ECE 656: Fall 2009
Lecture 29 Homework SOLUTION

1) Repeat the derivation of the zeroth and first moments of the BTE, but this time do it in 2D instead of 1D. You may assume parabolic energy bands.

HW29 Solution

Follow the approach of L28, but in 2D

i) $\phi(\vec{p}) = 1$

$$n_\phi = \frac{1}{A}\sum_{\vec{p}} 1 f(\vec{\rho},\vec{p},t) = n_s(\vec{\rho},t)$$

$$F_{\phi_i} = \frac{1}{A}\sum_{\vec{p}} 1 \cdot v_i f \qquad i = x, y$$

$F_{1i} = F_{n_i}$ = electron flux in x or y direction

G_ϕ involves $\frac{\partial \phi}{\partial p_i} = 0$

$R_\phi = 0$ because no explicit R-G

so $$\frac{\partial n_s}{\partial t} = -\frac{\partial}{\partial x_i} F_{n_i}$$

$$\boxed{\frac{\partial n_s}{\partial t} = -\nabla \cdot \vec{F}_n}$$

ii) $\phi(\vec{p}) = p_i \qquad i = x \text{ or } y$

$$n_\phi = \frac{1}{A}\sum_{\vec{p}} p_i f = P_i(\vec{\rho}, t) = n_s \langle p_i \rangle$$

↑ total momentum density in 2D in the ith direction

$$F_\phi = \frac{1}{A}\sum_{\vec{p}}' p_i' v_j f$$

↑ seeking balance eq. for ith component of $\vec{P}$

← jth component of $\vec{v}$

$$F_\phi = 2W_{ij} \qquad W_{ij} \equiv \frac{1}{A}\sum_{\vec{p}}' \frac{p_i v_j}{2} f$$

$$G_\phi = -q\,\mathcal{E}_j\left\{\frac{1}{A}\sum_{\vec{p}} \frac{\partial(p_i)}{\partial p_j} f\right\}$$

note repeated "j" because this is a dot product $\mathcal{E} \cdot \nabla_p \Phi(p)$

$$\frac{\partial p_i}{\partial p_j} = \delta_{ij}$$

2)

$$G_\phi = -q\mathcal{E}_i \frac{1}{A}\sum_{\vec{p}} f = -q\mathcal{E}_i n_s$$

$$R_\phi = \frac{P_i - P_i^0}{\langle \tau_m \rangle} \qquad P_i^0 = 0$$

so

$$\frac{\partial P_i}{\partial t} = -\frac{\partial}{\partial x_j}(2W_{ij}) - n_s q \mathcal{E}_i - \frac{P_i}{\langle \tau_m \rangle}$$

or symbolic notation:

$$\frac{\partial \vec{P}}{\partial t} = -\nabla \cdot (2\overleftrightarrow{W}) - n_s q \vec{\mathcal{E}} - \frac{\vec{P}}{\langle \tau_m \rangle}$$

we did not need to assume parabolic bands unless we want to write

$$W_{ij} = \frac{1}{A}\sum_{\vec{p}} \frac{1}{2} m v_i^2 f$$

also, to derive the current eqn., we would need an $E(k)$

3)