Exam 1 ECE-656 Fall 2009
(Revised: October 3, 2009)

NAME SOLUTION

ID________________________________

**This is a take home exam due at 10:30AM, Wednesday, October 7, 2009**

The exam consists of 10 questions on the attached pages.

1) Show your work for each problem CLEARLY.

2) Mark your answers CLEARLY.

3) Make reasonable assumptions when necessary, but be sure to state them.

DO NOT DISCUSS THIS EXAM WITH ANYONE. IT SHOULD BE YOUR WORK AND YOUR WORK ALONE.

When you hand in your exam, attach this sheet as a cover sheet stapled to your work.

You must also sign the following statement.

I attest that the attached work for Exam 1, ECE-656, Fall 2009 is my work and my work alone. I have not discussed this exam with anyone and received no help with the exam.

SIGNED:________________________________

DATE:________________________________

ECE-656 Take Home Exam 1: Fall 2009

1) For a 3D semiconductor with parabolic energy bands, the electron density is

$$n = N_C \mathcal{F}_{1/2}(\eta_F) \quad N_C = \frac{1}{4}\left(\frac{2m^* k_B T}{\pi\hbar^2}\right)^{3/2} .$$

Work out the corresponding result for 2D electrons.

2) For a nondegenerate, 3D semiconductor with parabolic energy bands, the average kinetic energy per electron is

$$\langle E - E_C \rangle = \frac{3}{2} k_B T .$$

Work out the corresponding result for 2D and $T = 0\text{K}$.

3) The average x-directed thermal velocity for 3D, non-degenerate electrons is

$$\langle \upsilon_x^+ \rangle = \sqrt{\frac{2k_B T}{\pi m^*}}$$

Work out the corresponding result for 2D electrons.

4) For a non-degenerate, 3D semiconductor with a constant mean-free-path, the diffusion coefficient is

$$D_n = \frac{\upsilon_T \lambda_0}{2}$$

Work out an expression for the 2D diffusion coefficient assuming a non-degenerate, parabolic band semiconductor with power law scattering.

5) Derive a drift-diffusion equation for a 2D semiconductor with parabolic energy bands assuming $T = 0\text{K}$.

6) Repeat problem 5) for electrons in the conduction band of graphene.

7) For a 3D, non-degenerate semiconductor, we found the mobility to be

$$\mu_n = \frac{q\langle\langle \tau_f \rangle\rangle}{m^*} \qquad \langle\langle \tau_f \rangle\rangle = \frac{\langle E\tau_f \rangle}{\langle E \rangle} \qquad \langle\langle \tau_f \rangle\rangle = \tau_0 \frac{\Gamma(s+5/2)}{\Gamma(5/2)}$$

Work out the corresponding results for a 2D, non-degenerate semiconductor.

8) The so-called transport distribution is

$$\Sigma(E) = \frac{h}{L^2}\sum_{\vec{k}} \upsilon_x^2 \tau_f \delta(E - E_k)$$

Work out this expression for a 2D semiconductor with parabolic energy bands.

9) The Seebeck coefficient is given by

$$S = \left(\frac{k_B}{-q}\right)\frac{I_1}{I_0}$$

where

$$I_j = \int \left(\frac{E-E_F}{k_B T}\right)^j M(E)T(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

Work out $S$ for a 2D semiconductor with parabolic energy bands and a constant mean-free-path.

10) Assume that the channel resistance and inversion layer density of a MOSFET are known. Assume that one subband is occupied. Develop a procedure to estimate the "average" or "effective" number of modes and mean-free-path from the measured data. Your procedure should be valid whether the semiconductor is non-degenerate, degenerate, or anywhere in between. Your procedure should be CLEARLY described.

1)

$n_{3D}$ : DOS $\sim E^{1/2}$ $\quad n_{3D} = N_{3D}\,\mathcal{F}_{1/2}(\eta_F)$

$n_{2D}$ $\quad\sim E^{0}$ $\quad n_{2D} = N_{2D}\,\mathcal{F}_{0}(\eta_F)$

$n_{1D}$ $\quad\sim E^{-1/2}$ $\quad n_{1D} = N_{1D}\,\mathcal{F}_{-1/2}(\eta_F)$

$$N_{2D} = \frac{m^*}{\pi\hbar^2}k_BT$$

$$n_s = \frac{1}{A}\sum_{\vec{k}}{}' f_0(\vec{k}) = \frac{1}{A}\frac{A}{(2\pi)^2}\cdot 2\int 2\pi k\,dk\, f_0(k)\,dk$$

$$= \frac{1}{\pi}\int f_0(k)\,k\,dk \qquad E = \hbar^2k^2/2m^* \qquad k\,dk = \frac{m^*}{\hbar^2}dE$$

$$= \frac{m^*}{\pi\hbar^2}\int_0^{\infty}\frac{dE}{1+e^{(E-E_F)/k_BT}} \qquad \eta = E/k_BT \quad \eta_F = E_F/k_BT \quad (E_C = 0)$$

$$= \frac{m^*k_BT}{\pi\hbar^2}\int_0^{\infty}\frac{d\eta}{1+e^{\eta-\eta_F}}$$

$$= N_{2D}\,\mathcal{F}_0(\eta_F)\ \checkmark$$

1)

2)

$$\langle E-E_c\rangle = \sum_k{}' (E-E_c) f_0 \Big/ \sum_k f_0$$

$$= \int k\,dk\,(E-E_c) f_0 \Big/ \int k\,dk\,f_0$$

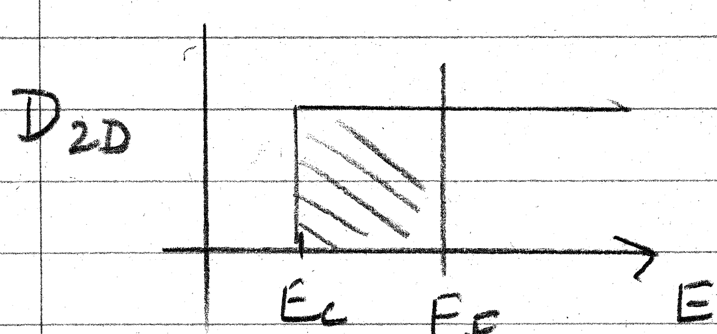
$E-E_c = \dfrac{\hbar^2 k^2}{2m}$

$$= \int_0^{k_F} k\,dk\left(\frac{\hbar^2 k^2}{2m}\right) 1 \Big/ \int_0^{k_F} k\,dk\ 1$$

$$= \frac{\hbar^2}{2m}\int_0^{k_F} k^3\,dk \Big/ \left(k_F^2/2\right)$$

$$= \frac{\hbar^2}{2m^*}\left(\frac{k_F^4}{4} \Big/ k_F^2/2\right)$$

$$= 1/2\left(\frac{\hbar^2 k_F^2}{2m}\right) = \frac{1}{2}\left(E_F - E_c\right) \checkmark$$




2)

3)

$$\langle v_x^+ \rangle = \underbrace{\sum_{k_y, k_x>0}' v_x f_0}_{NUM} \Big/ \underbrace{\sum_{k_y, k_x>0} f_0}_{\to \frac{N_{2D}}{2}\mathcal{F}_0(\eta_F)}$$

$$NUM = \frac{1}{(4\pi^2)} \cdot 2 \int_0^{\infty} \int_{-\pi/2}^{+\pi/2} k\,dk\,d\theta\, v\cos\theta\, f_0$$

$$= \frac{1}{2\pi^2} \int_0^{\infty} \int_{-\pi/2}^{\pi/2} k\,dk\,d\theta\, v\cos\theta\, f_0$$

$$= \frac{1}{2\pi^2} \int k\,dk\, f_0(k) \frac{\hbar k}{m^*} \underbrace{\int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta}_{\to 2}$$

$$= \frac{\hbar}{\pi^2 m^*} \int k^2 dk\, f_0$$

$$\frac{\hbar^2 k^2}{2m^*} = E \quad (E_C = 0)$$

$$k\,dk = \frac{m^*}{\hbar^2} dE$$

$$k^2 dk = \frac{\sqrt{2m^* E}}{\hbar} \cdot \frac{m^*}{\hbar^2} dE$$

$$= \frac{\sqrt{2m^*}}{\pi^2 \hbar^2} \int \frac{\sqrt{E}\, dE}{1 + e^{(E-E_F)/k_B T}}$$

$$= \frac{\sqrt{2m^*}(k_B T)^{3/2}}{\pi^2 \hbar^2} \underbrace{\int \frac{\eta^{1/2}}{1 + e^{\eta - \eta_F}}}_{\frac{\sqrt{\pi}}{2}\mathcal{F}_{1/2}(\eta_F)}$$

3)

3)
$$\langle v_x^+ \rangle = \frac{\frac{\sqrt{2m^*}}{\pi^2\hbar^2}(k_BT)^{3/2}\frac{\sqrt{\pi}}{2}\mathcal{F}_{1/2}(\eta_F)}{\frac{m^*}{\pi\hbar^2}k_BT\,\mathcal{F}_0(\eta_F)}$$

$$= \sqrt{\frac{2k_BT}{\pi m^*}}\,\cancel{\frac{\mathcal{F}_{1/2}(\eta_F)}{\mathcal{F}_0(\eta_F)}}$$

$\rightarrow 1$ non-degenerate

$$\langle v_x^+ \rangle = \sqrt{\frac{2k_BT}{\pi m^*}}$$ same in 2D as in 3D
(& 1D!)

4)

4)

$$J_x = D_n \frac{dn}{dx}$$ defines the diffusion coeff

so we must solve the BTE for $J_x$

$$\frac{\partial f}{\partial x} v_x = -\frac{f_A}{\tau_f} \longrightarrow f_A = -v_x \tau_f \frac{\partial f_S}{\partial x}$$

$$J_x = \frac{1}{A} \sum_{\vec{k}} (-q) v_x f_A = \frac{1}{A} \sum_{\vec{k}} q v_x^2 \tau_f \frac{\partial f_S}{\partial x}$$

$$= \frac{q}{A} \frac{\partial}{\partial x} \sum_{\vec{k}} v_x^2 \tau_f f_S \times \frac{1}{A} \sum_{k} f_S \times \frac{1}{\frac{1}{A} \sum_{k} f_S(k)}$$

$$J_x = q \frac{\partial}{\partial x} \underbrace{\frac{\sum_{k} v_x^2 \tau_f f_S}{\sum f_S}}_{} \cdot \sum_{\vec{k}} f_S$$

spatially uniform for $\tau_f$ uniform
<u>and</u> non-degenerate

$$J_x = q D_n \frac{\partial n}{\partial x} \qquad D_n \equiv \frac{\sum_{\vec{k}} v_x^2 \tau_f f_S}{\sum_{\vec{k}} f_S} = \langle v_x^2 \tau_f \rangle$$

5)

4)

$$\langle v_x^2 \tau_f \rangle = \sum_{\vec{k}} v^2 \cos^2\theta \, \tau_f f_s \Big/ \underbrace{\sum_{\vec{k}} f_0}_{\to N_{2D}}$$

$$\text{NUM} = \frac{1}{4\pi^2} \cdot 2 \underbrace{\int_0^{2\pi} \cos^2\theta \, d\theta}_{\pi} \int_0^{\infty} v^2 \tau_f f_s \, k\,dk$$

$$= \frac{1}{2\pi} \int v^2(E)\, \tau_f(E)\, k\,dk\, f_s(E)$$

recall: $\lambda(E) = \frac{\pi}{2} v(E) \tau_f(E)$

$$\text{NUM} = \frac{1}{2\pi} \cdot \frac{2}{\pi} \cdot \int \underbrace{\lambda(E)}_{\to \lambda_0 (E/k_BT)^r} \cdot \sqrt{\frac{2E}{m^*}} \, \underbrace{k\,dk}_{= \frac{m^*}{\hbar^2} dE} \, f_s(E)$$

$$\text{NUM} = \frac{\lambda_0}{\pi^2} \cdot \sqrt{\frac{2}{m^*}} (k_BT)^{1/2} \frac{m^*}{\hbar^2} \int (E/k_BT)^r (E/k_BT)^{1/2} f_s \, dE$$

$$\langle v_x^2 \tau_f \rangle = \frac{\lambda_0}{\sqrt{\pi}} \cdot \sqrt{\frac{2k_BT}{\pi m^*}} \underbrace{\int \frac{\eta^{r+1/2}}{1+e^{\eta-\eta_F}} d\eta}_{\Gamma(r+3/2)\,\mathcal{F}_{r+1/2}} \Big/ \mathcal{F}_0(\eta_F)$$

6)

4)

$$\langle v_x^2 \tau \rangle = \frac{v_T \lambda_0}{2} \cdot \frac{\Gamma(r + 3/2)}{\Gamma(1/2)} \cancel{\frac{\mathcal{F}_{r+1/2}(\eta_F)}{\mathcal{F}_0(\eta_F)}}$$

$=1$ non-degenerate

$$D_n = v_T \lambda_0 \frac{\Gamma(r + 3/2)}{\Gamma(1/2)}$$

check: $\lambda(E) = \text{constant} \longrightarrow r = 0$

$\Gamma(3/2) = ?$

$\Gamma(p+1) = p\Gamma(p) \quad \Gamma(3/2) = \frac{1}{2}\Gamma(1/2)$

$$D_n = v_T \lambda_0 \times \frac{1}{2} \quad \checkmark$$

7)

5)

solve the BTE for a small $\mathcal{E}$-field at $T=0K$

result: $J_n = q^2 D_{2D} \cdot D_n \mathcal{E}_x$

$\sigma_n = q^2 D_{2D} D_n \qquad D_{2D} = \frac{m^*}{\pi \hbar^2}$

$D_n = v^2(E_F)\tau_f(E_F)/2$

to include diff $q\mathcal{E}_x \rightarrow dF_n/dx$

$J_n = q D_{2D} D_n \cdot dF_n/dx \quad (1)$

need to find $dF_n/dx$

$n = \frac{m^*}{\pi \hbar^2}(F_n - E_c) \qquad F_n = E_c + \frac{n}{D_{2D}}$

$$\frac{dF_n}{dx} = \frac{dE_c}{dx} + \frac{1}{D_{2D}}\frac{dn}{dx}$$

insert in (1)

$$J_n = q^2 D_{2D} D_n \mathcal{E}_x + q D_n \frac{dn}{dx} \quad (2)$$

8) we have the diff. term, work on the drift term

5)

$$q^2 \underline{D_{2D}} \cdot D_n = q^2 \underbrace{D_{2D}(E_F - E_c)}_{n} \cdot \frac{D_n}{(E_F - E_c)}$$

$$= n q \, q \frac{D_n}{(E_F - E_c)}$$

$$\frac{1}{2} m^* v^2 = E_F - E_c$$

$$v^2 = \frac{2(E_F - E_c)}{m^*}$$

$$= n q \, q \frac{v^2 \tau_f(E_F)/2}{(E_F - E_c)}$$

$$= n q \, q \, \tau_f(E_F)/m^*$$

insert in (2)

$$J_n = n q \mu_n \mathcal{E}_x + q D_n \, dn/dx$$

$$\mu = q^2 \frac{\tau_f(E_F)}{m^*} \qquad D_n = \frac{v^2(E_F) \tau_f(E_F)}{m^*}$$

9)

6)

we still have

$$J_{nx} = q^2 D_{2D}(E_F) D_n(E_F) \frac{d(F_n/q)}{dx} \quad (1)$$

$$D_{2D} = \frac{2E}{\pi \hbar^2 v_F^2}$$

$$n = \int_0^{E_F} D(E)\,dE = \frac{E_F^2}{\pi \hbar^2 v_F^2}$$

$$E_F = E_C + \sqrt{\pi \hbar^2 v_F^2}\,\sqrt{n}$$

Dirac point

$$\frac{dF_n}{dx} = \frac{dE_c}{dx} + \sqrt{\pi \hbar^2 v_F^2}\, n^{-1/2}\, dn/dx \quad (2)$$

insert (2) in (1) and show

$$J_{nx} = nq\mu_n \mathcal{E}_x + qD_n dn/dx$$

$$D_n = v_F^2 \tau_f(E_F)/2$$

$$\mu_n = q\tau_f(E_F)/\left((E_F - E_C)/v_F^2\right)$$

10)

7)

start with a solution to the BTE in 2D

$$J_{nx} = \frac{1}{A} \sum_{\vec{k}} q^2 \tau_f v_x^2 \left(-\frac{\partial f_s}{\partial E}\right) \mathcal{E}_x$$

$$= +\frac{1}{k_B T} f_s \text{ (non-degenerate)}$$

$$J_{nx} = \frac{1}{A} \sum_{\vec{k}} q^2 \frac{\tau_f v_x^2}{k_B T} f_s \mathcal{E}_x$$

use $v_x^2 = v_y^2$ $v^2 = v_x^2 + v_y^2$ to show

$$\langle\langle \tau_f \rangle\rangle = \frac{\langle E \tau_f \rangle}{\langle E \rangle} \text{ in 2D}$$

now work out:

$$\langle\langle \tau_f \rangle\rangle = \sum_{\vec{k}} E \tau_f(E) f_s \Big/ \sum_{\vec{k}} E f_s$$

$$= \frac{\int dE\, E \cdot \tau_0 (E/k_B T)^s e^{-E/k_B T}}{\int dE\, E\, e^{-E/k_B T}}$$

$\eta = E/k_B T$

11)

7) $$\langle\langle T_f \rangle\rangle = \tau_0 \frac{\int \eta^{s+1} e^{-\eta} \, d\eta}{\int \eta e^{-\eta}}$$

$$\Gamma(p) \equiv \int_0^{\infty} y^{p-1} e^{-y} \, dy$$

$$\langle\langle T_f \rangle\rangle = \tau_0 \frac{\Gamma(s+2)}{\Gamma(2)}$$

8)

we know the answer $\Sigma(E) = M(E)T(E)$

$$T(E) = \lambda(E)/L$$

just need to work it out.

$$\Sigma(E) = \frac{h}{L^2}\sum_{\vec{k}} v_x^2 \tau_f \delta(E-E_k)$$

$$= \frac{h}{L^2}\cdot\frac{A}{(4\pi)^2}\underbrace{\int_0^{2\pi}\cos^2\theta\, d\theta}_{\pi}\int_0^{\infty} k\,dk\, v^2 \tau_f\, \delta(E-E_k)$$

$$\frac{h}{L^2}\cdot\frac{WL}{4\pi}\,\frac{m^*}{\hbar^2}\int v^2(E_k)\tau_f(E_k)\,\delta(E-E_k)\,dE_k$$

$$= \frac{W}{L}\cdot\frac{m^*}{2}\left(\frac{2}{\pi}\right)\frac{v(E)}{\hbar}\cdot\underbrace{v(E)\tau_f(E)\cdot\frac{\pi}{2}}_{\lambda(E)}$$

$$= \underbrace{\frac{W m^* v}{\pi\hbar}}_{M(E)?}\,\underbrace{\frac{\lambda(E)}{L}}_{T(E)} \checkmark$$

$v = \frac{\hbar k}{m}$ $\qquad M(E) = \frac{W}{(\lambda_B/2)} = \frac{Wk}{\pi} = \frac{Wmv}{\pi\hbar} \checkmark$

13)

9)

from L8

$$S = \frac{k_B}{(-q)}\left( \frac{\left(s + \frac{D+1}{2}\right) \mathcal{F}_{s+(D-1)/2}(\eta_F)}{\mathcal{F}_{s+\frac{(D-3)}{2}}(\eta_F)} - \eta_F \right)$$

$D = 2 \quad s = 0$ (constant mfp)

$$S_{2D} = \frac{k_B}{(-q)}\left( \frac{3\mathcal{F}_{1/2}(\eta_F)}{2\mathcal{F}_{-1/2}(\eta_F)} - \eta_F \right)$$ slide 33

so this is the answer we must get

$$I_0 = \int M(E) \cdot \frac{\lambda_0}{L}\left(-\frac{\partial f_0}{\partial E}\right)$$

$\hookrightarrow$ constant - will divide out num/denom

$$M(E) = W\frac{\sqrt{2m^*E}}{\pi\hbar}$$

$$-\frac{\partial f_0}{\partial E} = +\frac{\partial f_0}{\partial E_F}$$

$$I_0 = \frac{\lambda_0}{L}\frac{\partial}{\partial E_F}\frac{W\sqrt{2m^*}}{\pi\hbar}\int \frac{E^{1/2}dE}{1+e^{(E-E_F)/k_BT}}$$

9)

$$I_0 = \frac{\lambda_0}{L} \frac{W\sqrt{2m^* k_B T}}{\pi \hbar} \frac{\partial}{\partial \eta_F} \underbrace{\int \frac{\eta^{1/2} d\eta}{1+e^{\eta-\eta_F}}}_{\sqrt{\pi}/2\ \mathcal{F}_{1/2}(\eta_F)}$$

$$I_0 = \frac{W\lambda_0}{L} \frac{\sqrt{2m^* k_B T \pi}}{2\pi\hbar} \mathcal{F}_{-1/2}(\eta_F) \checkmark$$

$$I_1 = \frac{\lambda_0}{L} \int (\eta - \eta_F) \frac{W\sqrt{2m^* E}}{\pi \hbar} \left(-\frac{\partial f_0}{\partial E}\right)$$

$$= \frac{\lambda_0 W}{L} \frac{\sqrt{2m k_B T}}{\pi \hbar} \int (\eta - \eta_F) \eta^{1/2} \frac{\partial f_0}{\partial \eta_F}$$

$$(\eta - \eta_F)\eta^{1/2} \frac{\partial f_0}{\partial \eta_F} = \frac{\partial}{\partial \eta_F} \left[ (\eta - \eta_F)\eta^{1/2} f_0 \right] + \eta^{+1/2} f_0$$

$$I_1 = \frac{\lambda_0 W}{L} \frac{\sqrt{2m^* k_B T}}{\pi \hbar} \left[ \frac{\partial}{\partial \eta_F} \underbrace{\int (\eta - \eta_F)\eta^{1/2} f_0 d\eta}_{1} \right. \left. + \underbrace{\int \eta^{1/2} f_0 d\eta}_{2} \right]$$

now work out 1 & 2

15)

② $\int \frac{\eta^{1/2} d\eta}{1+e^{\eta-\eta_F}} = \frac{\sqrt{\pi}}{2} \mathcal{F}_{1/2}(\eta_F)$

① $\int \eta^{3/2} f_0 d\eta - \eta_F \int \eta^{1/2} f_0 d\eta$

$= \Gamma(5/2) \mathcal{F}_{3/2}(\eta_F) - \eta_F \frac{\sqrt{\pi}}{2} \mathcal{F}_{1/2}(\eta_F)$

$\frac{\partial}{\partial \eta_F}$ ① $= \Gamma(5/2) \mathcal{F}_{1/2}(\eta_F) - \frac{\sqrt{\pi}}{2} \mathcal{F}_{1/2} - \eta_F \frac{\sqrt{\pi}}{2} \mathcal{F}_{-1/2}(\eta_F)$

$I_1 = \frac{W\lambda_0}{L} \sqrt{\frac{2mk_BT}{\pi\hbar}} \left\{ \Gamma(5/2) \mathcal{F}_{1/2} - \cancel{\frac{\sqrt{\pi}}{2} \mathcal{F}_{1/2}} - \eta_F \frac{\sqrt{\pi}}{2} \mathcal{F}_{-1/2} + \cancel{\sqrt{\pi/2}\, \mathcal{F}_{1/2}} \right\}$

$\frac{I_1}{I_0} = \frac{2}{\sqrt{\pi}} \Gamma(5/2) \frac{\mathcal{F}_{1/2}}{\mathcal{F}_{-1/2}} - \eta_F$

$\Gamma(5/2) = \frac{3}{4} \Gamma(1/2) = \frac{3}{4} \sqrt{\pi}$

$\frac{I_1}{I_0} = \frac{3}{2} \frac{\mathcal{F}_{1/2}}{\mathcal{F}_{-1/2}} - \eta_F$

$S_{20} = \frac{k_B}{(-q)} \left\{ \frac{3}{2} \frac{\mathcal{F}_{1/2}(\eta_F)}{\mathcal{F}_{-1/2}(\eta_F)} - \eta_F \right\}$ ✓

16)

10)

Do T = 0K first (simple) then do T > 0K

$$G_{CH} = \frac{2q^2}{h} M(E_F) \frac{\lambda(E_F)}{L} = \frac{1}{R_{ch}} \quad (1)$$

$\leftarrow$ known

$$M(E_F) = \frac{W k_F}{\pi}$$

$$n_s = g_v \frac{k_F^2}{2\pi} \longrightarrow k_F = \sqrt{\frac{2\pi n_s}{g_v}}$$

$$M(E_F) = \frac{W}{\pi}\sqrt{\frac{2\pi n_s}{g_v}} \qquad n_s \text{ known } \checkmark \quad (2)$$

insert (2) in (1) and solve for $\lambda(E_F)$

T > 0K

in the ballistic case:

$$G_{ch} = \frac{2q^2}{h}\int M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$= \frac{2q^2}{h} \cdot \sqrt{\frac{2 m k_B T \pi}{2\pi\hbar}}\, g_v W\, \mathcal{F}_{-1/2} \quad (1)$$

17)

10)

from (1) $\langle M \rangle = \frac{\sqrt{2m^* k_B T \pi}}{2\pi\hbar} g_v W \mathcal{F}_{-1/2}(\eta_F)$ (2)

$G_{BALL} = \frac{2q^2}{h} \langle M \rangle$

we can compute $\langle M \rangle$ from (2) if we know $\eta_F$

$n_S = N_{2D} \mathcal{F}_0(\eta_F) = N_{2D} \ln(1 + e^{\eta_F})$

$\eta_F = \ln(e^{n_S/N_{2D}} - 1)$ (3)

use (3) in (2) to find $\langle M \rangle$

<u>non-ballistic</u>

$G_{CH} = \frac{2q^2}{h} \int M(E) \frac{\lambda(E)}{L} \left(-\frac{\partial f_0}{\partial E}\right) dE = \frac{1}{R_{ch}}$ (4)

$= \frac{2q^2}{h} \langle M(E) \rangle \frac{\langle \lambda(E) \rangle}{L}$

18)

10)

$$\langle \lambda(E) \rangle \equiv \frac{\int M(E)\,\lambda(E)\left(-\frac{\partial f_0}{\partial E}\right)dE}{\int M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE}$$

but we extract this from (4)
using the measured Rch
and compute $\langle M \rangle$

19)