Fundamentals of Nanoelectronics

ECE495 - Session 14, Sept 25, 2009

Law of Equilibrium

Ref: chapter 3.4

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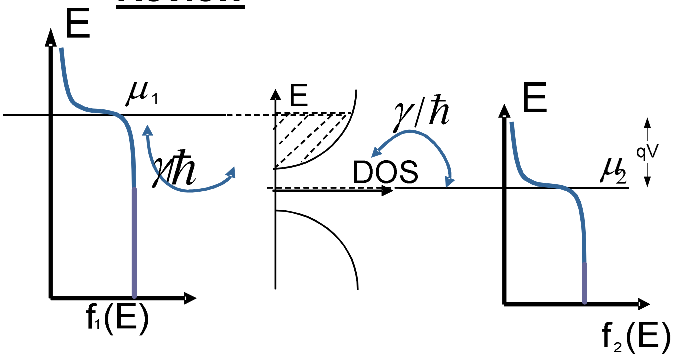
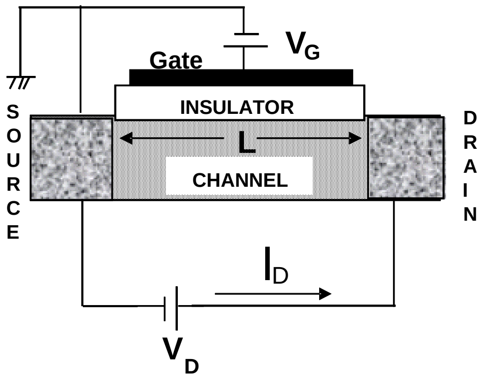
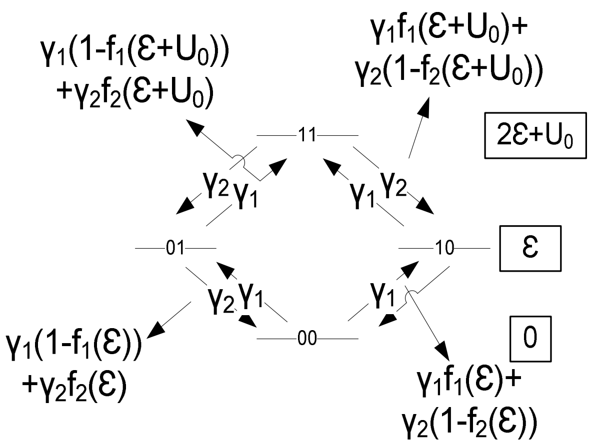
Review

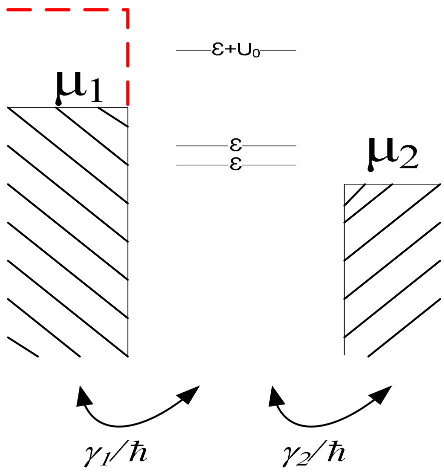




Multi-electron picture



One-electron picture



$$P_\alpha = \frac{1}{Z} e^{-(E_\alpha - \mu N_\alpha)/k_B T}$$

In equilibrium:

$$P_{00} = \frac{1}{Z}$$

$P_{01}=P_{10}=\frac{1}{Z}e^{-(\varepsilon-\mu)/k_BT}=\frac{1}{Z}e^{-x}$ where $x=\frac{\varepsilon-\mu}{k_BT}$, $N_\alpha = 1$ and $E_\alpha = \varepsilon$

$P_{11}=\frac{1}{Z}e^{-(2\varepsilon-U_0-2\mu)/k_BT}=\frac{1}{Z}e^{-2x}e^{-U_0/k_BT}$ where $x=\frac{\varepsilon-\mu}{k_BT}$, $N_\alpha = 2$ and $E_\alpha = 2\varepsilon - U_0$

Non-interacting Mode

$$\frac{U_0}{k_BT}=0 \xrightarrow{yields} P_{00}+P_{10}+P_{01}+P_{11}=1 \Rightarrow \frac{1}{Z}(1+2e^{-x}+e^{-2x})=1 \Rightarrow Z=(1+e^{-x})^2$$

$$\Rightarrow P_{00}=\frac{1}{(1+e^{-x})^2}=\frac{e^x}{1+e^x}\times\frac{e^x}{1+e^x}\equiv(1-f)\times(1-f)$$ f stands on Fermi function

$$\Rightarrow P_{01}=P_{10}=\frac{e^{-x}}{(1+e^{-x})^2}=\frac{e^{-x}}{1+e^{-x}}\times\frac{1}{1+e^{-x}}=\frac{1}{1+e^x}\times\frac{e^x}{1+e^x}\equiv f\times(1-f)$$

$$\Rightarrow P_{11}=\frac{e^{-2x}}{(1+e^{-x})^2}=\frac{e^{-x}}{1+e^{-x}}\times\frac{e^{-x}}{1+e^{-x}}=\frac{1}{1+e^x}\times\frac{1}{1+e^x}\equiv f\times f$$

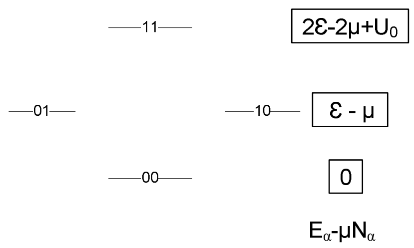
If we assume $P_{11}=0$

$$P_{00}+P_{10}+P_{01}=1 \Rightarrow \frac{1}{Z}(1+e^{-x}+e^{-x})=1 \Rightarrow Z=1+2e^{-x}$$

$$\Rightarrow P_{00}=\frac{1}{Z}=\frac{1}{1+2e^{-x}}$$

$$\Rightarrow P_{01}=P_{10}=\frac{1}{Z}e^{-x}=\frac{e^{-x}}{1+2e^{-x}}=\frac{1}{2+e^x}$$

Average Number of Electron: $<N>=P_{01}+P_{10}=\frac{2}{2+e^x}=\frac{1}{1+\frac{1}{2}e^x}$



If T=0 => $P_\alpha = 1$ for minimum $E_\alpha-\mu N_\alpha$

$P_\alpha = 0$ for others