ECE-656: Fall 2009

Lecture 30: Balance Equation Approach: III

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general approach

$$\frac{\partial f}{\partial t} + \upsilon_x \frac{\partial f}{\partial x} - \frac{q\mathcal{E}_x}{\hbar}\frac{\partial f}{\partial k_x} = \hat{C} f \quad \Rightarrow \quad \frac{\partial n_\phi}{\partial t} = -\nabla \bullet \vec{F}_\phi + G_\phi - R_\phi$$

$$n_\phi(x,t) = \frac{1}{L}\sum_p \phi(p)\, f\left(x,p,t\right) \qquad G_\phi = -q\mathcal{E}_x \left\{ \frac{1}{L}\sum_{p_x} \frac{\partial \phi}{\partial p_x} f \right\}$$

$$F_\phi \equiv \frac{1}{L}\sum_{p_x} \phi\left(p_x\right)\upsilon_x f\left(x,p_x,t\right) \qquad R_\phi \equiv \frac{n_\phi - n_\phi^0}{\left\langle \tau_\phi \right\rangle}$$

the four balance equations

$$\frac{\partial n_L(x,t)}{\partial t} = -\frac{d\left[I_{nx}/(-q)\right]}{dx}$$

0th moment of BTE

$$\langle \tau_m \rangle \frac{\partial I_x(x,t)}{\partial t} + I_x = n_L q \mu_n \mathcal{E}_x + 2\mu_n \frac{dW}{dx}$$

1st moment of BTE

$$\frac{\partial W(x,t)}{\partial t} = -\frac{dF_W}{dx} + I_x \mathcal{E}_x - \frac{(W - W_0)}{\langle \tau_E \rangle}$$

2nd moment of BTE

$$\frac{\partial F_W(x,t)}{\partial t} = -\frac{dX}{dx} - \frac{3q}{m^*} n_L u \mathcal{E}_x - \frac{F_W}{\langle \tau_{F_W} \rangle}$$

3rd moment of BTE

outline

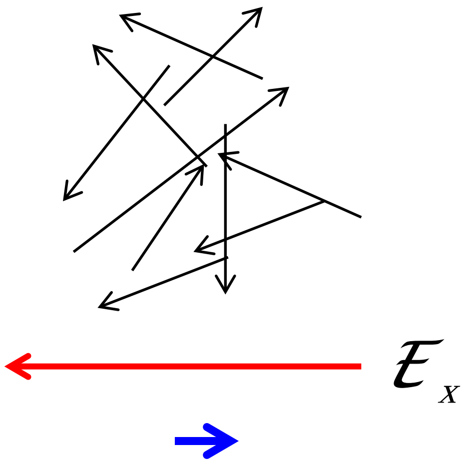
1) **Carrier Temperature and Heat Flux**
2) Balance equations in 3D
3) Heterostructures
4) Summary

electron temperature

- Electrons gain energy from the electric field.
- Electrons lose energy to the lattice by inelastic scattering.
- If the electric field is high, electrons will gain energy faster than they lose it to the lattice.
- Under such conditions, the electron energy > lattice energy.
- If we measure the electron energy by a “temperature,” then the electrons are ***hot.***

electron temperature

random thermal motion of electrons



$$\langle \upsilon_x \rangle = \upsilon_{dx}$$

$$W = \frac{1}{L}\sum_{k_x} E(k) f = \frac{1}{L}\sum_{k_x} \frac{1}{2} m^* \upsilon_x^2 f$$

$$W = n_L \frac{1}{2} m^* \langle \upsilon_x^2 \rangle$$

$$\upsilon_x = \upsilon_{dx} + c \qquad \langle c \rangle \equiv 0$$

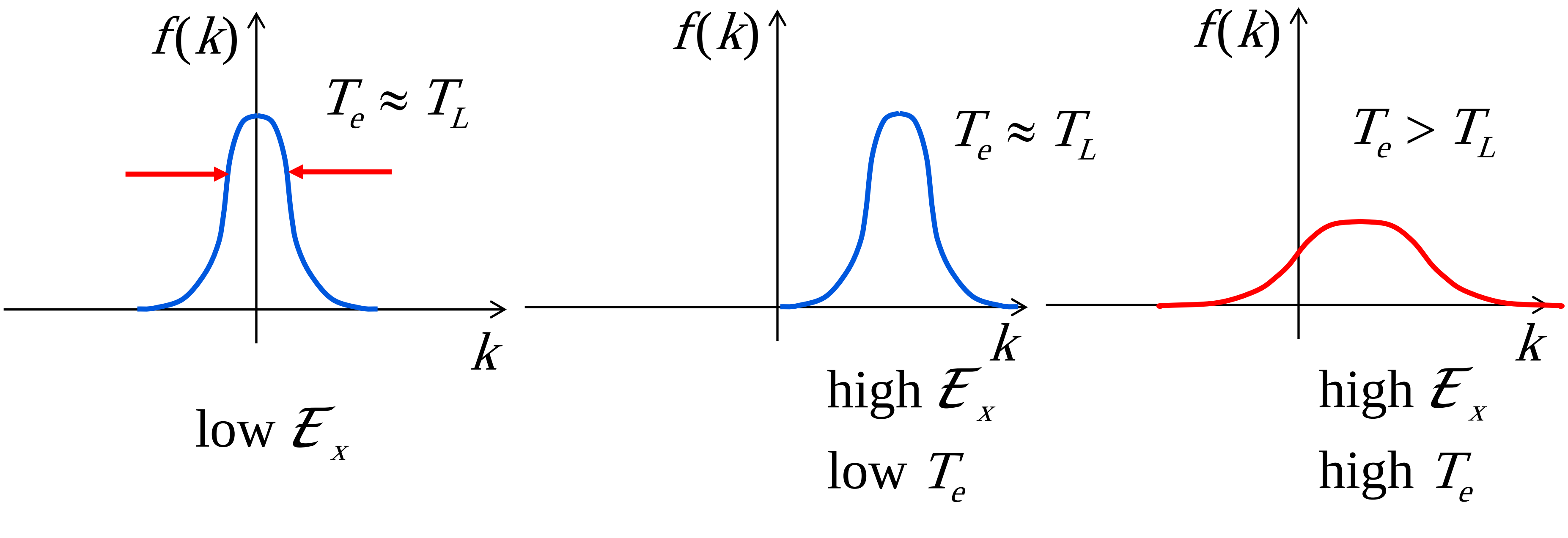
$$\upsilon_x^2 = \upsilon_{dx}^2 + 2c\upsilon_{dx} + c^2$$

$$W = n_L \left(\frac{1}{2} m^* \upsilon_{dx}^2 + \frac{1}{2} m^* \langle c^2 \rangle \right)$$

$$W = W_{drift} + W_{thermal}$$

electron temperature

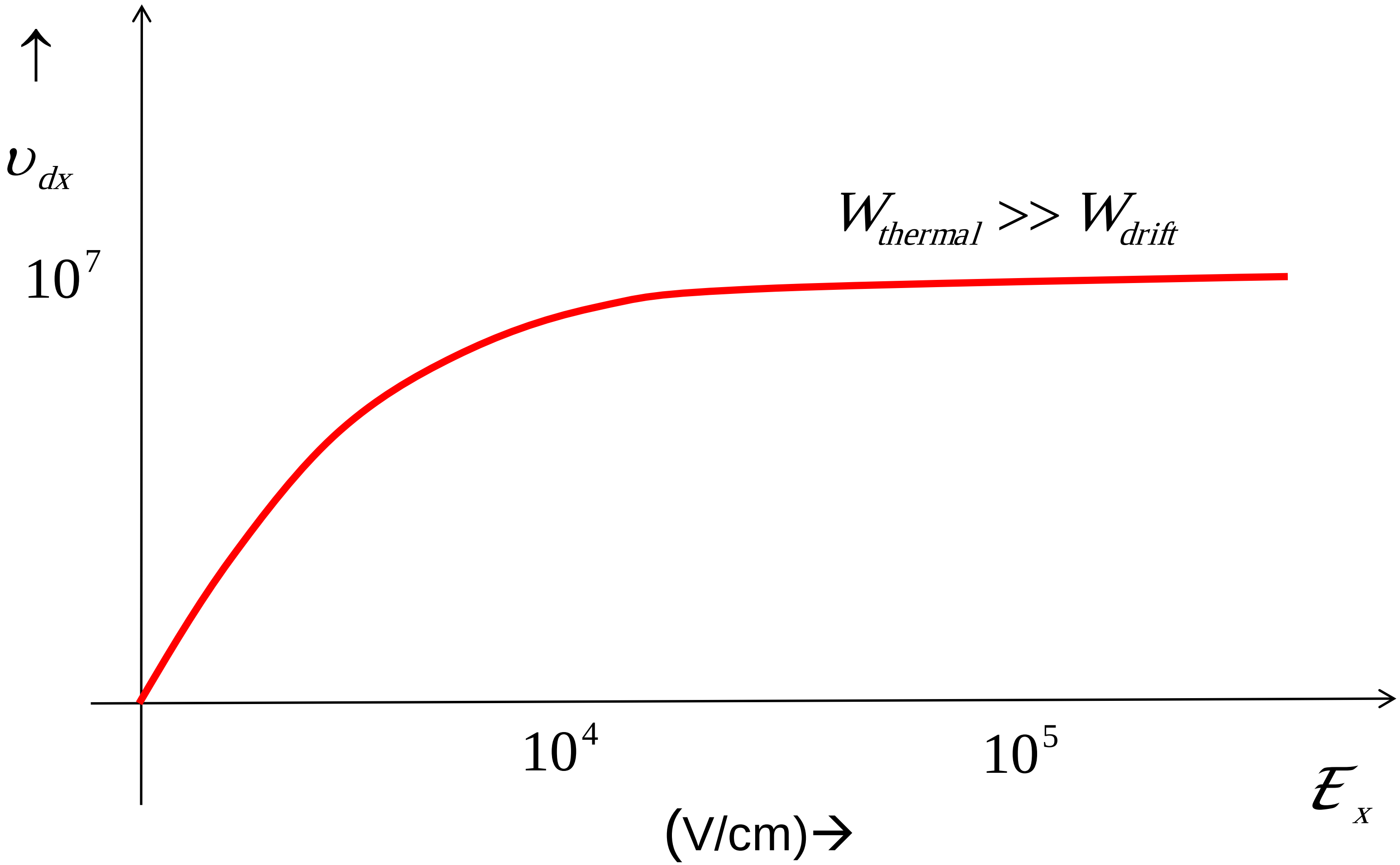
$$W = n_L\left(\frac{1}{2}m^*\upsilon_{dx}^2 + \frac{1}{2}m^*\left\langle c^2\right\rangle\right) \qquad \frac{1}{2}k_B T_e \equiv \frac{1}{2}m^*\left\langle c^2\right\rangle$$



$$f(k) \sim e^{-\hbar^2 k^2/2m^* k_B T_L}$$

$$f(k) \sim e^{-\hbar^2 k^2/2m^* k_B T_e}$$

example



heat flux

$$F_W = \frac{1}{L}\sum_{p_x} E(p_x)\upsilon_x f(x, p_x, t)$$

$$F_W = \frac{1}{L}\sum_{p_x}\left(\frac{1}{2}m^*\upsilon_x^2\right)\upsilon_x f(x, p_x, t)$$

$$F_W = \frac{1}{2}m^* n_L \langle \upsilon_x^2 \upsilon_x \rangle$$

$$\upsilon_x = \upsilon_{dx} + c$$

$$F_W = \frac{1}{2}m^* n_L \langle \upsilon_x^2 (\upsilon_{dx} + c) \rangle$$

$$F_W = \frac{1}{2}m^* n_L \langle \upsilon_x^2 \rangle \upsilon_{dx} + \frac{1}{2}m^* n_L \langle \upsilon_x^2 c \rangle$$

heat flux

$$F_W = \frac{1}{2} m^* n_L \left\langle \upsilon_x^2 \right\rangle \upsilon_{dx} + \frac{1}{2} m^* n_L \left\langle \upsilon_x^2 c \right\rangle \qquad \upsilon_x = \upsilon_{dx} + c$$

$$F_W = W\upsilon_{dx} + \frac{1}{2} m^* n_L \left\langle \left(\upsilon_{dx}^2 + 2\upsilon_{dx} c + c^2 \right) c \right\rangle$$

$$F_W = W\upsilon_{dx} + \frac{1}{2} m^* n_L \left[\upsilon_{dx}^2 \left\langle c \right\rangle + 2 \left\langle c^2 \right\rangle \upsilon_{dx} + \left\langle c^2 c \right\rangle \right]$$

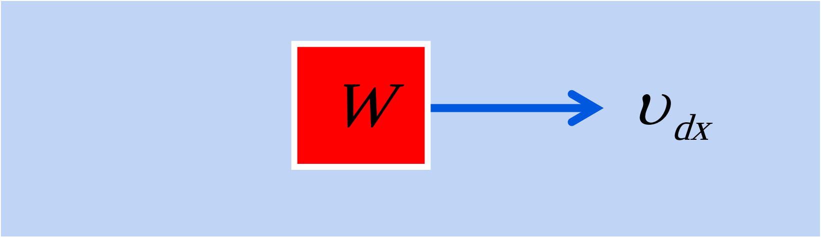
$$F_W = W\upsilon_{dx} + n_L m^* \left\langle c^2 \right\rangle \upsilon_{dx} + n_L \frac{1}{2} m^* \left\langle c^2 c \right\rangle$$

heat flux

$$F_W = W\upsilon_{dx} + n_L m^* \left\langle c^2 \right\rangle \upsilon_{dx} + n_L \frac{1}{2} m^* \left\langle c^2 c \right\rangle \qquad \upsilon_x = \upsilon_{dx} + c$$

$$k_B T_e \equiv m^* \left\langle c^2 \right\rangle \qquad Q_x \equiv n_L \frac{1}{2} m^* \left\langle c^2 c \right\rangle$$ "heat flux"

$$F_W = W\upsilon_{dx} + n_L k_B T_e \upsilon_{dx} + Q_x$$



$$P\Omega = N k_B T$$

energy balance equations

$$\frac{\partial n_L(x,t)}{\partial t} = -\frac{d\left[I_{nx}/(-q)\right]}{dx}$$ electron continuity equation

$$I_x = n_L q \mu_n \mathcal{E}_x + 2\mu_n \frac{dW}{dx}$$ drift-diffusion equation

$$\frac{\partial W(x,t)}{\partial t} = -\frac{dF_W}{dx} + I_x \mathcal{E}_x - \frac{(W - W_0)}{\tau_E}$$ energy-balance equation

$$F_W = -3\mu_E W \mathcal{E}_x - 3\mu_E \frac{d(W k_B T / q)}{dx}$$ energy-flux equation

energy balance equations

$$\frac{\partial n_L(x,t)}{\partial t} = -\frac{d\left[I_{nx}/(-q)\right]}{dx}$$

electron continuity equation

$$I_x = n_L q \mu_n \mathcal{E}_x + 2\mu_n \frac{dW}{dx}$$

drift-diffusion equation

$$\frac{\partial W(x,t)}{\partial t} = -\frac{dF_W}{dx} + I_x \mathcal{E}_x - \frac{(W - W_0)}{\tau_E}$$

energy-balance equation

$$F_W = W\upsilon_{dx} + n_L k_B T_e \upsilon_{dx} + Q_x$$

energy-flux equation

heat flux

$$F_W = W\upsilon_{dx} + n_L k_B T_e \upsilon_{dx} + Q_x$$

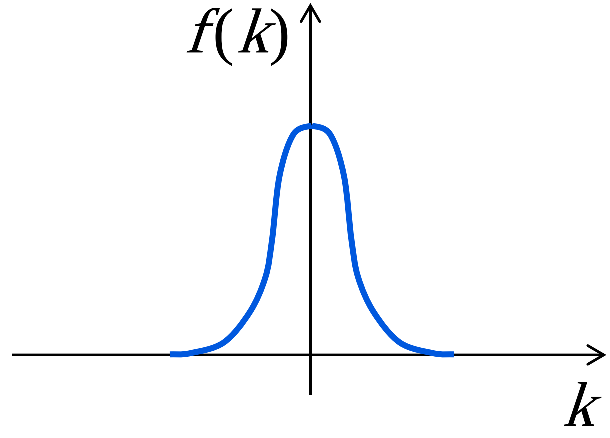
$$Q_x \equiv n_L \frac{1}{2} m^* \left\langle c^2 c \right\rangle$$

$$W = n_L\left(\frac{1}{2} m^* \upsilon_{dx}^2 + \frac{1}{2} k_B T_e\right)$$

$$Q_x \approx -\kappa_e \frac{dT}{dx}$$

but more generally

$$I_{Q_x} \approx \pi I_x - \kappa_e \frac{dT_e}{dx}$$

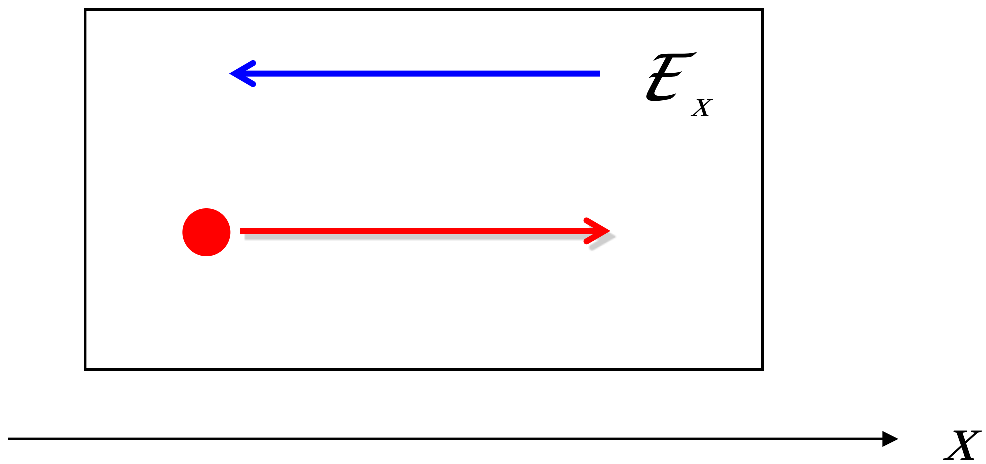


heat flux of a Maxwellian (or displaced Maxwellian) is 0

Q and I_q

does $Q_x = I_q$?

N-type semiconductor



$I_q = \pi I > 0$ $\quad$ $Q_x < 0$

Mark A. Stettler, Muhammad A. Alam, and Mark S. Lundstrom, "A Critical Examination of the Assumptions Underlying Macroscopic Transport Equations for Silicon Devices," *IEEE Trans. on Electron Devices*, **40**, 733, 1993.

outline

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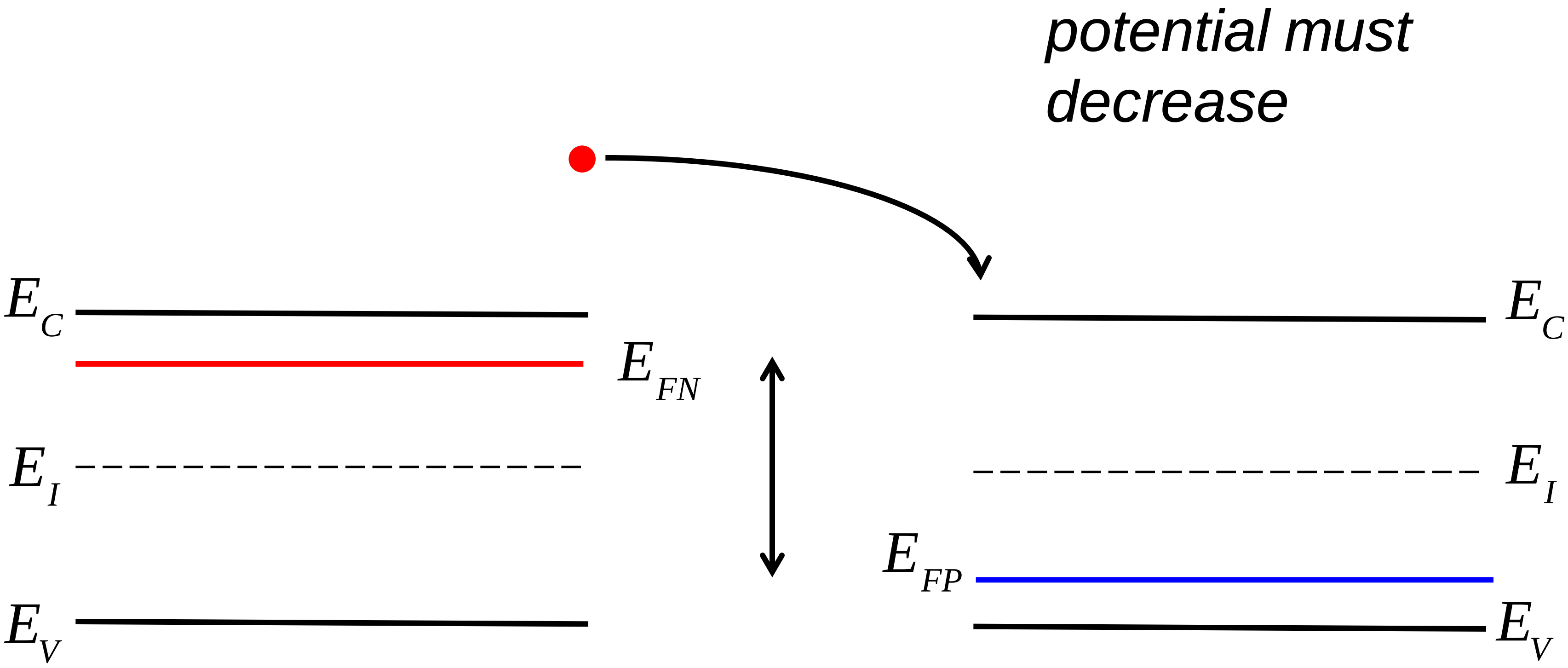
2) Balance equations in 3D

3) Heterostructures

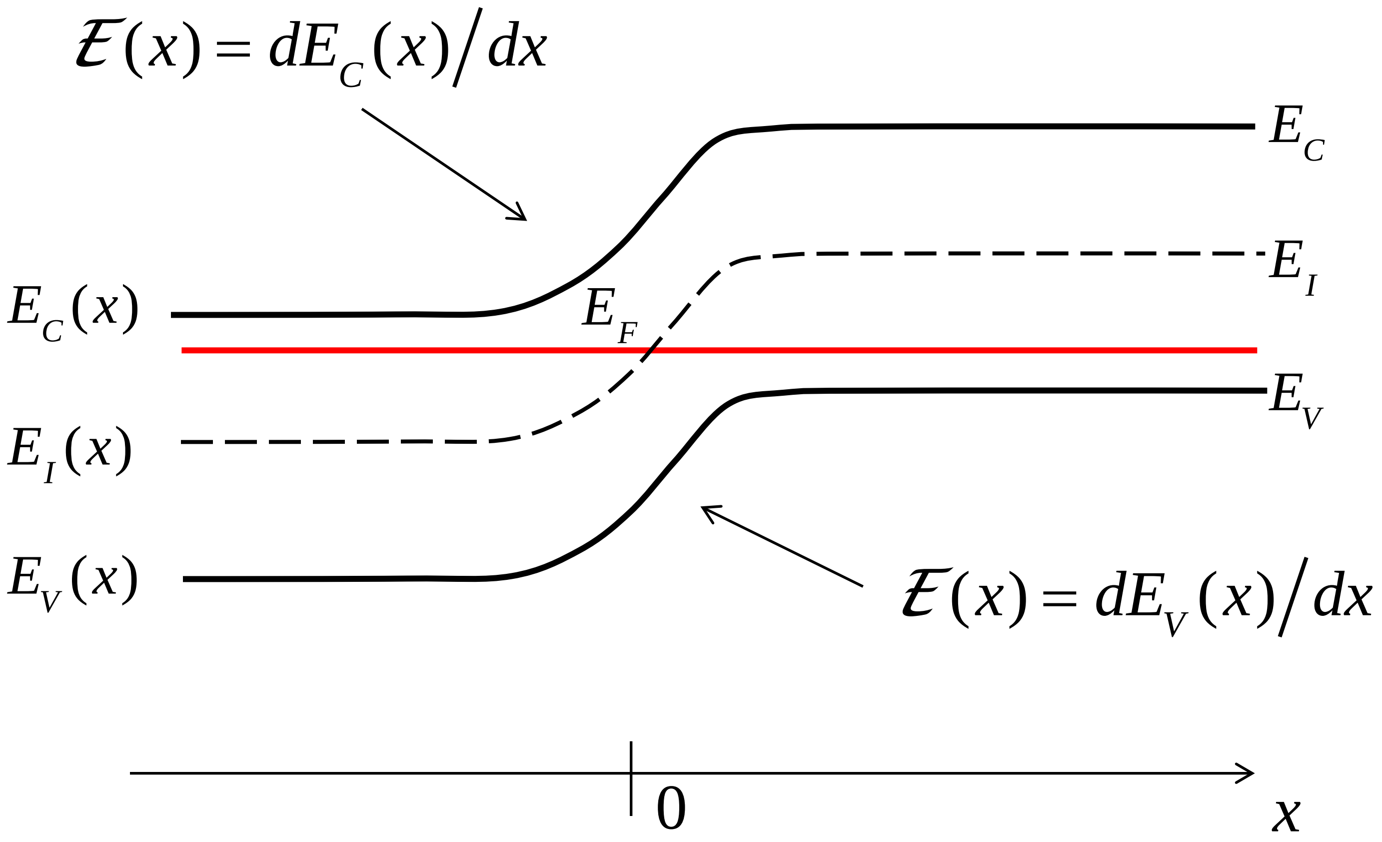
4) Summary

review: pn homojunctions

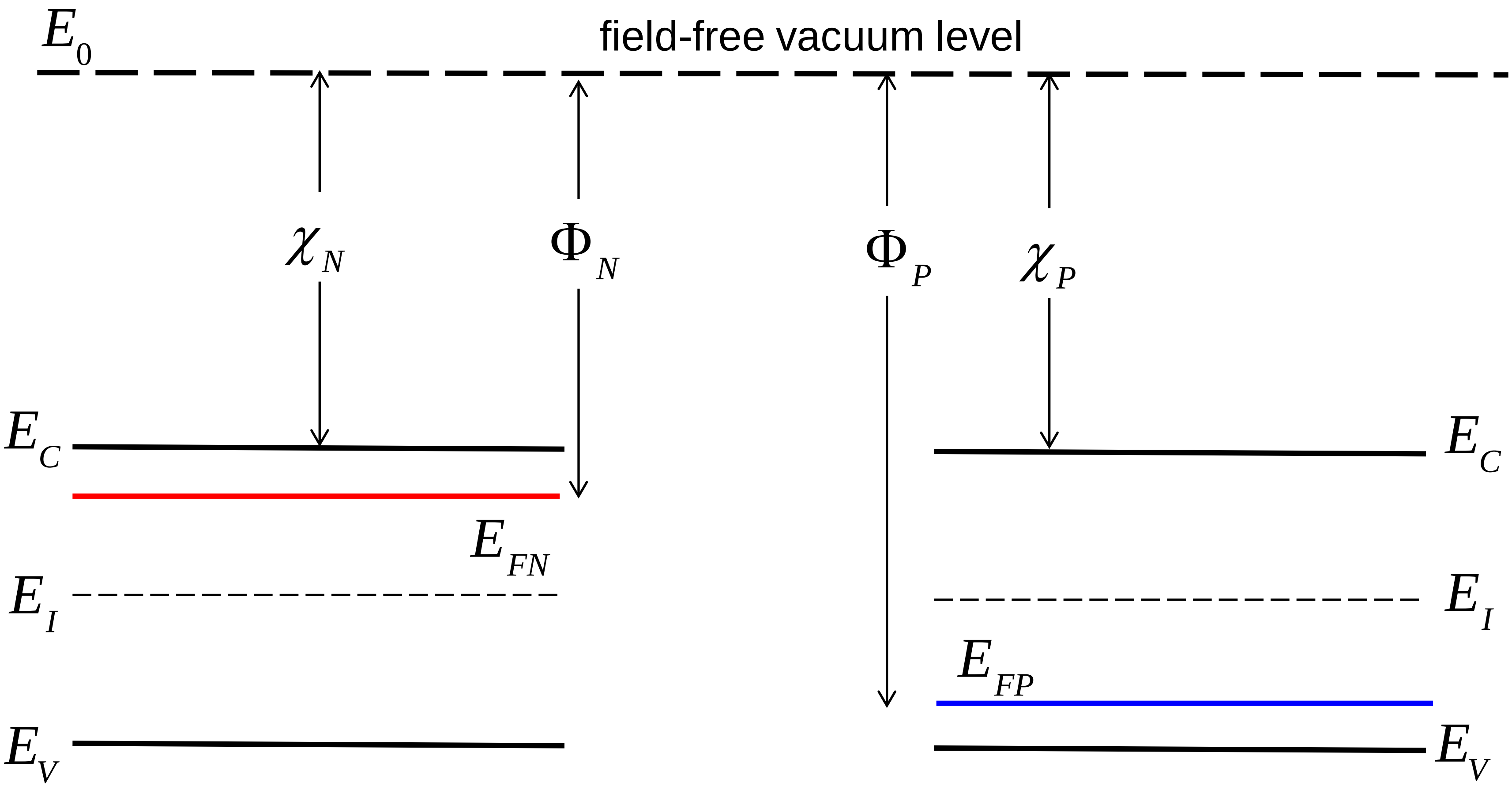


$$qV_{BI} = \left(E_{FN} - E_{FP}\right)/q$$

review: pn homojunctions



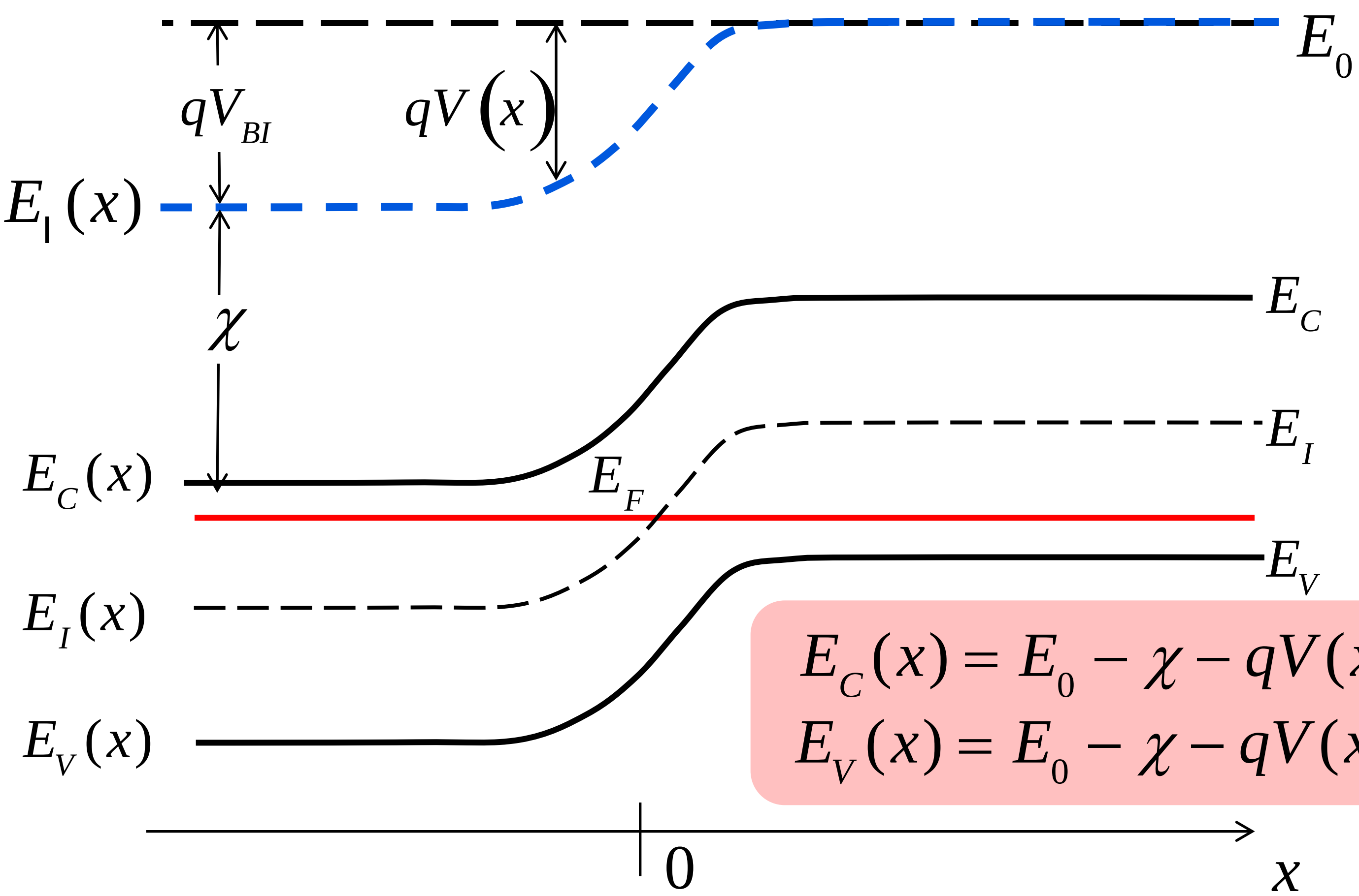
reference for the energy bands



$$E_C = E_0 - \chi$$
$$E_V = E_0 - \chi - E_G$$

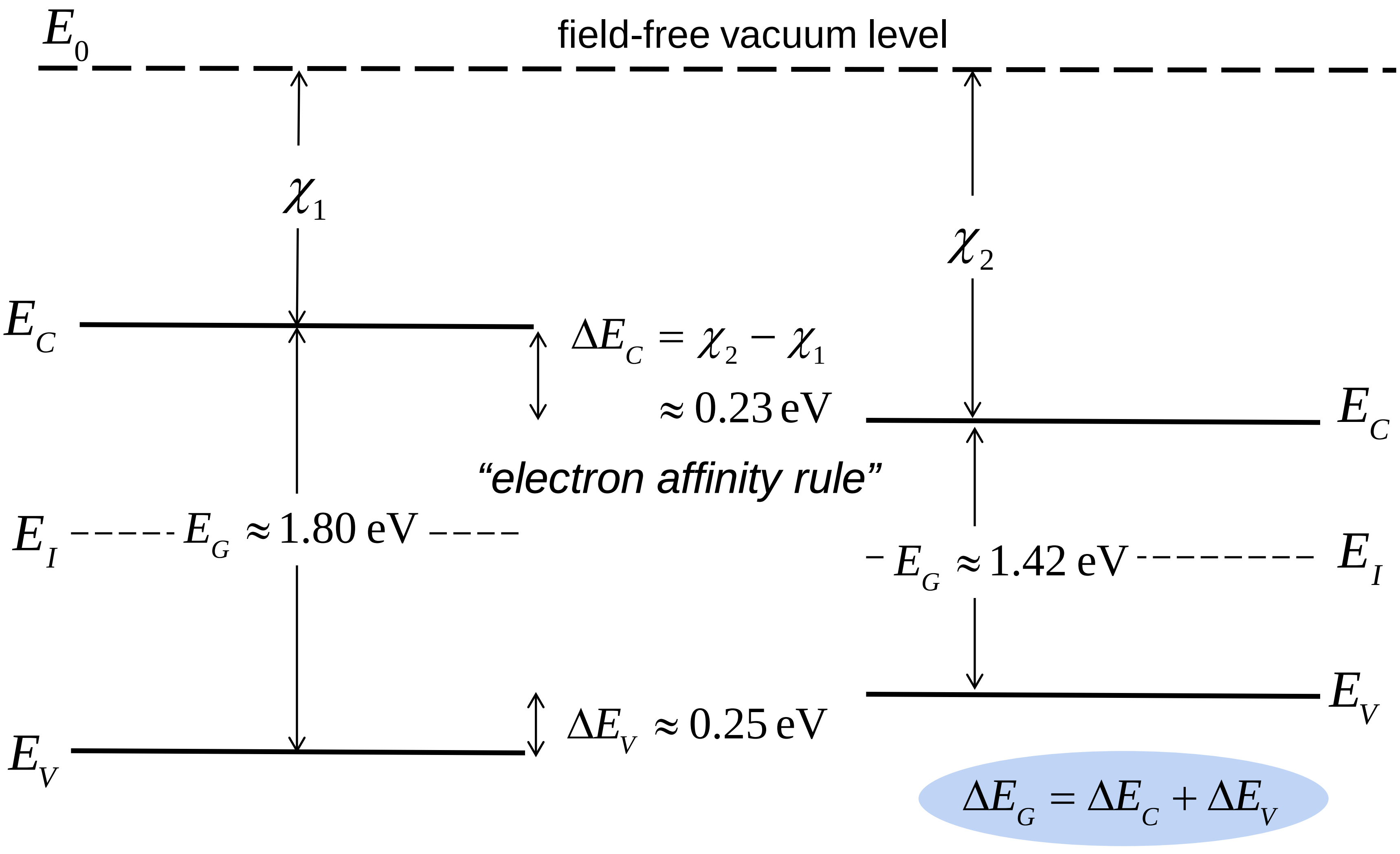
$$qV_{BI} = \left(\Phi_P - \Phi_N\right)$$

local vacuum level

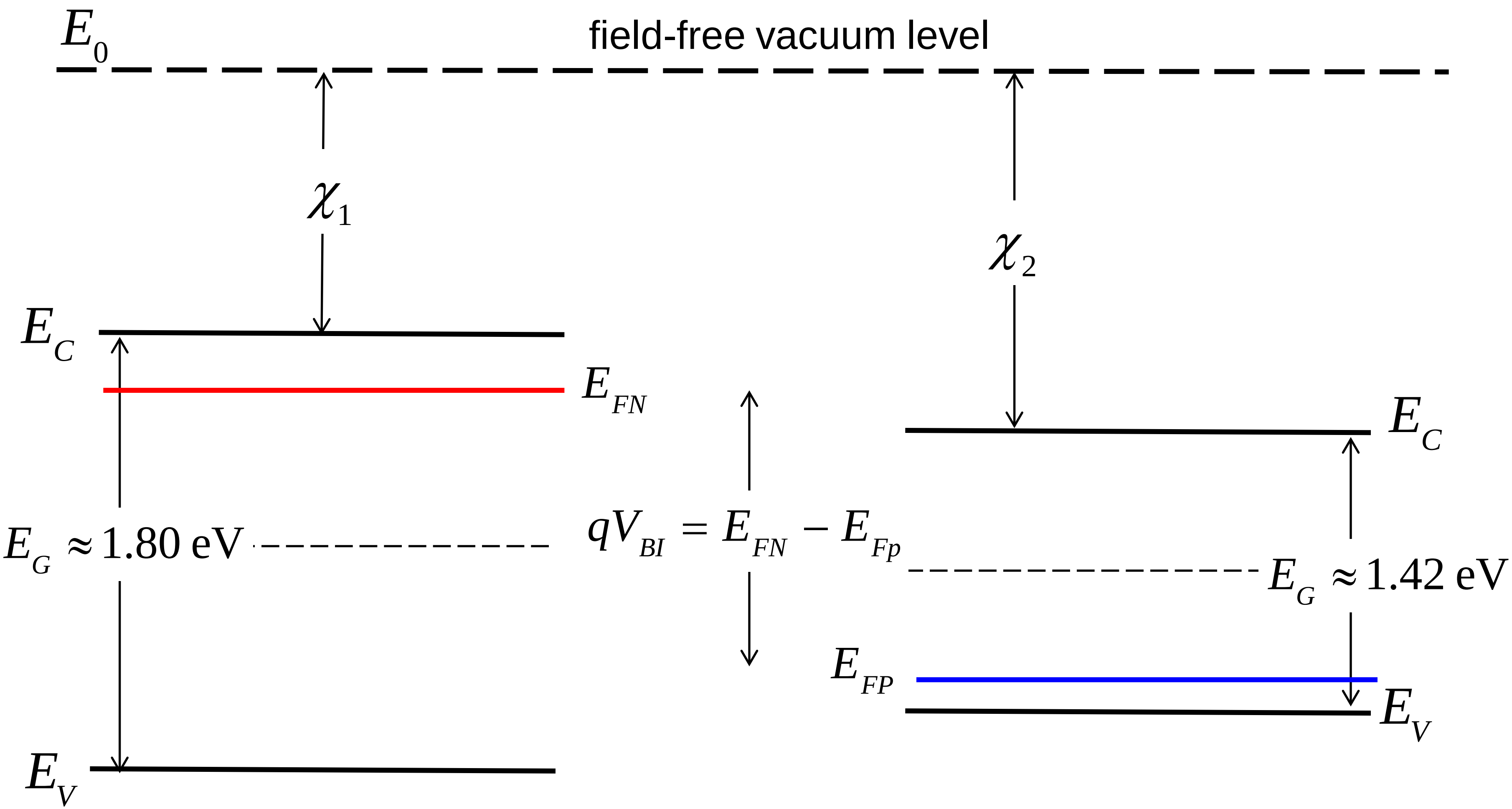


$$E_C(x) = E_0 - \chi - qV(x)$$
$$E_V(x) = E_0 - \chi - qV(x) - E_G$$

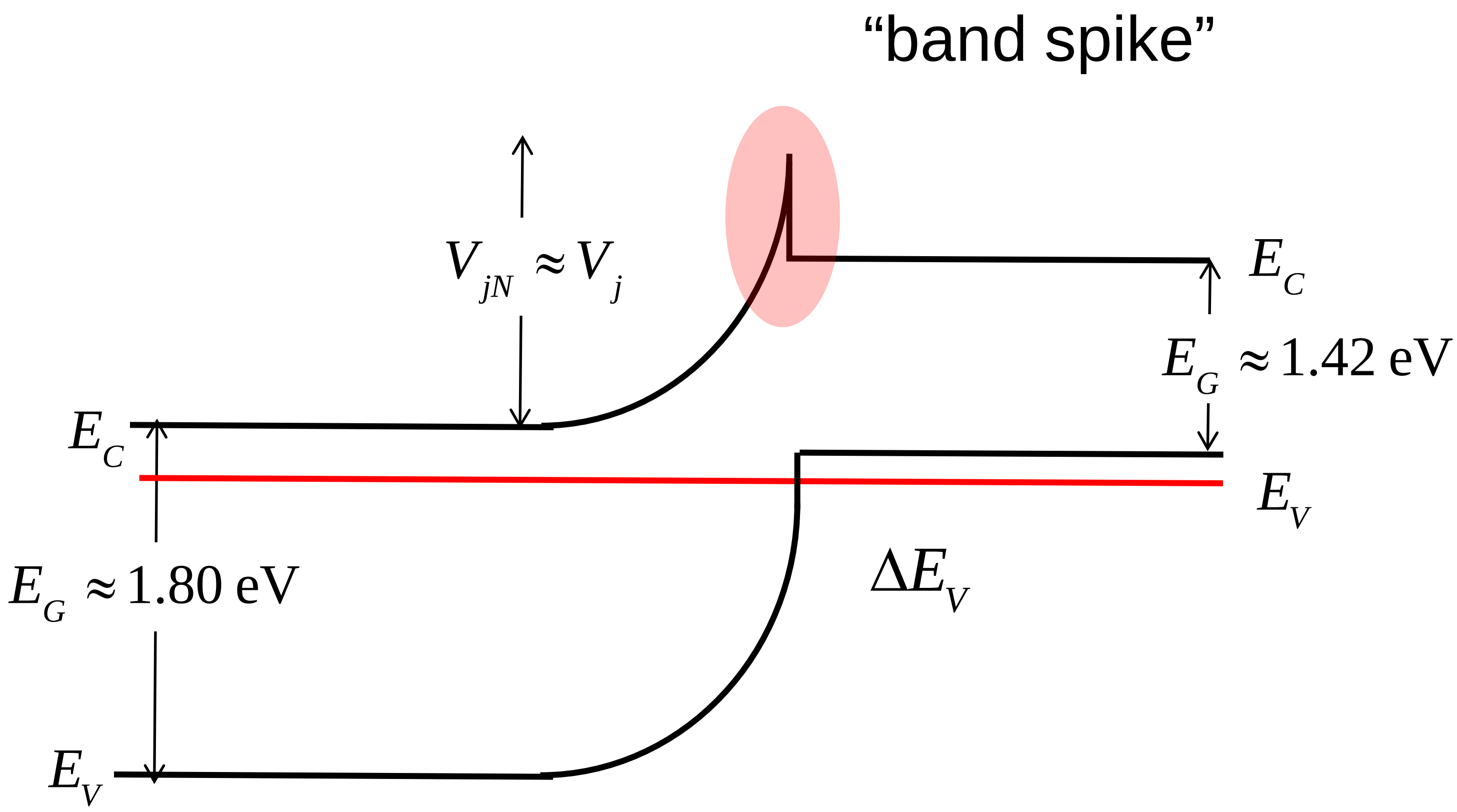
$Al_{0.3}Ga_{0.7}As$: GaAs (Type I HJ)



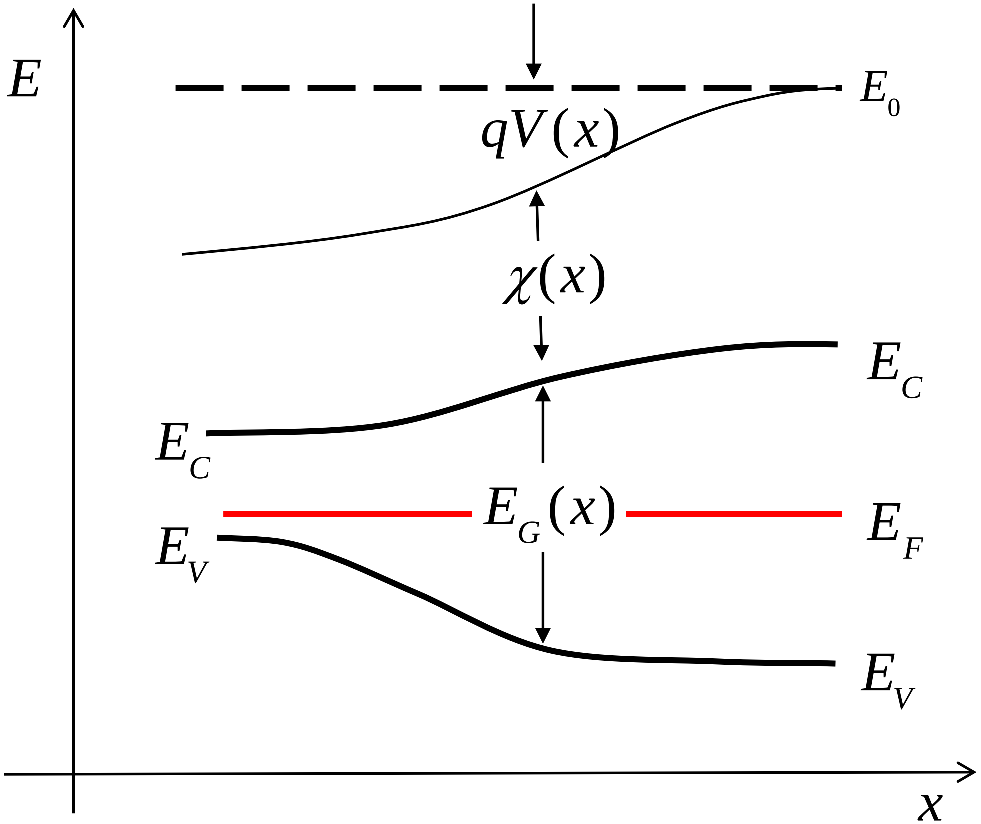
N-$Al_{0.3}Ga_{0.7}As$: p^+-GaAs (Type I HJ)



N-$Al_{0.3}Ga_{0.7}As$: p^+-GaAs (Type I HJ)



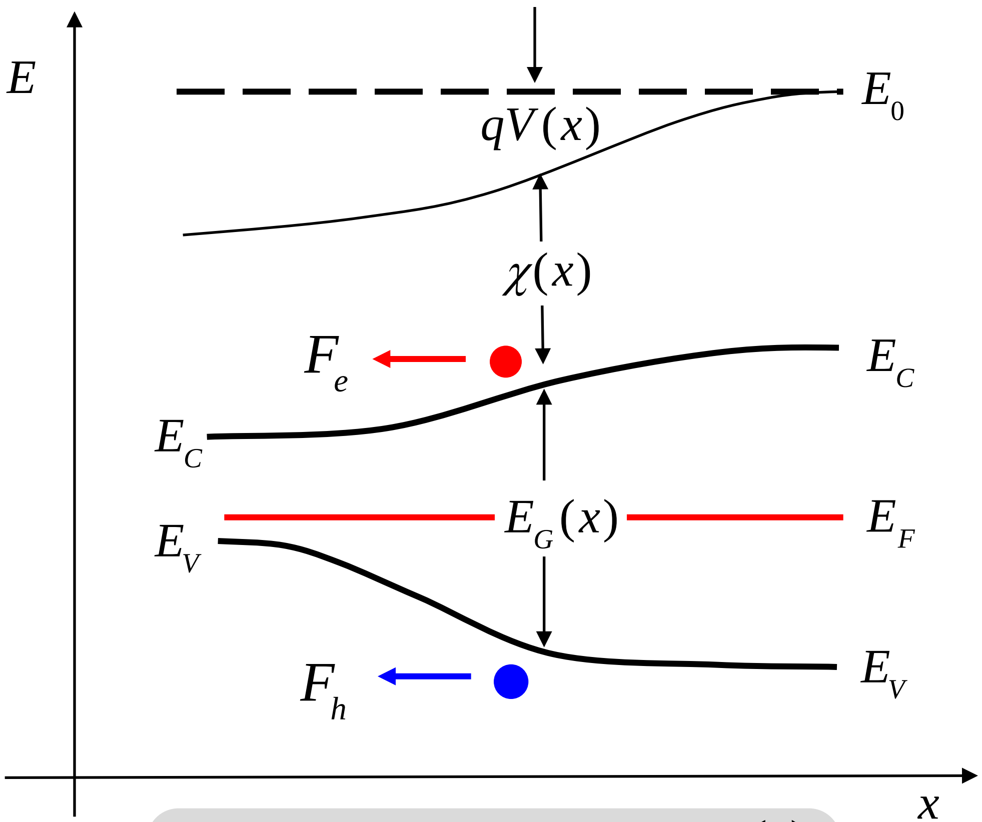
general, graded heterostructure



$$E_C(x) = E_0 - \chi(x) - qV(x)$$

$$E_V(x) = E_C(x) - E_G(x)$$

quasi-electric fields



$$E_C(x) = E_0 - \chi(x) - qV(x)$$
$$E_V(x) = E_C(x) - E_G(x)$$

$$F_e = -\frac{dE_C}{dx} = q\frac{dV}{dx} + \frac{d\chi}{dx}$$

$$F_e = -q\mathcal{E}(x) - q\mathcal{E}_{QN}(x)$$

$$\mathcal{E}_{QN} \equiv -\frac{1}{q}\frac{d\chi}{dx}$$

$$F_h = +\frac{dE_V}{dx} = -q\frac{dV}{dx} - \frac{d\left(\chi + E_G\right)}{dx}$$

$$F_h = +q\mathcal{E}(x) + q\mathcal{E}_{QP}(x)$$

$$\mathcal{E}_{QP} \equiv -\frac{1}{q}\frac{d\left(\chi + E_G\right)}{dx}$$

BTE

$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \frac{d\,\vec{p}}{dt} \bullet \nabla_p f = 0$$

$$\frac{d\,\vec{p}}{dt} = \frac{d\left(\hbar\vec{k}\right)}{dt} = -\nabla_r E_C(\vec{r}) = -q\vec{\mathcal{E}}(\vec{r})$$

(constant effective mass)

This equation does not hold when the effective mass is position dependent. See HW 12.

alternative approach: hole current

$$J_p = p\mu_p \frac{dF_p}{dx} \qquad p = N_V(x)\, e^{\left(E_V - F_p\right)/k_B T} \qquad F_p = E_V(x) - k_B T \ln\left(p/N_V\right)$$

$$\frac{dF_p}{dx} = \frac{dE_V(x)}{dx} - k_B T\left[\frac{1}{p}\frac{dp}{dx} - \frac{1}{N_V}\frac{dN_V}{dx}\right]$$

$$J_p = p\mu_p\left[\frac{dE_V(x)}{dx} + \frac{k_B T}{N_V}\frac{dN_V}{dx}\right] - k_B T \mu_p \frac{dp}{dx}$$

$$\frac{dE_V(x)}{dx} = \frac{d}{dx}\left[E_0 - \chi(x) - E_G(x) - qV(x)\right] = q\left(\mathcal{E}(x) + \mathcal{E}_{QP}\right)$$

hole and electron currents

$$J_p = pq\mu_p\left[\mathcal{E} + \mathcal{E}_{QP} + \frac{k_B T}{q}\frac{1}{N_V}\frac{dN_V}{dx}\right] - qD_p\frac{dp}{dx}$$

"DOS effect"

$$J_p = nq\mu_n\left[\mathcal{E} + \mathcal{E}_{QN} - \frac{k_B T}{q}\frac{1}{N_C}\frac{dN_C}{dx}\right] + qD_n\frac{dn}{dx}$$

quasi-electric fields

$$\mathcal{E}_{QP} \equiv -\frac{1}{q}\frac{d\left(\chi + E_G\right)}{dx} \qquad \mathcal{E}_{QN} \equiv -\frac{1}{q}\frac{d\chi}{dx}$$

questions

1) Carrier Temperature and Heat Flux
2) Balance equations in 3D
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