Exam 2 ECE-656 Fall 2009

NAME SOLUTIONS

ID

This is a take home exam due at 10:30AM, Monday, November 23, 2009

The exam consists of 10 questions on the attached pages.

1) Show your work for each problem CLEARLY.

2) Mark your answers CLEARLY.

3) Make reasonable assumptions when necessary, but be sure to state them.

DO NOT DISCUSS THIS EXAM WITH ANYONE. IT SHOULD BE YOUR WORK AND YOUR WORK ALONE.

When you hand in your exam, attach this sheet as a cover sheet stapled to your work.

You must also sign the following statement.

I attest that the attached work for Exam 2, ECE-656, Fall 2009 is my work and my work alone. I have not discussed this exam with anyone and received no help with the exam.

SIGNED:

DATE:

ECE-656 Take Home Exam 2: Fall 2009

1) In HW 16, I asked you to show that the procedure used in L16 to solve the Boltzmann Transport Equation for small magnetic fields would not work for graphene. The solution that I provided was NOT CORRECT – the procedure of L16 may work (but there may also be a better way to solve the problem). Explain why my solution was not correct. Hint: Examine the HW16 solution for question 2).

Extra credit if you can solve the BTE for graphene in the presence of a z-directed B-field.

2) It is commonly stated that for "short range scattering" in graphene, the conductivity displays the unusual characteristic of being independent of the carrier density. The same also applies to acoustic phonon scattering at room temperature where it is elastic. Provide a simple, physical **explanation** for this effect. (**Hint:** Imagine yourself explaining this to a student who has not taken ECE-656.)

3) In L8, we derived the heat current by arguing that electrons at energy, E, needed to absorb $\left(E - E_F\right)$ of heat from the contact in order to flow across the device. Using this idea, define the appropriate $\phi\left(p\right)$ and derive a balance equation for the heat flux in 1D (you may assume parabolic bands). Compare your result to that obtained in L8. Also check the result of L28 to see if the Kelvin relation is satisfied.

4) When deriving balance equations in 3D, we obtained a tensor

$$W_{ij} = \frac{1}{\Omega}\sum_{\vec{p}} \frac{p_i \upsilon_j}{2} f\left(\vec{r},\vec{p},t\right)$$

In general:

$$W_{ij} = \begin{bmatrix} W_{xx} & W_{xy} & W_{xz} \\ W_{yx} & W_{yy} & W_{yz} \\ W_{zx} & W_{zy} & W_{zz} \end{bmatrix}$$

Evaluate this tensor in equilibrium for a non-degenerate semiconductor and show that:

$$W_{ij}^0 = \frac{W_0}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What is W_0?

5) Assume scattering by optical phonon <u>emission</u> as given by the expression in L24:

$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\left(\frac{\hbar D_0^2}{2\rho\omega_0}\right)(N_0+1)\frac{D_{3D}(E-\hbar\omega_0)}{2}$$

Review L19 and then derive an expression for the microscopic <u>energy flux relaxation rate</u> (i.e. what I called the out-scattering rate in L19).

6) The mobility of electrons in intrinsic silicon at room temperature is about 1360 cm^2/V-s. Compute the average mean-free-path for backscattering. (**Hint:** You may find it useful to refer to L20, but remember that this problem is a 3D one - not 2D.)

7) In L27, we worked out the scattering rate for ADP intra-scattering of electrons in graphene. Follow a similar approach and derive an expression for the ODP inter-valley scattering rate for electrons in graphene.

8) In HW 26, we worked out the ODP scattering rate for 2D electrons and found:

$$\left.\frac{1}{\tau_{n,n'}}\right)^{a,e} = \frac{\pi}{\hbar}\frac{D_0^2\hbar}{\rho\omega_0}\left(N_0+\frac{1}{2}\mp\frac{1}{2}\right)\frac{D_{2D}(E\pm\hbar\omega_0)}{2}\frac{(2+\delta_{n,n'})}{2}$$

Let $\hbar\omega_0 = 1.1k_BT$ and plot the absorption and emission scattering rates vs. energy for an electron in subband 1 (include 2 subbands). (**Hint:** Your plot should look something like the one in Sec. 3 of L26.)

9) Derive an energy balance equation for graphene, and then simplify it to terminate the balance equation hierarchy.

10) Use the balance equation approach and derive a current equation for a semiconductor nanowire. You may assume that only 1 subband is occupied, but DO NOT ASSUME PARABOLIC ENERGY BANDS.

Exam 2 Solutions

1) Following the method of L16, it is necessary to evaluate:

$$\vec{\nabla}_p(\vec{v}\cdot\vec{\mathcal{E}})$$

the HW16 solution says this $=0$, but this is NOT correct.

$$\vec{\nabla}_p(\vec{v}\cdot\vec{\mathcal{E}}) = \frac{\partial}{\partial p_x}\left(v_x\mathcal{E}_x + v_y\mathcal{E}_y\right)\hat{x}$$

$$+\frac{\partial}{\partial p_y}\left(v_x\mathcal{E}_x + v_y\mathcal{E}_y\right)\hat{y}$$

consider the component due to $\mathcal{E}_x$

$$= \frac{\partial}{\partial p_x}(v_x\mathcal{E}_x)\hat{x} + \frac{\partial}{\partial p_y}(v_x\mathcal{E}_x)\hat{y} \qquad (1)$$

as shown in the HW16 solution

$$v_x = p_x/(E/v_F^2) \qquad v_y = p_y/(E/v_F^2) \qquad (2)$$

$$\frac{\partial}{\partial p_x}\left(p_x \cdot v_F^2 E^{-1}\right) = \frac{1}{(E/v_F^2)} - \frac{p_x v_F^2}{E^2}\cdot v_x \qquad (3)$$

1) cont

$$\frac{\partial}{\partial p_x}(v_x \mathcal{E}_x) = \left(\frac{1}{(E/v_F^2)} - \frac{v_F^2}{E^2}\, p_x v_x \right) \mathcal{E}_x$$

use (2)

$$p_x v_x = p_x^2 \cdot v_F^2 / E$$

$$\frac{\partial}{p_x}(v_x \mathcal{E}_x) = \left(\frac{1}{(E/v_F^2)} - \frac{v_F^4}{E^3}\, p_x^2 \right) \mathcal{E}_x$$

the second term in (1)

$$\frac{\partial}{\partial p_y}(v_x \mathcal{E}_x) = -\frac{p_x v_F^2}{E^2}\, v_y$$

use (2)

$$p_x v_y = p_x p_y v_F^2 / E$$

$$\frac{\partial}{\partial p_y}(v_x \mathcal{E}_x) = -\frac{v_F^4}{E^3}\, p_x p_y$$

so

$$\frac{\partial}{\partial p_x}(v_x \mathcal{E}_x)\hat{x} + \frac{\partial}{\partial p_y}(v_x \mathcal{E}_x)\hat{y} = \frac{1}{(E/v_F^2)}\, \mathcal{E}_x \hat{x}$$

$$- \frac{v_F^4}{E^3}\, p_x^2 \mathcal{E}_x \hat{x} - \frac{v_F^4}{E^3}\, p_x p_y \mathcal{E}_x \hat{y}$$

2)

1) continued

$$\vec{\nabla}_p(\vec{v}\cdot\vec{\mathcal{E}}) = (\qquad)\hat{x} + (\qquad)\hat{y}$$

we have shown that the term due to $\mathcal{E}_x \neq 0$
similarly, the term due to $\mathcal{E}_y \neq 0$.

so $\boxed{\vec{\nabla}_p(\vec{v}\cdot\vec{\mathcal{E}}) \neq 0 \quad \text{QED}}$

for graphene, it may be easier to solve the BTE exactly, for any strength of $\vec{B}$.

see:

N.M.R. Peres, J.M.B. Lopes dos Santos, "Phenomenological study of the electronic transport coefficients of graphene," Phys. Rev. B, 76, 073412, 2007.

see eqs. 28 - 34.

3)

2)

606 students understand $\sigma = n_s q \mu_n$ (1)

consider $T = 0K$, which is a good approx for graphene.

$$n_s \sim \int_0^{E_F} D(E)\, dE$$

$$\mu_n = q\,\tau(E_F)/m^* \qquad \tau(E_F) \sim 1/D(E_F)$$

case i) parabolic bands $D_{2D} = \text{const}$ $\tau \sim \text{const}$

$n_s \sim E_F$

$$\sigma \sim E_F/m^*$$

case ii) graphene $D_{2D} \sim E$

$n_s \sim E_F^2$

$\tau \sim 1/E_F$

$$\sigma \sim E_F/m^*$$

for parabolic bands, $m^* \simeq$ const $\sigma \sim E_F \sim n_s$ ✓

for graphene, $m^* = E_F/v_F^2$

$\sigma \sim E_F/E_F \sim$ const independent of E_F or n_s! This occurs because of the unusual m^* $(E(k))$ for graphene.

4)

3)

$\underline{\Phi(p) = (E - E_F)\, v_x}$ $\qquad F_n \simeq E_F$ near-equilibrium

$\Phi(p) = (E(p) + E_C - E_F)\, v_x$

a) $n_\Phi = \frac{1}{L} \sum_p \Phi(p) f = I_Q$ ✓

b) $F_\Phi = \frac{1}{L} \sum_p \left[E(p) + E_C - E_F\right] v_x \cdot v_x\, f$

$= \frac{1}{L} \sum_p E(p)\, v_x^2 f + \frac{1}{L}(E_C - E_F) \sum_p v_x^2 f$

$= \frac{2}{m^*} \left\{ \underbrace{\frac{1}{L} \sum_p E^2(p) f}_{n_L \langle E_k^2 \rangle} + (E_C - E_F) \underbrace{\frac{1}{L} \sum_{\vec{p}} E(p) f}_{n_L \langle E_k \rangle} \right\}$

E_k = kinetic energy per electron
$= E(p)$

$F_\Phi = \frac{2}{m^*} n_L \left\{ \langle E_k^2 \rangle + (E_C - E_F) \langle E_k \rangle \right\}$ ✓

5)

3)

c) $G_\Phi = -q\,\mathcal{E}_x \frac{1}{L}\sum_p \underbrace{\frac{\partial}{\partial p_x}\left(E(p)+E_c-E_F\right)v_x}\, f$

$$\frac{\partial}{\partial p_x}\left(E(p)v_x + (E_c-E_F)v_x\right)$$

$$= \left(v_x^2 + \frac{E(p)}{m^*} + \frac{(E_c-E_F)}{m^*}\right)$$

$$= \frac{2}{m^*}E(p) + \frac{E(p)}{m^*} + \frac{(E_c-E_F)}{m^*}$$

$$G_\Phi = -q\,\mathcal{E}_x \cdot \frac{1}{L}\sum_p \left(\frac{3E(p)}{m^*} + \frac{(E_c-E_F)}{m^*}\right) f$$

$$= -q\,\mathcal{E}_x \frac{n_L}{m^*}\left(3\langle E_k\rangle + (E_c-E_F)\right) \checkmark$$

d) $R_\Phi = \frac{I_Q - I_Q^0}{\langle \tau_{I_Q}\rangle} = \frac{I_Q}{\langle \tau_{I_Q}\rangle}$ ✓

So

$$\boxed{\frac{\partial I_Q}{\partial t} = -\frac{\partial}{\partial x}\left\{\frac{2}{m^*}n_L\langle E_k^2\rangle + (E_c-E_F)\langle E_k\rangle\right\} - q\,\mathcal{E}_x\left(3\langle E_k\rangle + (E_c-E_F)\right)\frac{n_L}{m^*} - I_Q/\langle \tau_{I_Q}\rangle}$$

b)

3) <u>now compare with L28</u>

steady-state $\rightarrow \partial/\partial t = 0$
spatially uniform $\rightarrow \partial/\partial x = 0$
$\left.\begin{array}{l}\text{non-degenerate} \\ \text{near-equilibrium}\end{array}\right\} \rightarrow \langle E_k \rangle = k_B T/2$

we will be ignoring temp gradients and will only check to see if $\Pi = TS$

from the previous balance eqn.

$$I_Q = -n_L q \frac{\langle \tau_{IQ} \rangle}{m^*}\left(\frac{3}{2}k_B T + (E_C - E_F)\right)\mathcal{E}_x$$

$$= -n_L q \frac{\langle \tau_m \rangle}{m^*} \cdot \frac{\langle \tau_{IQ} \rangle}{\langle \tau_m \rangle} \cdot \left(\frac{3}{2} - \frac{(E_F - E_C)}{k_B T}\right) \cdot k_B T \, \mathcal{E}_x$$

$$= \left(\frac{k_B}{-q}\right) \frac{n_L q^2 \langle \tau_m \rangle}{m^*} \frac{\langle \tau_{IQ} \rangle}{\langle \tau_m \rangle} \left(\frac{3}{2} - \eta_F\right) T \, \mathcal{E}_x$$

$$= \left(\frac{k_B}{-q}\right) \sigma_n \mathcal{E}_x \frac{\langle \tau_{IQ} \rangle}{\langle \tau_m \rangle} \left(\frac{3}{2} - \eta_F\right) T$$

$\sigma_n \mathcal{E}_x = I_n$ for the assumed uniform cond.

7)

3)

so we get

$$I_Q = \Pi I_n \quad \text{(we have ignored the gradient)}$$

$$\Pi = T\left\{\left(\frac{k_B}{-q}\right)\frac{\langle \tau_{IQ}\rangle}{\langle \tau_m\rangle}\left(\frac{3}{2}-\eta_F\right)\right\} \quad (1)$$

is $\{\ \} = S$?

the RTA answer is in L8

$S_{1D} = \left(\frac{k_B}{-q}\right)\left(s+1-\eta_F\right)$ which is different

But we can ask: do the Π and S from the balance eqn. approach satisfy the Kelvin relation?

from L28, $S = \left(\frac{k_B}{-q}\right)\left(\frac{3}{2}-\eta_F\right)$ (2)

* NOTE the typo in L28 $2 \longrightarrow 3/2$!

(1) $\neq$ $T \times$(2) unless $\langle \tau_{IQ}\rangle = \langle \tau_m\rangle$

is this the case?

8)

3) - cont

using the approach of prob. 5), we can show the for isotropic scattering,

$\langle \tau_{I\omega} \rangle = \langle \tau_m \rangle = \langle \tau \rangle$

so

$$S = \left(\frac{k_B}{-q}\right)\left(\frac{3}{2} - \eta_F\right)$$

$$\Pi = T \times \left(\frac{k_B}{-q}\right)\left(\frac{3}{2} - \eta_F\right)$$

the balance eq. results satisfy the Kelvin relation

but we have lost the information about the energy-dependence of scattering.

8A)

4)

diagonal component: $W_{zz} = \frac{1}{\Omega} \sum_{\vec{p}} \frac{p_z v_z}{2} f_0$

f_0 is even in p_z
p_z is odd v_z is odd, so the integral is finite

$$W_{zz} = n_L \langle \frac{p_z v_z}{2} \rangle$$

similar result for W_{xx}, W_{yy}

off-diagonal component: $W_{zx} = \frac{1}{\Omega} \sum_{\vec{p}} p_z v_x f_0$

f is even in p_x, p_y, p_z
p_z odd
v_x odd

$$W_{zx} = \frac{1}{\Omega} \underbrace{\sum_{p_x}}_{=0} \underbrace{\sum_{p_y}}_{\neq 0} \underbrace{\sum_{p_z}}_{=0} p_z v_x f_0 = 0$$

all diagonal components are finite, all off-diagonal components are zero.

9)

so

$$\overleftrightarrow{W} = \frac{n_L}{2}\begin{bmatrix} \langle p_x v_x \rangle & 0 & 0 \\ 0 & \langle p_y v_x \rangle & 0 \\ 0 & 0 & \langle p_z v_z \rangle \end{bmatrix}$$

for parabolic bands:

$$\frac{\langle p_x v_x \rangle}{2} = \frac{\langle p_y v_y \rangle}{2} = \frac{\langle p_z v_z \rangle}{2} = \frac{k_B T}{2}$$

$$W_0 = \frac{p_i v_i}{2} = \frac{3}{2} k_B T$$

$$\text{so} \quad \overleftrightarrow{W} = \frac{3}{2} n \cdot k_B T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ ✓

---

10)

5)

$$\frac{1}{\tau_{FW}} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')\underbrace{\left[\frac{F_W(\vec{p})-F_W(\vec{p}')}{F_W(\vec{p})}\right]}_{=[\cdot]}$$

$$[\cdot] = \frac{E v_z - (E-\hbar\omega_0)v_z'}{E v_z} = 1 - \frac{(E-\hbar\omega_0)}{E}\frac{v_z'}{v_z}$$

$$[\cdot] = 1 - \left(1-\frac{\hbar\omega_0}{E}\right)\frac{v_z'}{v_z} = 1 - \frac{v_z'}{v_z} + \frac{\hbar\omega_0}{E}\frac{v_z'}{v_z}$$

$$= \left(1-\frac{v_z'}{v_z}\right) - \frac{\hbar\omega_0}{E}\left(1-\frac{v_z'}{v_z}\right) + \frac{\hbar\omega_0}{E}$$

assume m^* const

$$[\cdot] = \left(1-\frac{p_z'}{p_z}\right) + \frac{\hbar\omega_0}{E} - \frac{\hbar\omega_0}{E}\left(1-\frac{p_z'}{p_z}\right)$$

$$\frac{1}{\tau_{FW}} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}')\cdot[\cdot]$$

$$= \frac{1}{\tau_m} + \frac{\hbar\omega_0}{E}\frac{1}{\tau} - \frac{\hbar\omega_0}{\tau_m}\frac{1}{\tau_m}$$

11)

5) cont.

for ODP scattering $1/\tau = 1/\tau_m$

$$\frac{1}{\tau_{FW}} = \frac{1}{\tau} + \frac{\hbar\omega_0}{E}\frac{1}{\tau} - \frac{\hbar\omega_0}{E}\frac{1}{\tau} = \frac{1}{\tau}$$

$$\frac{1}{\tau_{FW}} = \frac{1}{\tau}$$ ✓

12)

6)

$$G = \frac{2q^2}{h} \langle M' \rangle \langle \lambda \rangle \frac{A}{L}$$ 3D L20 p.17

$$G = nq\mu \frac{A}{L}$$

from these two eqns $\rightarrow \langle \lambda \rangle = \dfrac{n\mu_n}{(2q/h)\langle M' \rangle}$ (1)

$$\langle M \rangle = \int M'(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

work out (1) in 3D – for non-deg. conditions the result is the same in all dimensions

$$\langle \lambda \rangle = 2(k_B T/q)\mu_n / v_T$$

$$v_T = \sqrt{\frac{2k_B T}{\pi m^*}}$$ use conductivity eff mass $m_c^* = 0.26$

$v_T = 1.08 \times 10^7$ cm/s.

$\langle \lambda \rangle = 63$ nm ✓

one might ask: does any thing change for ellipsoidal band like Si

13)

7) following L27, we have

$$|H_{p'p}|^2 = \frac{1}{A} U_{op}^{a,e} \delta_{p'_{\parallel}, p_{\parallel} \pm \hbar\beta_{\parallel}}$$

$$S(\vec{p}, \vec{p}\,') = \frac{2\pi}{\hbar} \frac{U_{op}^{a,e}}{A} \delta_{p'_{\parallel}, p_{\parallel} \pm \hbar\beta_{\parallel}} \delta(E' - E \mp \hbar\omega_0)$$

note that I am not worrying about the graphene wavefunction (e.g. L27) because we are treating inter-valley scattering and $\theta_{k,k'}$ is set by the initial and final valleys, so its effect can be absorbed in D_o.

also, we will note worry about $\delta_{p'_{\parallel}, p_{\parallel} \pm \hbar\beta_{\parallel}}$

because there is no β dependence in the integral

$$\frac{1}{\tau} = \sum_{\vec{p}\,'} \frac{2\pi}{\hbar} \frac{U_{op}^{a,e}}{A} \delta(E' - E \mp \hbar\omega_0)$$

$$= \frac{2\pi}{\hbar} U_{op}^{a,e} \cdot \underbrace{\frac{1}{A} \sum_{\vec{p}\,' \uparrow} \delta(E' - E \mp \hbar\omega_0)}_{D_f/2}$$

14)

7)

$$\frac{D_f}{2} = \frac{D(E)}{4} = \frac{(E \pm \hbar\omega_0)}{2\pi \hbar^2 v_F^2}$$

where D had a valley deg. of 2, but now we have only 1 final valley

$$W_{op}^{a,e} = \frac{D_0^2 \hbar}{2\rho m \omega_0}\left(N_0 + \frac{1}{2} \mp \frac{1}{2}\right) \quad L24$$

$$\boxed{\frac{1}{\tau} = \frac{D_0^2}{2\rho m \omega_0 (\hbar v_F)^2}\left[N_0(E+\hbar\omega_0) + (N_0+1)(E-\hbar\omega_0)\right]} \checkmark$$

this agrees with

J. Chauhan and J. Guo, Appl. Phys. Lett., 98, 023120, 2009.

8)

$$N_0 = \frac{1}{e^{\hbar\omega_0/kT}-1} = \frac{1}{3-1} = \frac{1}{2}$$

$$\left.\frac{1}{\tau_{nn'}}\right)^{a,e} = \underbrace{\frac{\pi D_0^2 \hbar}{\hbar \rho\omega_0}\left(\frac{m^*}{\pi\hbar^2}\right)}_{\Gamma_0}\left(N_0+\frac{1}{2}\mp\frac{1}{2}\right)\left(\frac{2+\delta_{nn'}}{2}\right)$$

$$= \Gamma_0\left(\frac{1}{2}+\frac{1}{2}\mp\frac{1}{2}\right)\left(\frac{2+\delta_{n,n'}}{2}\right)$$

$$\left.\frac{1}{\tau_{1,1}}\right)^{a} = \Gamma_0\cdot\left(\frac{1}{2}\right)\cdot\frac{3}{2} = \frac{3}{4}\Gamma_0$$

$$\left.\frac{1}{\tau_{1,1}}\right)^{e} = \Gamma_0\left(3/2\right)\cdot 3/2 = \frac{9}{4}\Gamma_0$$

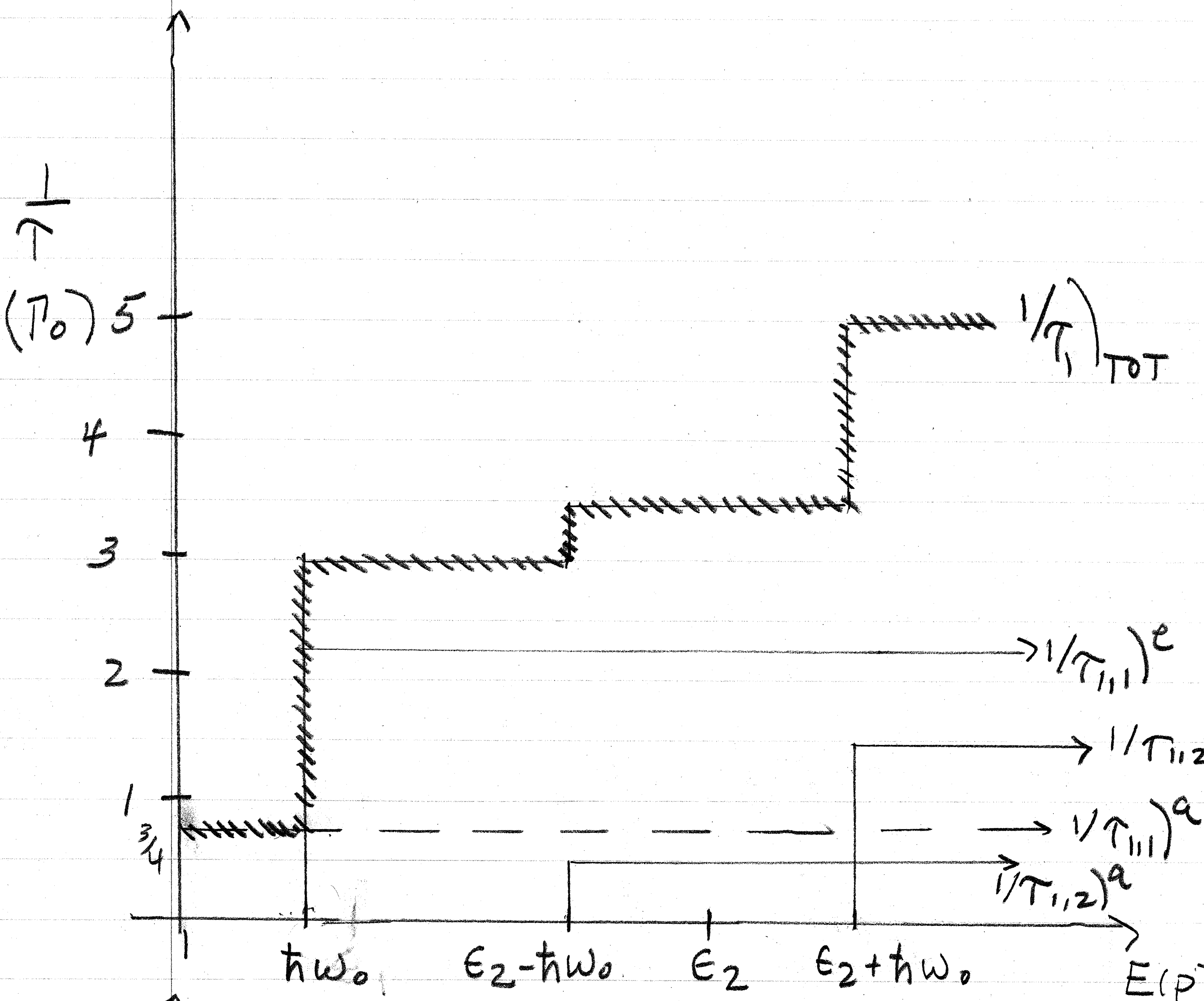
$$\left.\frac{1}{\tau_{1,2}}\right)^{a} = \Gamma_0\left(\frac{1}{2}\right)\cdot 1 = \frac{2}{4}\Gamma_0$$

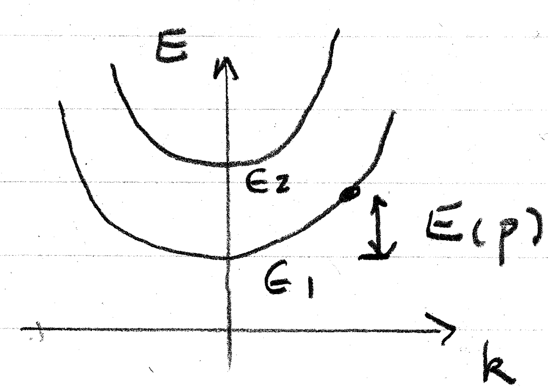
$$\left.\frac{1}{\tau_{1,2}}\right)^{e} = \Gamma_0\left(3/2\right)\cdot 1 = \frac{6}{4}\Gamma_0$$

$$\left.\frac{1}{\tau_1}\right)_{TOT} = \Gamma_0\left(\frac{3}{4}+\frac{9}{4}+\frac{2}{4}+\frac{6}{4}\right) = 5\Gamma_0$$

16)

8) cont





have set the zero of energy at the bottom of the first subband

note that this is the K.E. of electrons in the first subband.

17)

9) $\Phi(p) = E(p) \quad E_C = E_D = 0$

a) $n_\Phi = \frac{1}{A}\sum_{\vec{p}} \Phi(p) f = n_S u = W \quad u = \langle E(p) \rangle$

b) $F_{\Phi_i} = \frac{1}{A}\sum_{\vec{p}} E(\vec{p}) v_i f = F_{W_i}$

c) $G_\Phi = \frac{1}{A}\sum_{p} \frac{\partial E}{\partial p} f \cdot (-q)\vec{\mathcal{E}}$

$$= (-q)\frac{1}{A}\sum_{\vec{p}} \vec{v} f \cdot \vec{\mathcal{E}} = \vec{J}_n \cdot \vec{\mathcal{E}}$$

d) $R_\Phi = (W - W_0)/\langle \tau_W \rangle$

$$\longrightarrow \quad \boxed{\frac{\partial W}{\partial t} = -\nabla \cdot \vec{F}_W + \vec{J}_n \cdot \vec{\mathcal{E}} - \frac{(W - W_0)}{\langle \tau_W \rangle}} \qquad (1)$$

now we need to simplify this

To do so, derive an energy balance eqn. for $\vec{F}_W$ and simplify it.

a) $\phi(p) = E(\vec{p})\vec{v} \neq E(\vec{p}) v_i$

a) $n_\Phi = \vec{F}_W$

18)

b)

$$F_{\phi_j} = \frac{1}{A}\sum_{\vec{p}} E(\vec{p})\, v_i v_j f \equiv X_{ij}$$

c) $$G_\phi = -q\,\mathcal{E}_j \cdot \frac{1}{A}\sum_{\vec{p}} \frac{\partial \phi(\vec{p})}{\partial p_j} f$$

$$= -q\,\mathcal{E}_j \cdot \frac{1}{A}\sum_{\vec{p}} \frac{\partial}{\partial p_j}\left(E(p)\, v_i\right)$$

need to evaluate $\frac{\partial}{\partial p_j}\left(E(p)\, v_i\right)$

$$E = \hbar v_F \sqrt{k_x^2 + k_y^2}$$

$$v_x = \frac{1}{\hbar}\frac{\partial E}{\partial k_x} = v_F \frac{k_x}{\sqrt{k_x^2+k_y^2}} \longrightarrow v_i = \frac{k_i}{(E/\hbar v_F^2)}$$

$$v_i = \frac{p_i}{(E/v_F^2)} \qquad E(p)\, v_i = v_F^2 p_i$$

$$\frac{\partial}{\partial p_j}\left(E(p)\, v_i\right) = v_F^2 \delta_{ij}$$

$$G_\phi = -q\,\mathcal{E}_i \frac{1}{A}\sum_{\vec{p}} v_F^2 f = -q n_s v_F^2 \mathcal{E}_i$$

d) $R_\phi = F_W / \langle \tau_{FW} \rangle$

19)

9) cont

$$\frac{\partial F_{W_i}}{\partial t} = -\frac{\partial}{\partial x_j}\left(X_{ij}\right) - q n_s v_F^2 \mathcal{E}_i - \frac{F_W}{\langle \tau_{FW} \rangle}$$

now simplify: $\partial/\partial t \simeq 0$

$$X_{ij} = \frac{1}{A}\sum_{\vec{p}} E(\vec{p}) v_i v_j f$$

assume diagonal
$X_{xx} = X_{yy}$
$X_{xy} = X_{yx} = 0$

$$X_{xx} = \frac{1}{A}\sum_{\vec{p}} E(\vec{p}) v_x^2 f$$

$v_x = p_x / (E/v_F^2)$
$v_x^2 + v_y^2 = v_F^2$

$$= \frac{1}{A}\sum E(p) f \cdot \frac{v_F^2}{2}$$

near eq. $v_x^2 \simeq v_y^2 = \frac{v_F^2}{2}$

$$= n_s \langle E_k \rangle \cdot v_F^2/2$$

so

$\times \langle \tau_{FW} \rangle$

$$F_{W_i} = -n_s q \langle \tau_{FW} \rangle \cdot v_F^2 \mathcal{E}_i - \frac{\partial}{\partial x_i}\left(\frac{v_F^2}{2} n_s \langle E_k \rangle\right)$$

$$= -\frac{n_s q \langle \tau_{FW} \rangle}{\langle E_k \rangle / v_F^2} \langle E_k \rangle \mathcal{E}_i - \frac{v_F^2}{2} \langle \tau_{FW} \rangle \frac{\partial}{\partial x_i}\left(n_s \langle E_k \rangle\right)$$

20)

9) cont

$$F_{W_i} = -\mu_E W \mathcal{E}_i - D_E \frac{\partial W}{\partial x_i} \qquad (2)$$

$$\mu_E = q\frac{\langle \tau_{FW} \rangle}{\langle m^* \rangle} \qquad \langle m^* \rangle = \langle E_k \rangle / v_F^2$$

$$W = n_S \langle E_k \rangle$$

$$D_E = \frac{v_F^2}{2} \langle \tau_{FW} \rangle$$

might be able to assume $\langle \tau_{FW} \rangle = \langle \tau_m \rangle$

if we insert (2) in (1) we have a <u>closed system</u> – 1 eqn. for W.

21)

10) $\Phi(p) = (-q)v_x$

a) $n_\Phi = \frac{1}{L}\sum_{\vec{p}} (-q)v_x f = I_n$

b) $F_\Phi = \frac{1}{L}\sum_{\vec{p}} (-q)v_x^2 f = (-q)n_L\langle v_x^2\rangle$

NOT $\frac{2}{m^*}\langle E_k\rangle$

c) $G_\Phi = \frac{1}{L}\sum_{\vec{p}} \frac{\partial}{\partial p_x}\left((-q)v_x\right) f \times (-q)\mathcal{E}_x$

$= q^2\mathcal{E}_x \frac{1}{L}\sum_p \frac{\partial}{\partial p_x}\left(\frac{\partial E}{\partial p_x}\right) f$

$= q^2\mathcal{E}_x \sum_p \left(\frac{\partial^2 E}{\partial p_x^2}\right) f = q^2\mathcal{E}_x n_L\langle 1/m^*\rangle$

← inverse eff. mass

d) $R_\Phi = I_n/\langle\tau_v\rangle$

$$\boxed{\frac{\partial I_n}{\partial t} = -\frac{\partial}{\partial x}\left((-q)n_L\langle v_x^2\rangle\right) + q^2 n_L\langle 1/m^*\rangle\mathcal{E}_x - \frac{I_n}{\langle\tau_v\rangle}}$$

steady-state

$I_n = n_L q\mu_n\mathcal{E}_x + q\langle\tau_v\rangle\frac{\partial}{\partial x}\left(n_L\langle v_x^2\rangle\right)$

$\mu_n = q\langle\tau_v\rangle \times \langle 1/m^*\rangle$

22)

10)

assume $\langle v_x^2 \rangle$ is spatially uniform

$$I_n = n_L q \mu_n \mathcal{E}_x + q \underbrace{\langle \tau_v \rangle \langle v_x^2 \rangle}_{\equiv D_n} \frac{\partial n_L}{\partial x}$$

23)