

Fundamentals of Nanoelectronics

ECE495 - Session 4, August 30, 2009

Charging Effect

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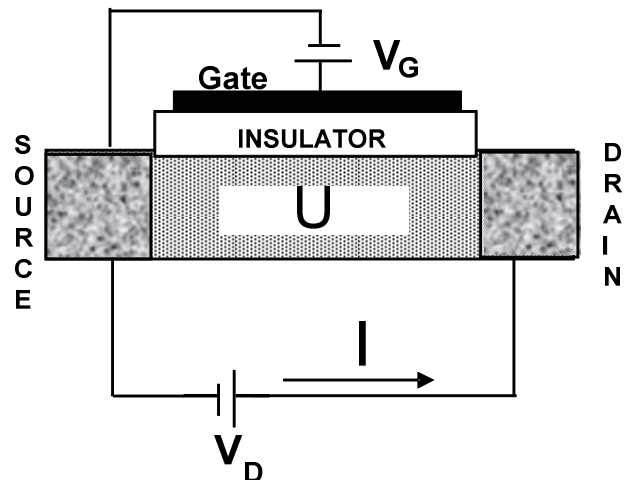
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Current Calculation

$$I = \int \frac{q}{\hbar} dE D(E-U) \frac{\gamma}{2} (f_1(E) - f_2(E)) \quad (I)$$

$$N = \int dE D(E-U) \frac{f_1(E) + f_2(E)}{2} \quad (II)$$

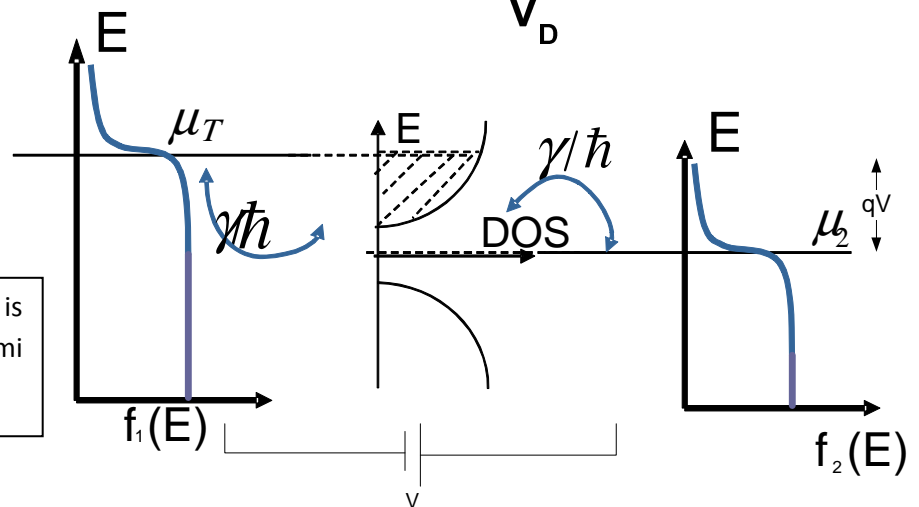
$$U = U_L + U_0(N - N_0) \quad (III)$$



The basic quantities that we want to know are:

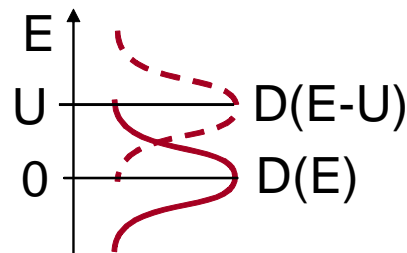
$\frac{\gamma}{2}$ and $D(E)$

The only reason for current flows is the difference between two Fermi functions (f_1 and f_2).



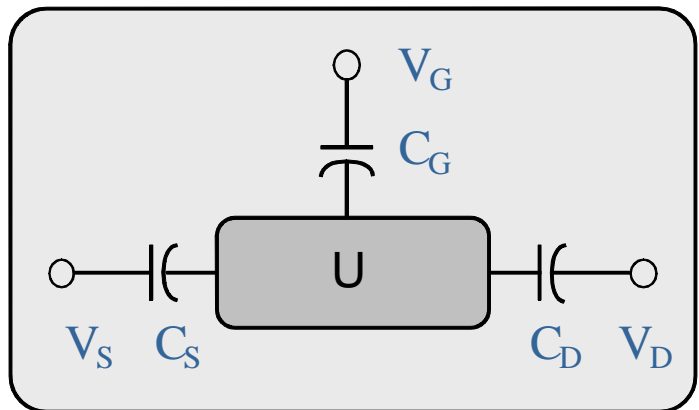
Channel Potential and Charging effect

U in the channel represents the channel's potential. Thinking in a simple way, the effect of the potential U is to move the entire density of states up or down (depending on the sign of U). Notice that the correct expression is $D(E-U)$ and not $D(E+U)$ because the peak of the density of states should appear when its argument is 0.



Due to the channel potential, we have to modify our equation for current in order to take it into account then we will achieve **equation (I)**.

Gate voltage moves levels up and down to control the transistor. Actually, a good transistor is which controls the channel potential from the gate and not from drain or



The figure is a model for calculating the U in the channel. To control the channel potential with gate voltage C_G should be very big then insulator should be very thin and/or the insulator material should be high k dielectric.

Suppose also that in equilibrium we have 100 electrons in the channel. In non-equilibrium one gets in and one gets out, so in average the number of electrons is not anymore 100 but something like 50. So, the potential in the channel will become more positive. And positive potential means the $D(E)$ down a little bit.

$$U = U_L + U_0(N - N_0) \quad \text{The first part } (U_L) \text{ is average potential and the second part is for electron-electron interactions and it is achieved from Poisson equation } (\nabla^2 \phi = -\frac{\rho}{\epsilon_0}).$$

U_L is Laplace potential ($\nabla^2 \phi = 0$ Laplace Equation)

U_0 is average potential per electron

N is number of electron in the channel

N_0 (or N_{eq}) is number of electron in the channel in equilibrium

It is important how big U_0 is. It is called **single electron charging energy** and it tells us how much channel's potential changes when one electron is added to it.

$$U_0 = \frac{q^2}{2C} = \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19} \text{ coulomb}}{2 \times 10^{-15} \text{ F}} \approx 10^{-4} \text{ eV for adding one electron}$$

$k_B T = 25 \text{ meV} \Rightarrow U_0 \ll k_B T$ then for one electron it is not a big effect.

Single electron charging effect is something that can arise in small structures under certain conditions. Under such situations, what we've done thus far will not capture the right physics. One has to consider what's called **Coulomb Blockade** regime of operation.

Electron-electron interaction

Electron-electron interaction is a very important issue and it is not solved totally yet. To solve it the most usual approach is self-consistent. The approach for solving equations II and III is as below:

- I) Assume an U (by guess),
- II) Find a N using equation (II),
- III) Find a new U using equation (III),
- IV) Do step II and III until converge,
- V) Using equation (I) calculate current.

Number of Electron

$N = 2 \int dE \cdot D(E) f(E)$ this is for equilibrium, and 2 is multiplied for spin.

$N = 2 \int dE \cdot D_{\varepsilon}(E) \left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$ this is for non-equilibrium and here we assume escape rate γ is different for both sides (γ_1 and γ_2).

$N = 2 \int dE \cdot D_{\varepsilon}(E) \left[\frac{f_1(E) + f_2(E)}{2} \right]$ this is for non-equilibrium and here we assume escape rate γ is equal for both sides ($\gamma = \gamma_1 = \gamma_2$).