ECE-656: Fall 2009

Lecture 34: Ensemble Effects in Non-Local Transport

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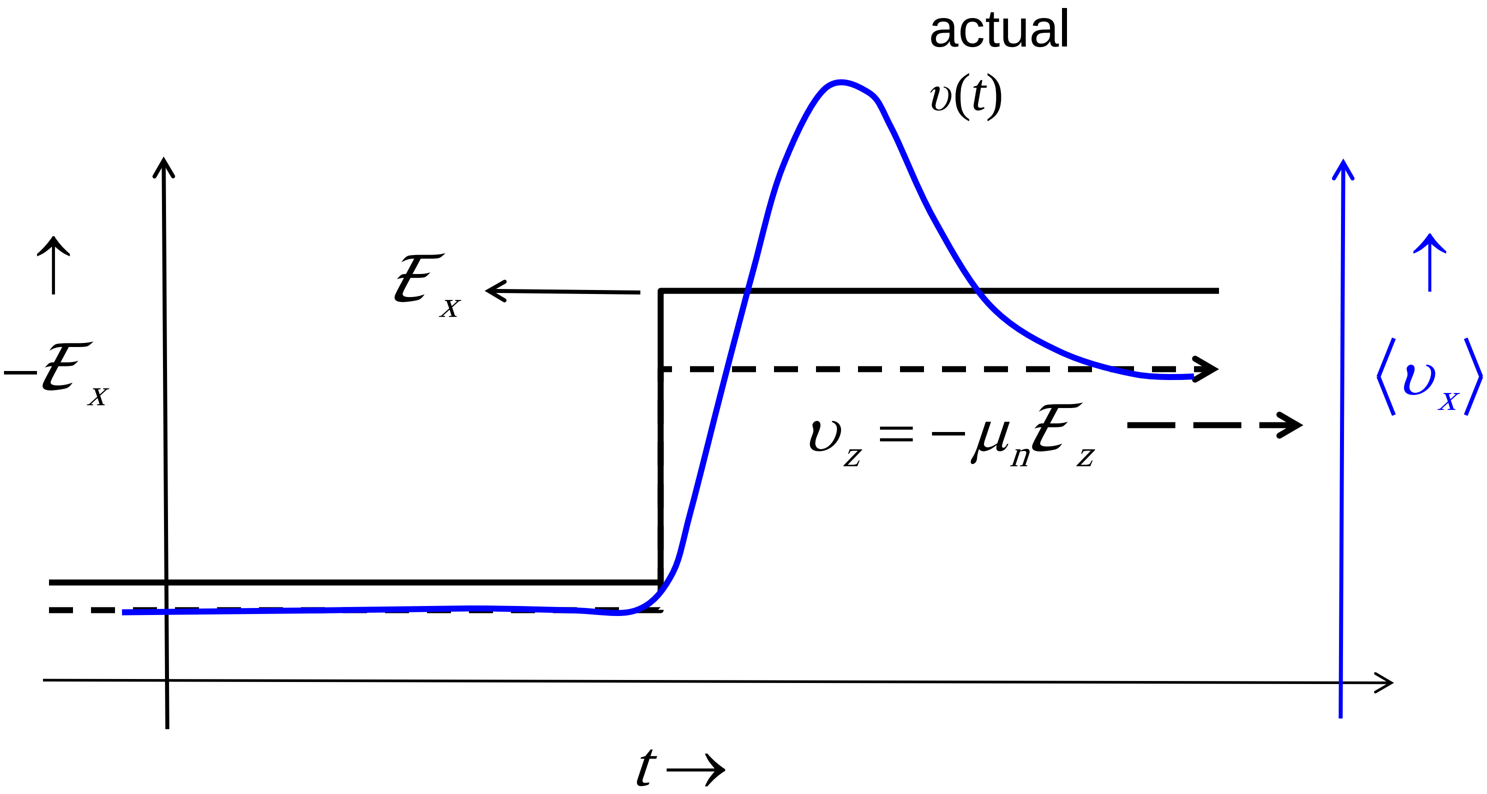
outline

1) **Review of velocity overshoot**
2) Steady-state, spatial transients
3) Heterojunction launching ramps
4) Repeated velocity overshoot
5) Questions?

rapidly varying electric fields



Monte Carlo simulation

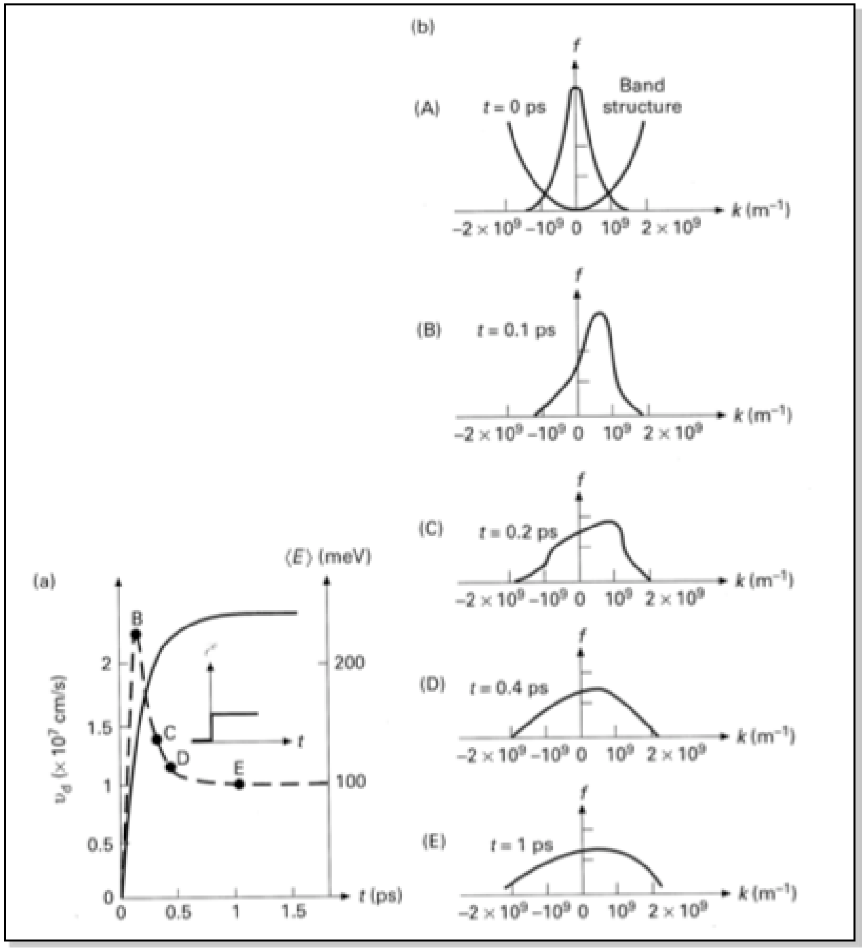


Fig. 8.9 Evolution of the distribution function during a velocity overshoot transient. The average drift velocity and energy are shown in (a), and the evolution of the corresponding distribution function is shown in (b). The results were obtained by Monte Carlo simulation of electron transport in silicon by E. Constant [8.10].

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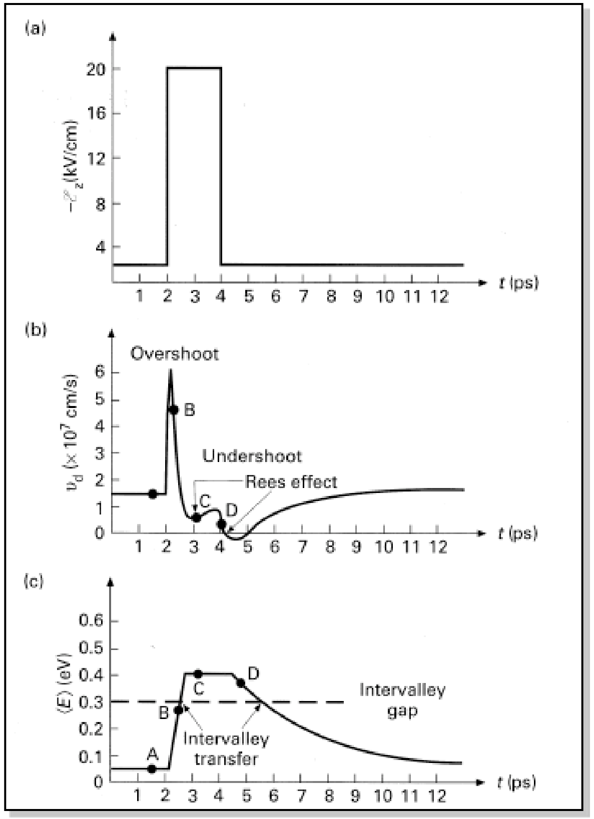
Rees effect (GaAs)



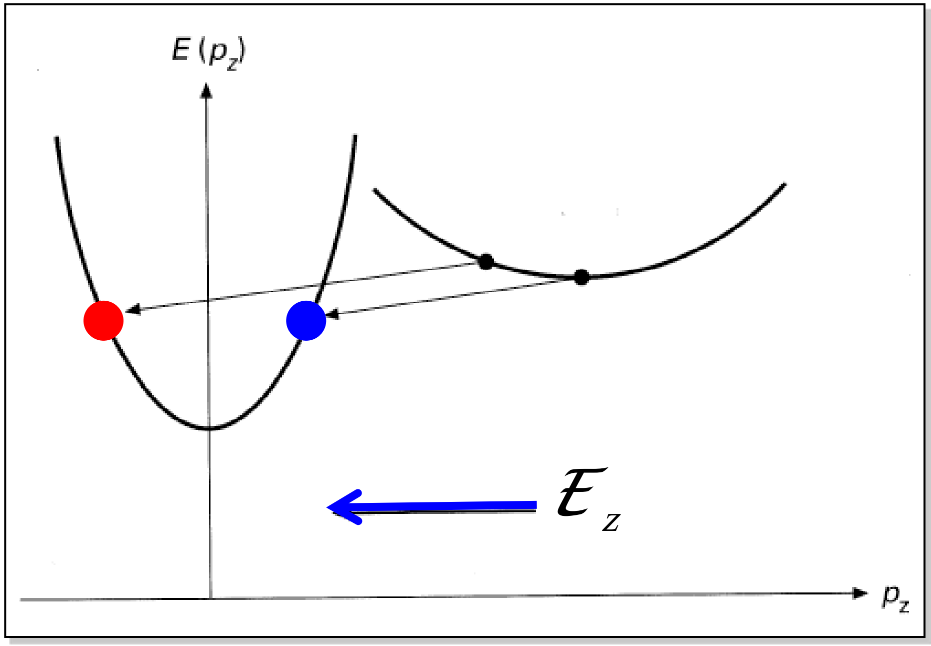


Fig. 8.10 (a) Applied electric field vs. time. (b) Ave. drift velocity vs. time. (c) Ave. electron energy vs. time. (Monte Carlo simulations from E. Constant [8.10].

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velocity overshoot

$p_{dx} = \langle p_x \rangle$ Let's find an equation for the ave. *x*-directed momentum.

$$\frac{dp_{dx}}{dt} = -q\mathcal{E}_x - \frac{p_{dx}}{\langle \tau_m \rangle}$$

$u = \langle E(p) \rangle$ Let's find an equation for the ave. *x*-directed momentum.

$$\frac{du}{dt} = -q\upsilon_{dx}\mathcal{E}_x - \frac{(u - u_0)}{\langle \tau_E \rangle}$$

$$\upsilon_{dx}(t) = -\mu_n \mathcal{E}_x \left(1 - e^{-t/\langle \tau_m \rangle}\right)$$

$$u(t) = u_0 + q\langle \tau_E \rangle \mu_n \mathcal{E}_x^2 \left(1 - e^{-t/\langle \tau_E \rangle}\right)$$

But, we have ignored diffusion (an ensemble effect).

momentum and energy balance

$$\frac{dp_{dx}}{dt} = -q\mathcal{E}_x - \frac{p_{dx}}{\langle \tau_m \rangle}$$

$$\frac{du}{dt} = -q\upsilon_{dx}\mathcal{E}_x - \frac{(u - u_0)}{\langle \tau_E \rangle}$$

$$\frac{dP_x}{dt} = -\frac{d(2W_{xx})}{dx} - qn\mathcal{E}_x - \frac{P_x}{\langle \tau_m \rangle}$$

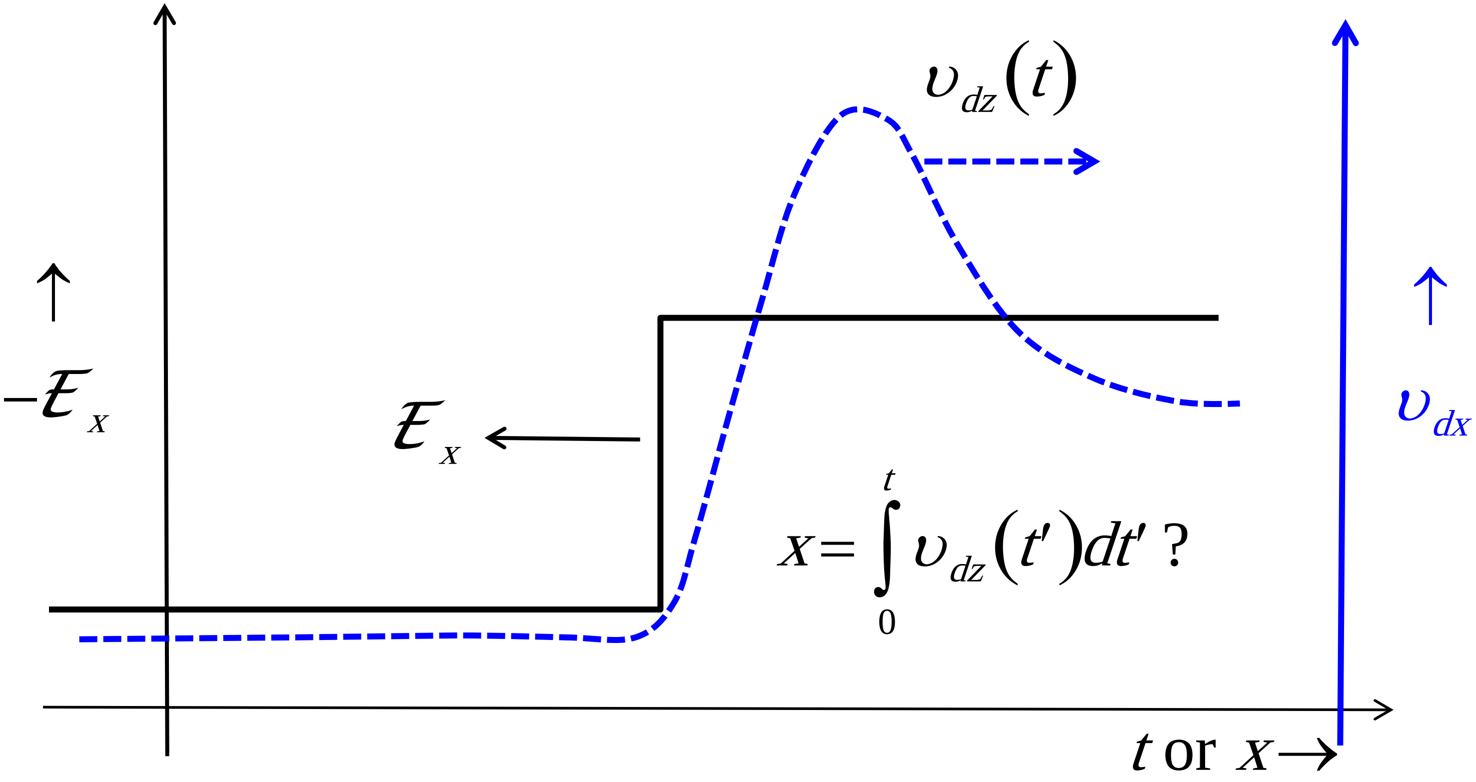
$$P_x = n\langle p_x \rangle = np_{dx}$$

$$\frac{dW}{dt} = -\frac{d(F_W)}{dx} + J_x\mathcal{E}_x - \frac{W - W_0}{\langle \tau_E \rangle}$$

$$W = n\langle E(p) \rangle = nu$$

What effect do these spatial gradients have?

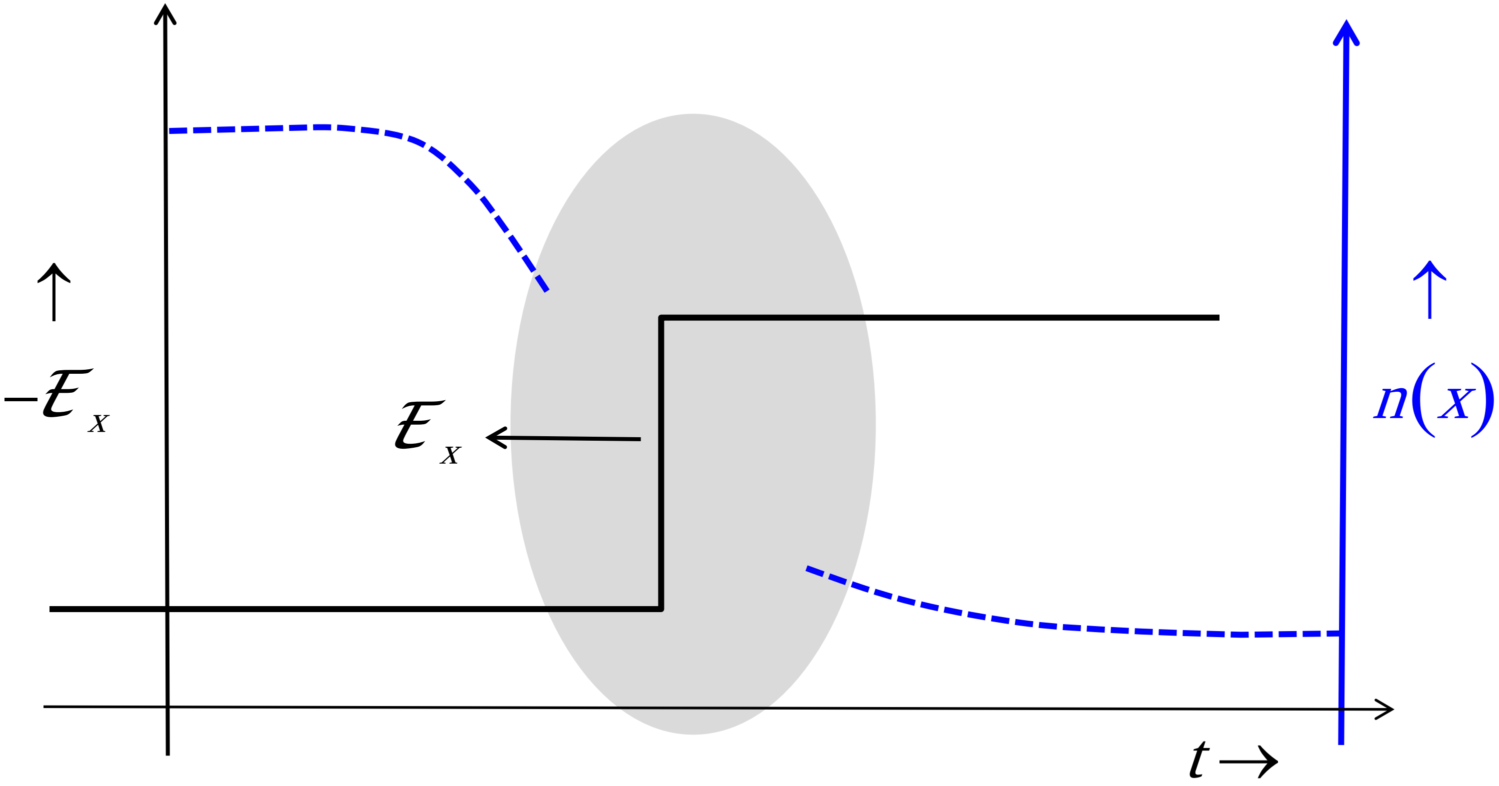
temporal vs. spatial transients



$$x = \int_0^t \upsilon_{dz}(t')dt' \,?$$

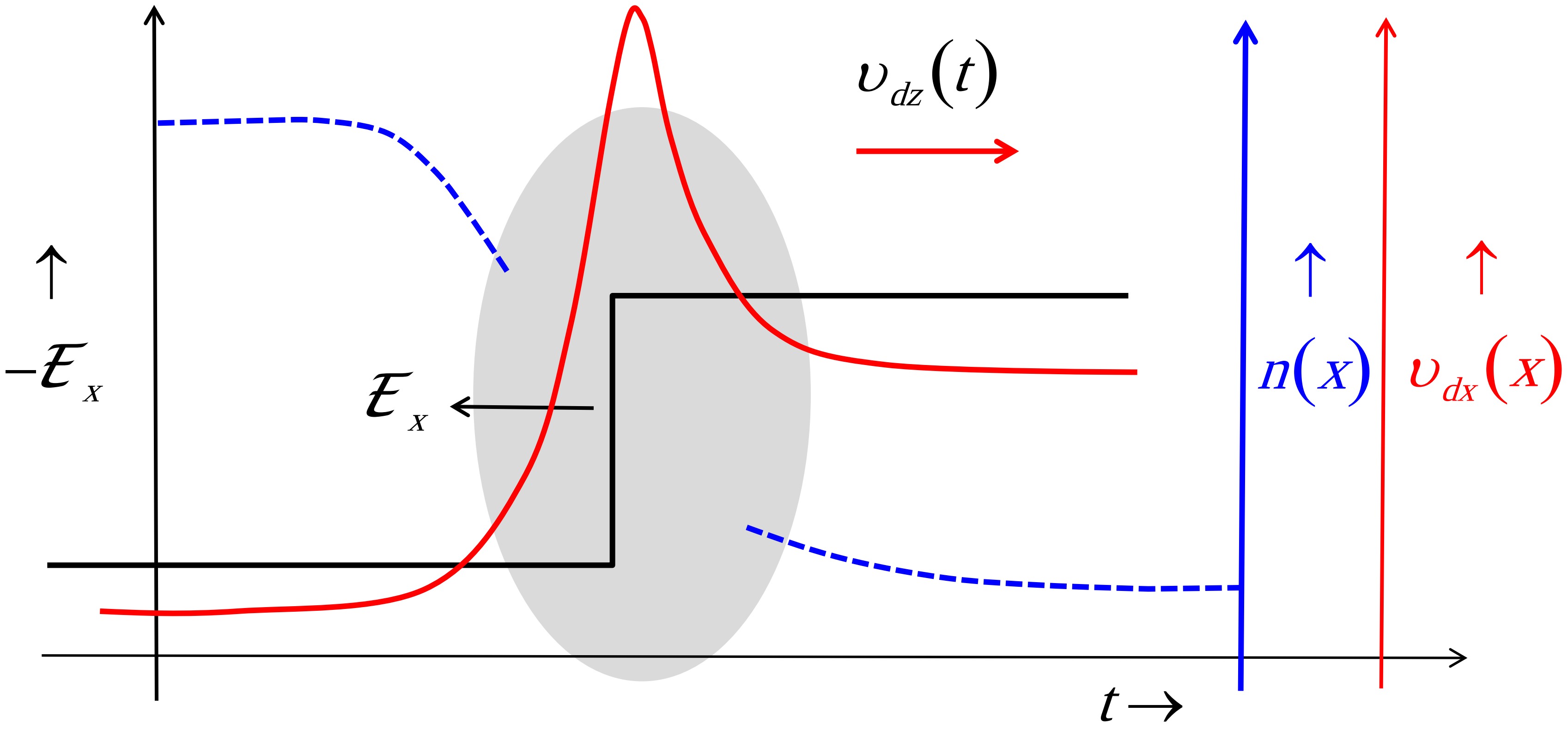
Question: If we change the horizontal axis to distance, what does the steady-state velocity vs. position characteristic look like?

carrier density vs. position



Steady-state current is constant: $J_{nx} = n(x)q\langle \upsilon_x(x)\rangle$

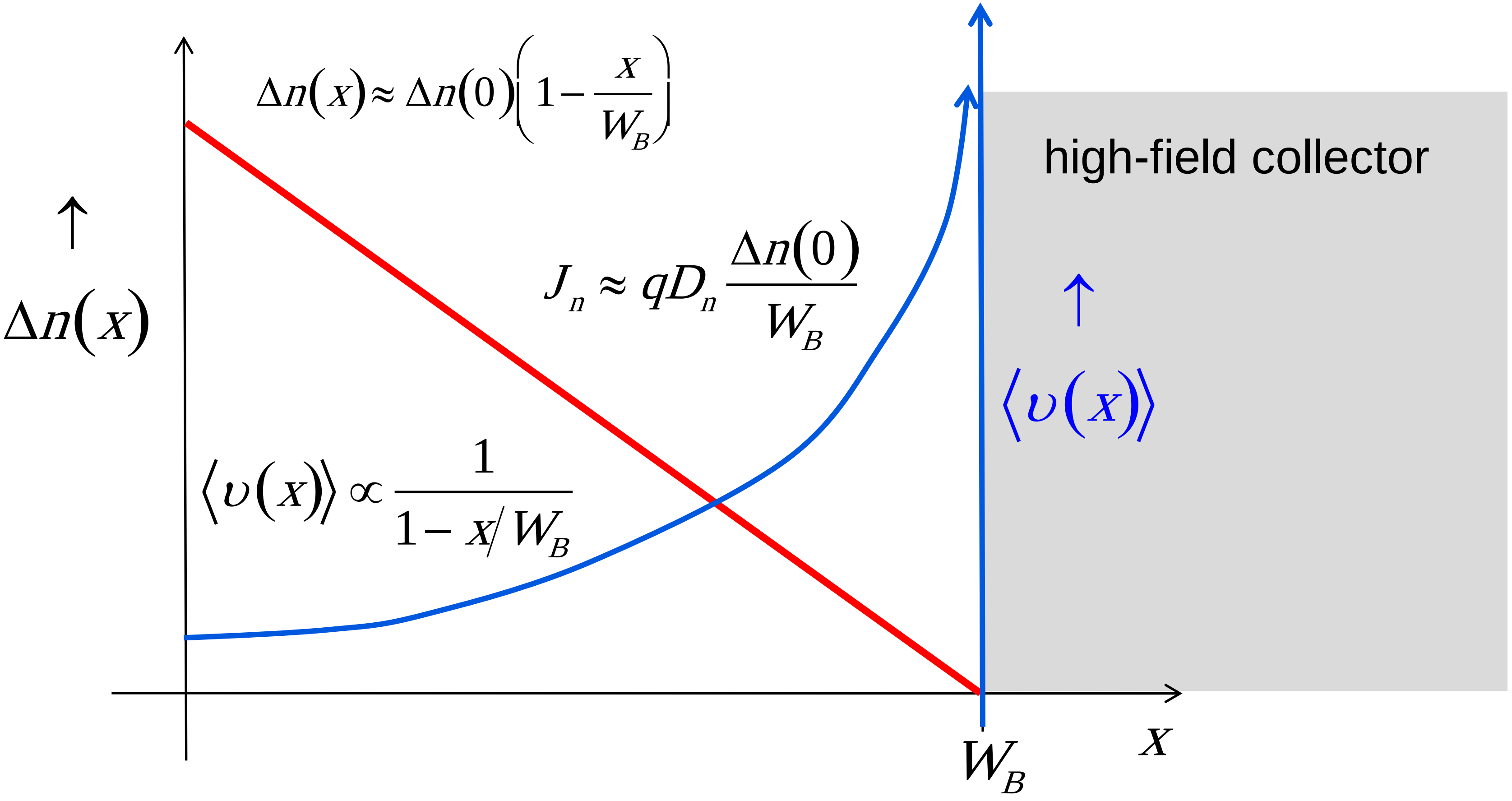
carrier velocity vs. position



outline

1) Review of velocity overshoot
2) **Steady-state, spatial transients**
3) Heterojunction launching ramps
4) Repeated velocity overshoot
5) Questions?

a familiar example



$$\Delta n(x) \approx \Delta n(0)\left(1 - \frac{x}{W_B}\right)$$

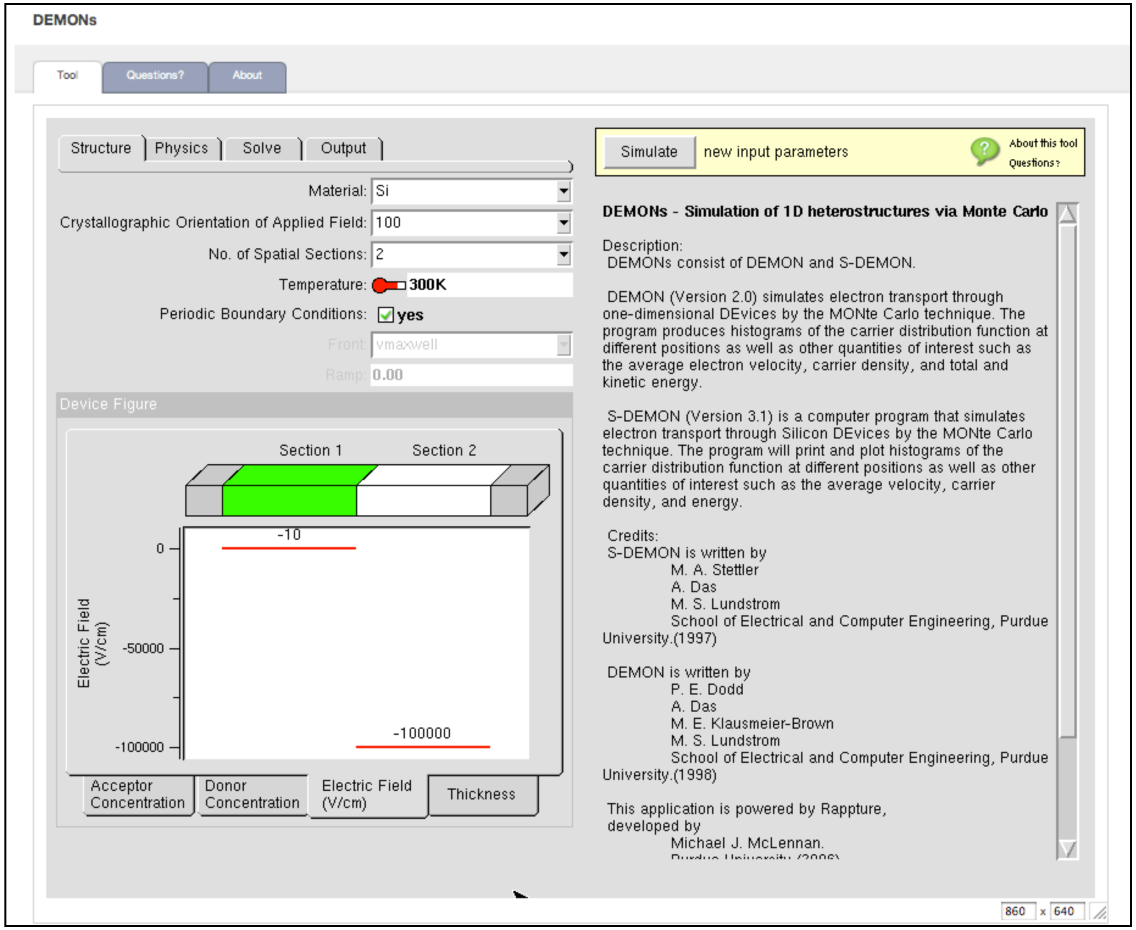
$$J_n \approx qD_n \frac{\Delta n(0)}{W_B}$$

$$\langle \upsilon(x) \rangle \propto \frac{1}{1 - x/W_B}$$

$$J_{nx} = n(x)q\langle \upsilon_x(x) \rangle$$

example Monte Carlo simulations

DEMONs



Material: Si

Crystallographic Orientation of Applied Field: 100

No. of Spatial Sections: 2

Temperature: 300K

Periodic Boundary Conditions: ☑ yes

Front: vmaxwell

Ramp: 0.00

DEMONs - Simulation of 1D heterostructures via Monte Carlo

Description:
DEMONs consist of DEMON and S-DEMON.

DEMON (Version 2.0) simulates electron transport through one-dimensional DEvices by the MONte Carlo technique. The program produces histograms of the carrier distribution function at different positions as well as other quantities of interest such as the average electron velocity, carrier density, and total and kinetic energy.

S-DEMON (Version 3.1) is a computer program that simulates electron transport through Silicon DEvices by the MONte Carlo technique. The program will print and plot histograms of the carrier distribution function at different positions as well as other quantities of interest such as the average velocity, carrier density, and energy.

Credits:
S-DEMON is written by
M. A. Stettler
A. Das
M. S. Lundstrom
School of Electrical and Computer Engineering, Purdue University.(1997)

DEMON is written by
P. E. Dodd
A. Das
M. E. Klausmeier-Brown
M. S. Lundstrom
School of Electrical and Computer Engineering, Purdue University.(1998)

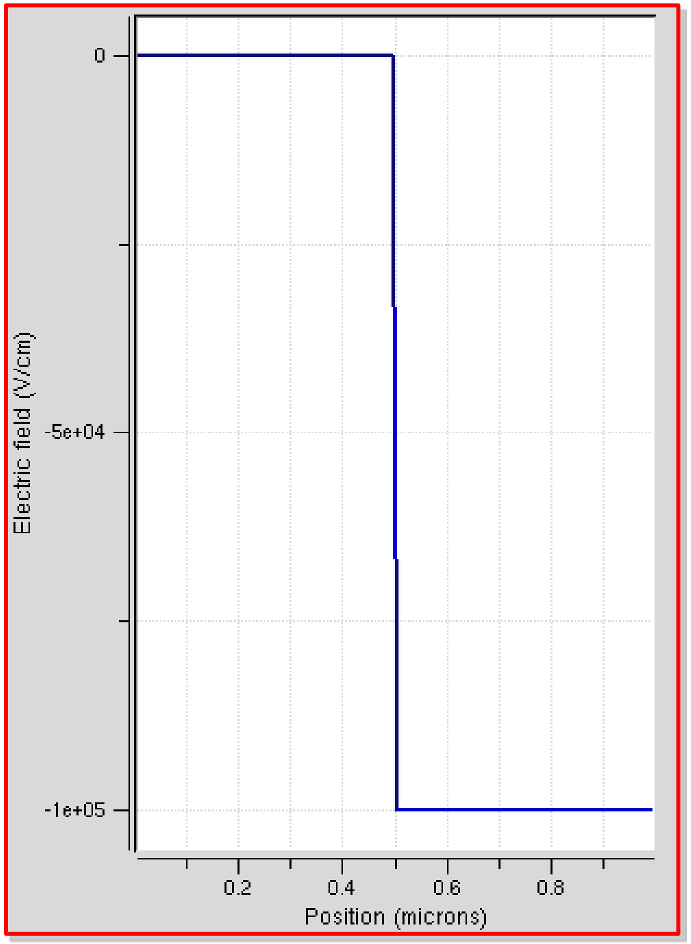
This application is powered by Rappture,
developed by
Michael J. McLennan.

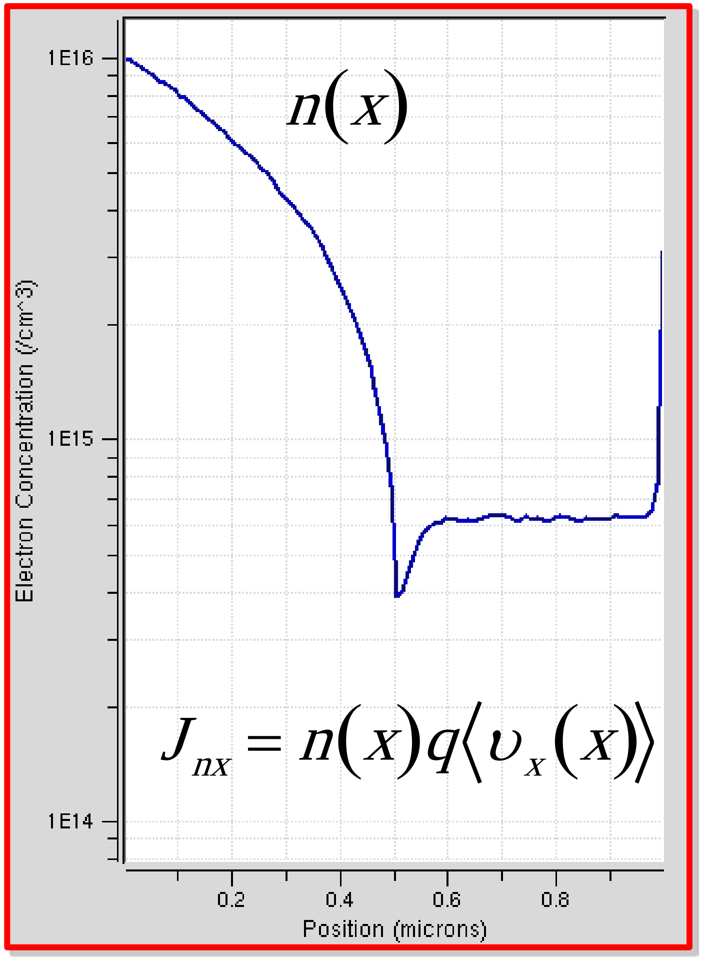
1D, steady-state Monte Carlo simulation for Si and GaAs

Piecewise constant electric field profiles.

http://nanohub.org/resources/1934

low-high field structure



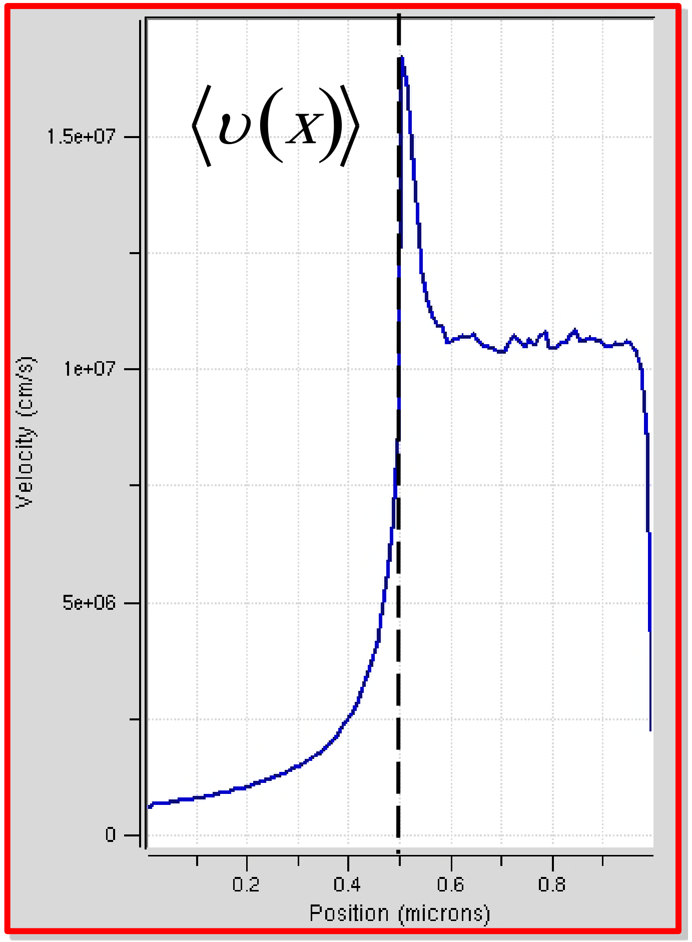


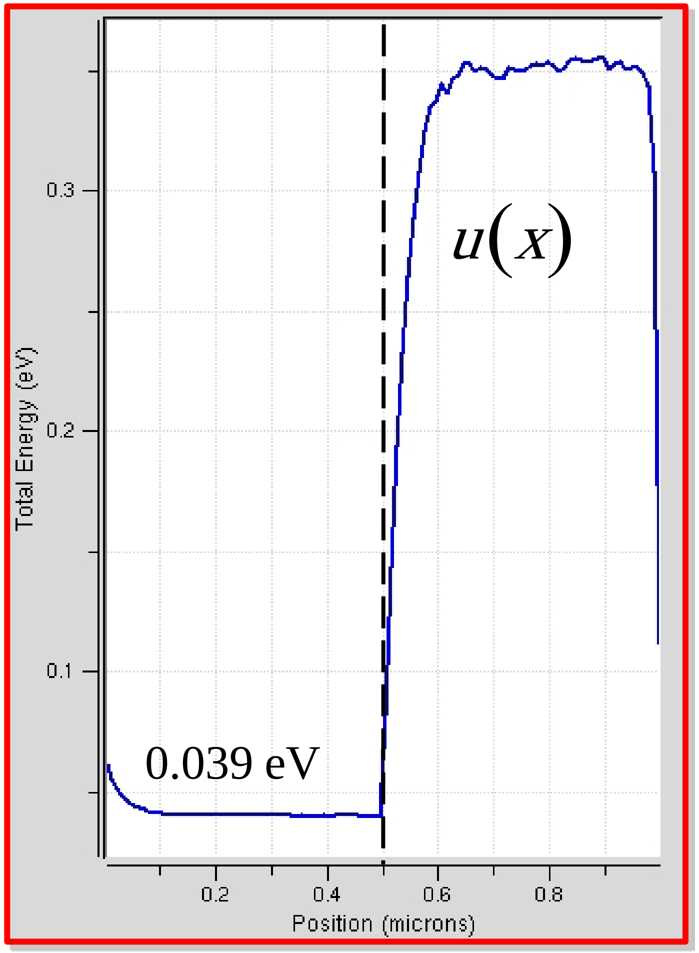
$$n(x)$$

$$J_{nx} = n(x) q \langle \upsilon_x(x) \rangle$$

periodic boundary conditions

low-high field structure

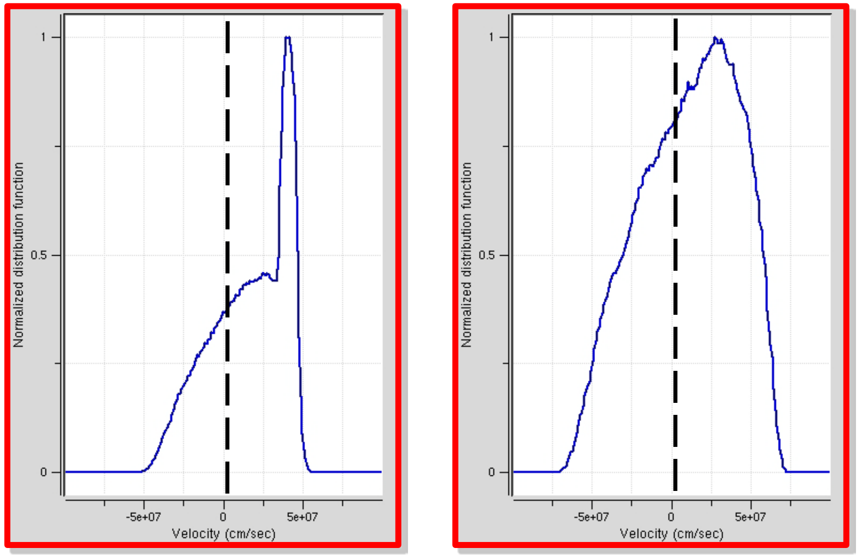




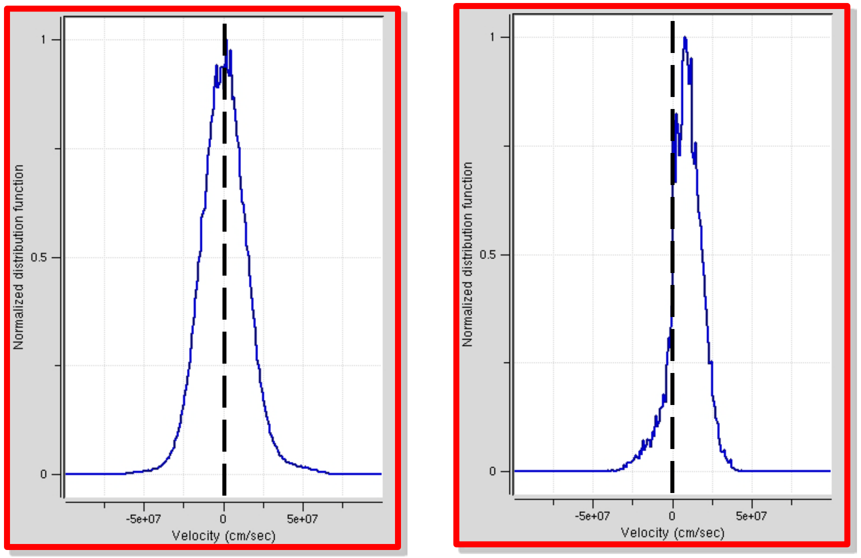
periodic boundary conditions

velocity histograms

$\mathcal{E} = -10^5 \text{ V/cm}$



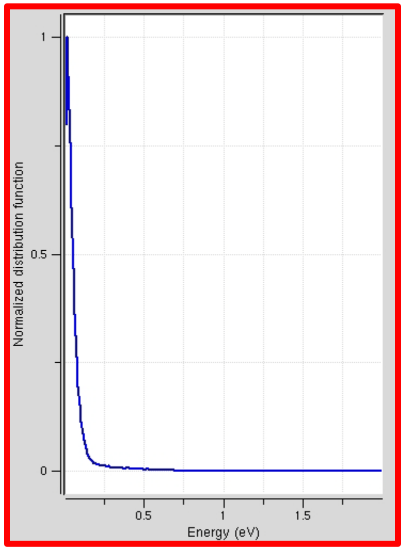
$\mathcal{E} = -10 \text{ V/cm}$

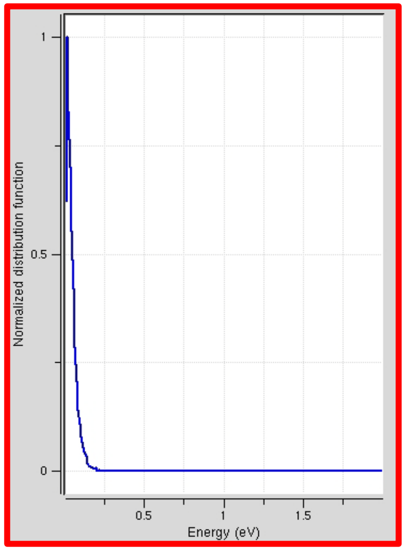


energy histograms

$$n(E) = f(E)D(E)$$

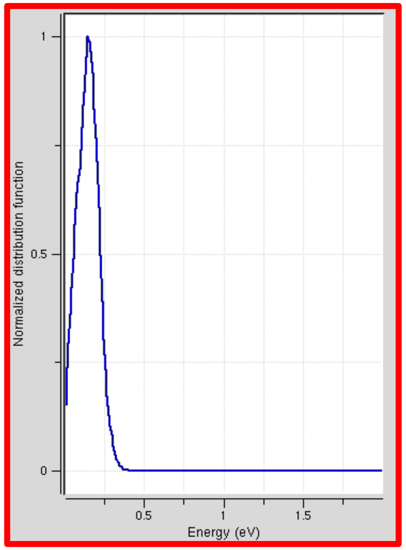
$$\mathcal{E} = -10 \text{ V/cm}$$

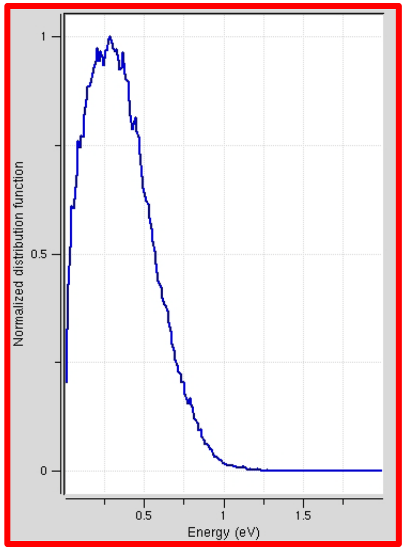




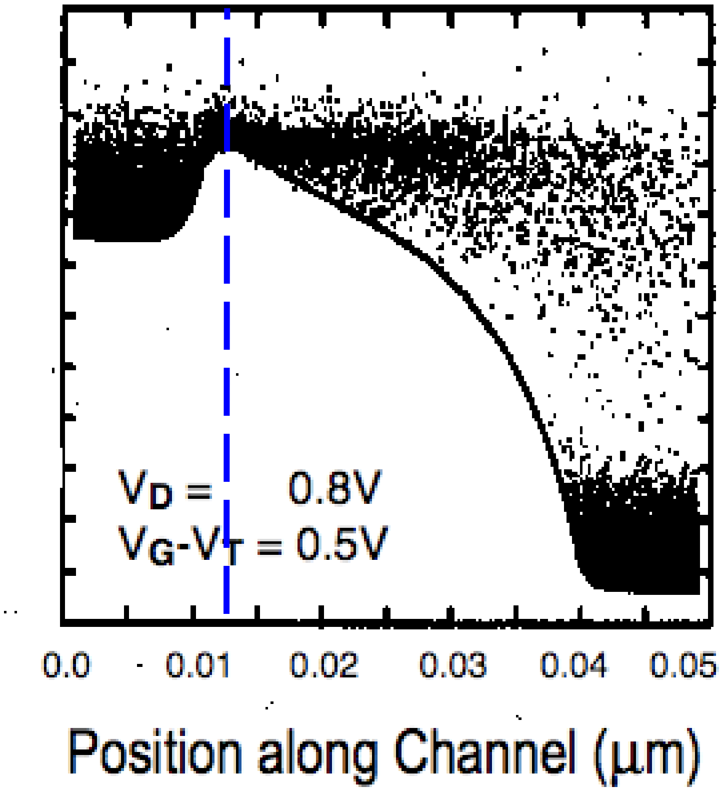
$$n(E) \propto e^{-E/k_BT}\sqrt{E}$$

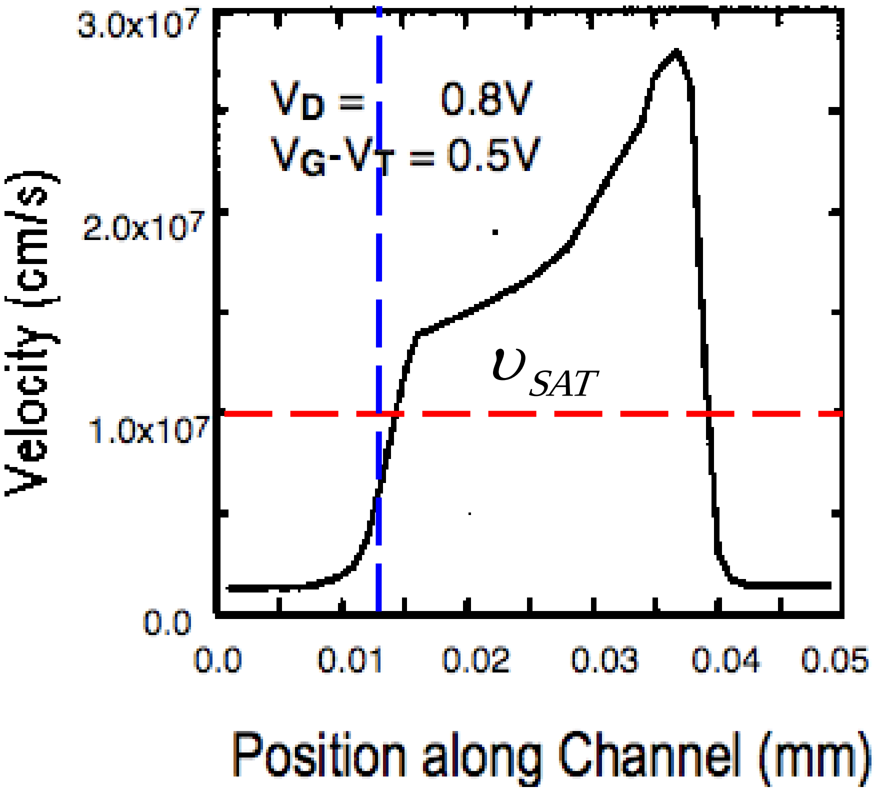
$$\mathcal{E} = -10^5 \text{ V/cm}$$





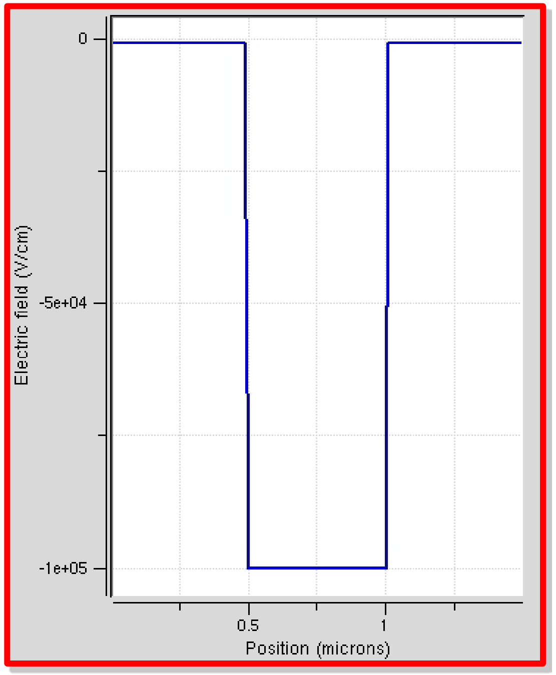
off-equilibrium nanoscale MOSFETs



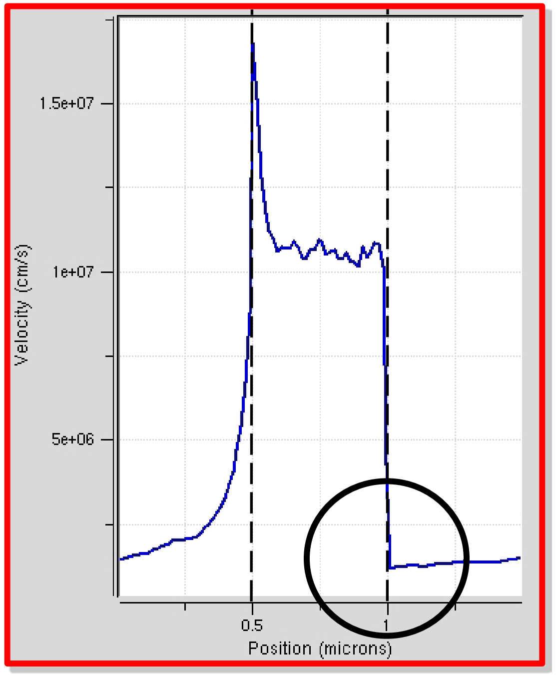


Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992

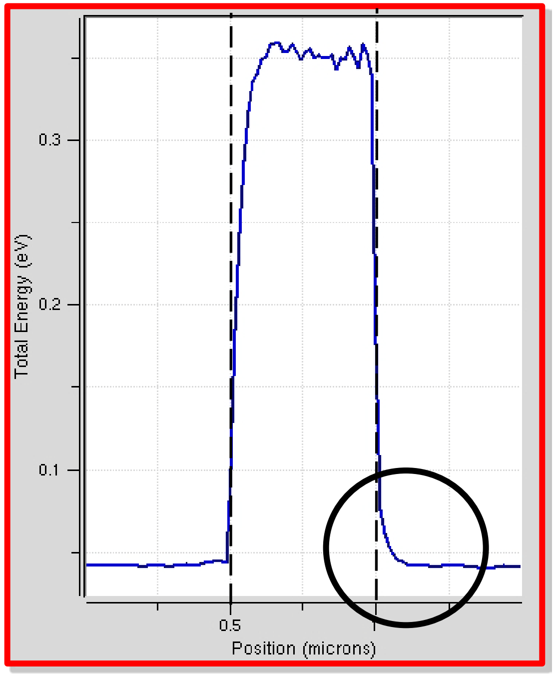
low-high-low field structure



periodic
boundary
conditions



velocity
undershoot



$u(x) > u_0$

temporal vs. spatial transients

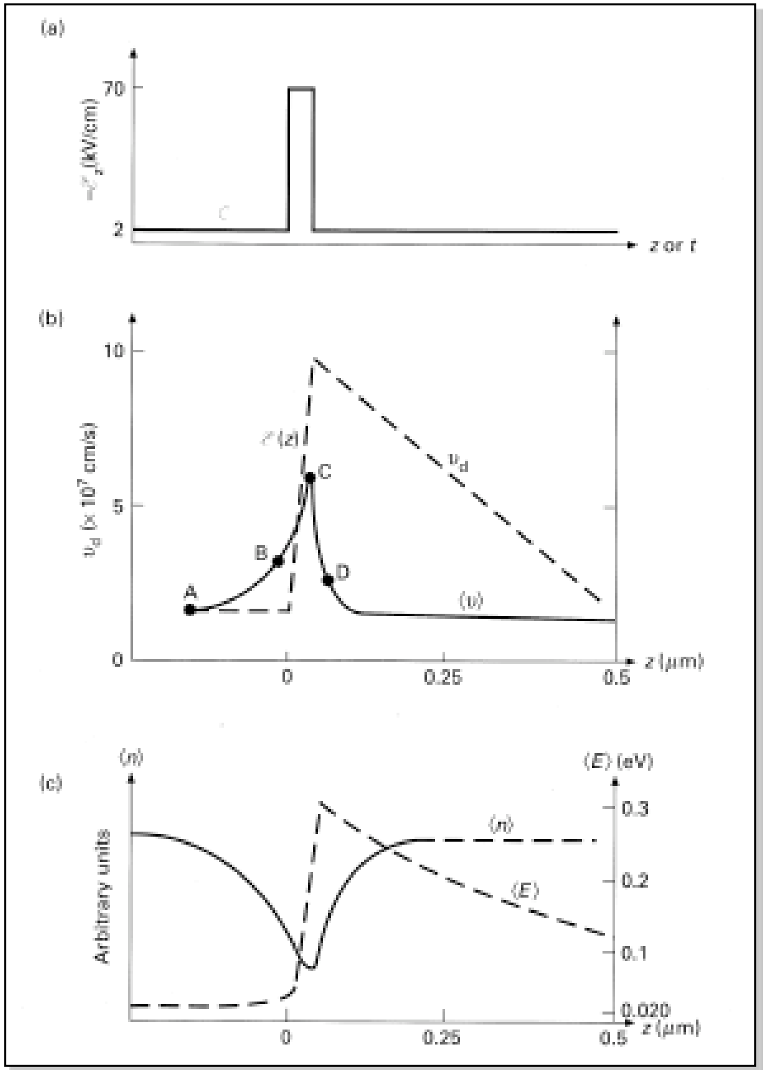


Fig. 8.13 (a) Applied electric field in time and space. (b) Average velocity versus position for a pulse applied in space (solid line) and time (dashed line). (c) Steady-state carrier density (solid line() and energy (dashed line.) The results were obtained by Monte Carlo simulation of electron transport in GaAs by E. Constant [8.10].

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$$z = \int_0^t \upsilon(t')\,dt'$$

outline

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ballistic launching ramps in HBTs

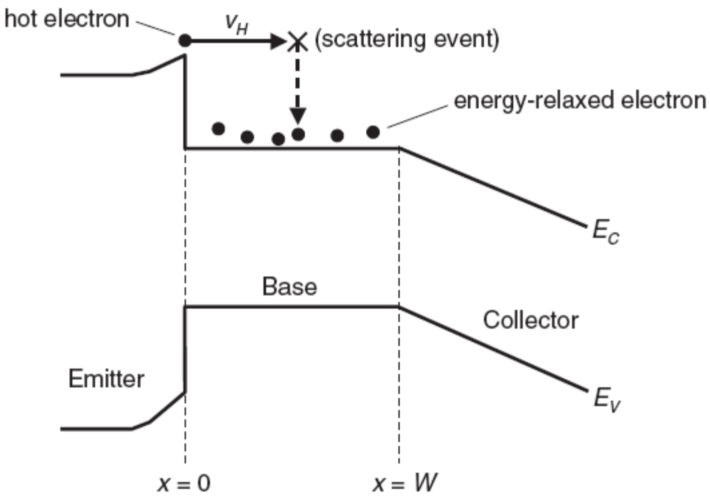


Fig. 1. Modeling of electron transport in the base of InP/InGaAs HBTs. A hot electron having a constant velocity of $v_H = 1 \times 10^8$ cm/s is assumed to be thermalized to an energy-relaxed electron after one scattering event. The energy-relaxed electron has a diffusivity D_T and a recombination life time τ_T.

Hiroki Nakajima, *Jap. J. of Appl. Phys.*, **46**, pp. 485–490, 2007.

Question: How do we experimentally tell whether base transport is ballistic or diffusive?

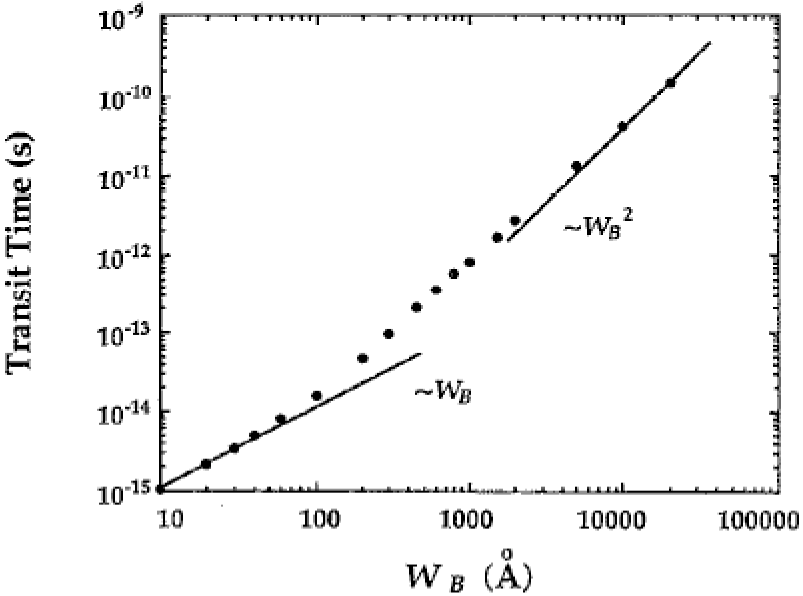
Answer: Look at the base current.

$$I_B \propto \frac{t_t}{\tau_n}$$

ballistic: $t_t = W_B / \upsilon_{ball}$

diffusive: $t_t = W_B^2 / 2D_n$

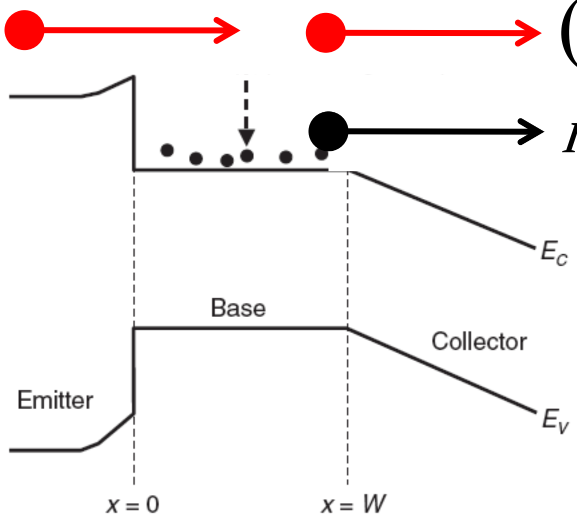
base transit time scaling



~80% traverse a 300A base ballisticaly or quasi-ballisticaly

P.E. Dodd and M.S. Lundstrom, “Minority electron transport in InP/InGaAs heterojunction bipolar transostors,” *Appl. Phys. Lett*., **61**, 27,1992

base transit time scaling



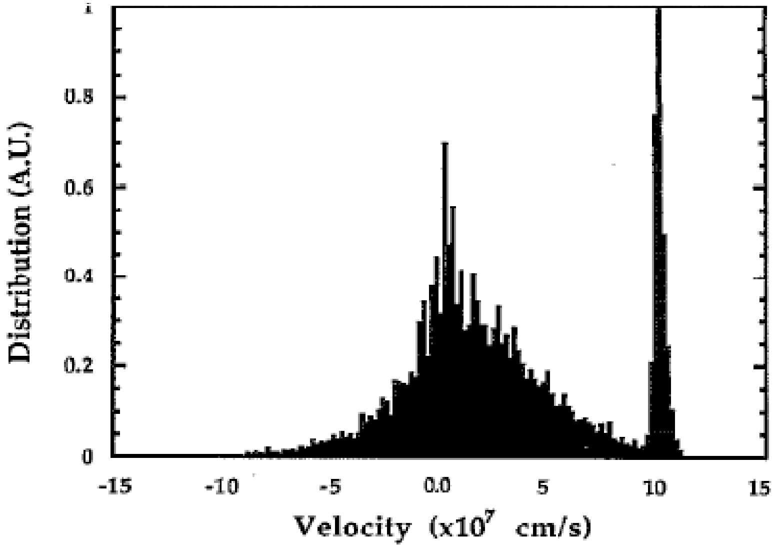
$n_0 \upsilon_{inj}$

$(1-\Gamma)n_0 \upsilon_{inj}$

$n_{mw}(D_n/W)$

$$n_0 \upsilon_{inj} = (1-\Gamma)n_0 \upsilon_{inj} + n_{mw}(D_n/W)$$

$$\frac{n_0}{n_{mw}} = \Gamma \frac{\upsilon_{inj}}{(D_n/W)}$$



Dodd and Lundstrom, *APL*, **61**, 27,1992

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