ECE 656: Fall 2009
Lecture 30 Homework
(Revised 11/15/09)

1) Work out the energy flux balance equation in 3D, and then make appropriate simplifications to simplify it. Specifying the energy flux relaxation time is a challenge – you do not need to address the question of how to specify it in this assignment.

Following L29 in 1D

$\Phi(p) = E(p_x) v_x$

$n_\Phi = \bar{F}_W$

$F_\Phi = n_L \langle E v_x^2 \rangle$ or $\frac{2}{m^*} n_L \langle E^2 \rangle = X$

$G_\Phi = -\frac{3q}{m^*} \mathcal{E}_x n_L u$ $\quad u$ = ave energy per electron

$R_\Phi = \frac{\bar{F}_W}{\langle \tau_{FW} \rangle}$

$\frac{\partial \bar{F}_W}{\partial t} = -\frac{dX}{dt} - \frac{3q}{m^*} n_L u \mathcal{E}_x - \frac{\bar{F}_W}{\langle \tau_{FW} \rangle}$

$n_L u = W$, and we already have a balance eqn. for that

Some how, we need to specify $\langle \tau_{FW} \rangle$ – that is always a complicated topic.

Let's focus on X

$$\chi = \frac{2}{m^*} \eta_L \langle E^2 \rangle \qquad \langle E^2 \rangle = \sum_p E^2 f \Big/ \sum_p f$$

assume a Maxwellian: $f \sim e^{-E/k_B T}$

$\langle E^2 \rangle$ = NUM/DEN

$$\text{NUM} = \int_{-\infty}^{\infty} E^2 e^{-E/k_B T} \, dp \qquad dp = \sqrt{\frac{2m}{2}} E^{-1/2} \, dE$$

$$= \int E^{3/2} e^{-E/k_B T} \, dE$$

constants will divide out

let $E' = E/k_B T = x$

$$\text{NUM} = (k_B T)^{5/2} \underbrace{\int_{-\infty}^{+\infty} x^{3/2} e^{-x} \, dx}_{= 2 \times \Gamma(5/2)}$$

$$\text{DEN} = \int_{-\infty}^{+\infty} e^{-E/k_B T} \, dp = \int e^{-E/k_B T} \sqrt{\frac{2m}{2}} E^{-1/2} \, dE$$

$$= (k_B T)^{1/2} \underbrace{\int x^{-1/2} e^{-x} \, dx}_{= 2 \times \Gamma(1/2)}$$

so

$$\langle E^2 \rangle = (k_B T)^2 \frac{\Gamma(5/2)}{\Gamma(1/2)}$$

$$\Gamma(5/2) = 3/4\, \Gamma(1/2) \rightarrow \langle E^2 \rangle = \frac{3}{4}(k_B T)^2$$

so

$$X = \frac{2}{m^*} n_L \langle E^2 \rangle = \frac{3}{2m^*} n_L (k_B T)^2$$

recall in 1D: $W = n_L \frac{k_B T}{2}$

$$X = 3\frac{(k_B T)}{m^*} W \quad \checkmark$$

so we can terminate the hierarchy by assuming f is a Maxwellian!

But there is still a problem to specify $\langle \tau_{FW} \rangle$!