly because they have a different ADP scattering rate)

The point is that there is some disagremeewnt on the material parameters. For this exercise, I will use the values of Chen, et al.

$\upsilon_s = 2.1 \times 10^4 \text{ m/s}$

$\rho_m = 7.6 \times 10^{-7} \text{ kg/m}^2$

$\upsilon_F = 1 \times 10^6 \text{ m/s}$

$D_A = 18 \text{ eV}$

The degenerate approximation is good for graphene, so I will use

$$\mu_n = \frac{q\tau_m(E_F)}{m^*}$$

$$\lambda(E_F) = \frac{\pi}{2}\upsilon_F \tau(E_F)$$

where the "effective mass" of graphene is:

$$m^*(E_F) = \frac{E_F}{\upsilon_F^2}$$

So we first need to determine the Fermi level. From L4

$$D(E) = \frac{2E}{\pi\hbar^2\upsilon_F^2} \quad E > 0$$

so for $n_S = 10^{12}\ \text{cm}^{-2}$

$$n_s = \frac{E_F^2}{\pi\hbar^2\upsilon_F^2} \qquad\qquad E_F = \hbar\upsilon_F\sqrt{\pi n_s} = 0.12\ \text{eV}$$

$$m^*(E_F) = \frac{\hbar\sqrt{\pi n_s}}{\upsilon_F} = \frac{E_F}{\upsilon_F^2} = 1.9\times 10^{-31}\ \text{kg} = 0.02 m_0$$

$$\tau_m(E_F) = \frac{4\hbar^3\rho_m(\upsilon_F\upsilon_S)^2}{D_A^2 k_B T}\frac{1}{E_F} = \frac{4\hbar^2\rho_m\upsilon_F\upsilon_S^2}{D_A^2 k_B T}\frac{1}{\sqrt{\pi n_S}} = 2.5\ \text{ps}$$

so, finally,

$$\mu_n = \frac{q\tau_m(E_F)}{m^*} = 210{,}000\ \text{cm}^2/\text{V-s}$$

$$\lambda(E_F) = \frac{\pi}{2}\upsilon_F \tau(E_F) = 3.9\ \mu\text{m}$$