Fundamentals of Nanoelectronics

ECE495 - Session 23, Oct 19, 2009

Density of States II

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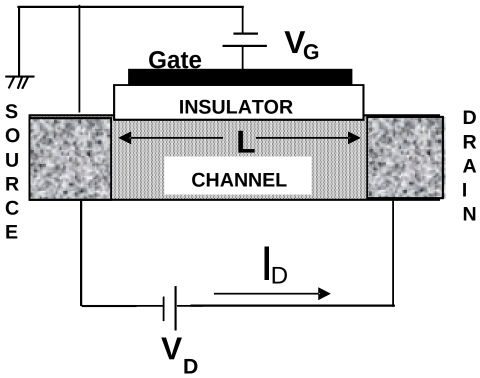
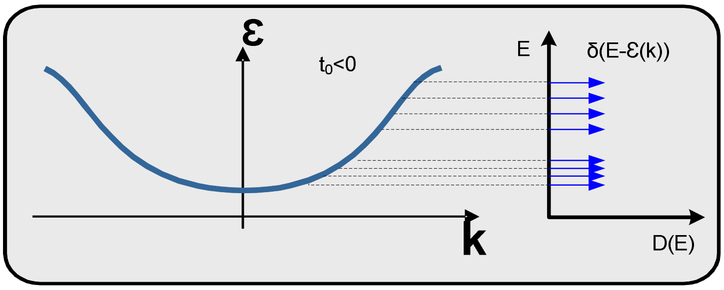
Review

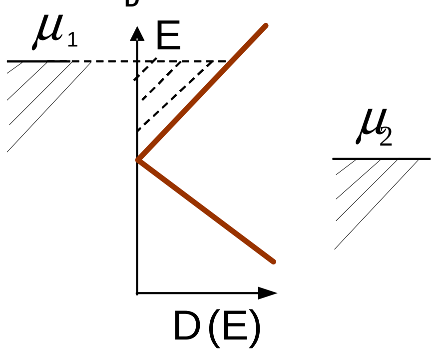
$$h(\vec{k}) = \sum_m [H_{nm}] e^{i\vec{k}(\vec{d}_m - \vec{d}_n)}$$

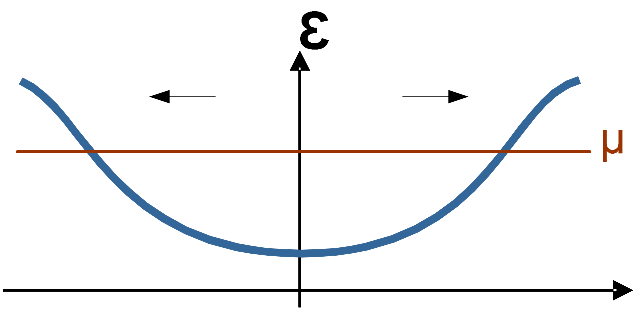
$$D(E) = \sum_{\vec{k}} \delta(E - \varepsilon(\vec{k}))$$

$$M(E) = \sum_{\vec{k}} \delta(E - \varepsilon(\vec{k})) \frac{\pi \hbar v_z}{L}$$







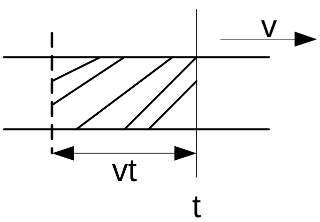


$$N = \int dE D(E) f(E)$$

N: Total number of electrons

$$n = \frac{N}{L} = \frac{1}{L} \int dE D(E) f(E)$$

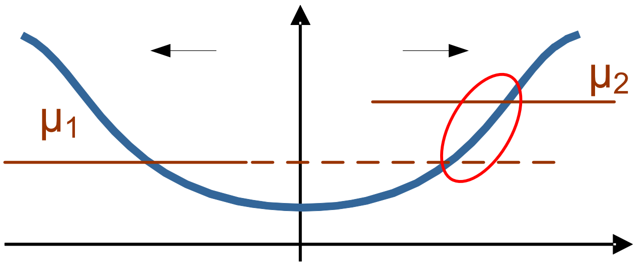
$$v = \frac{1}{\hbar} \frac{d\varepsilon}{dk}$$



$$I = q \frac{nvt}{t} = qnv$$

$$I = \frac{q}{L} \int dE D(E) v(E) f(E) = 0$$

Due to equilibrium we will not have any current!



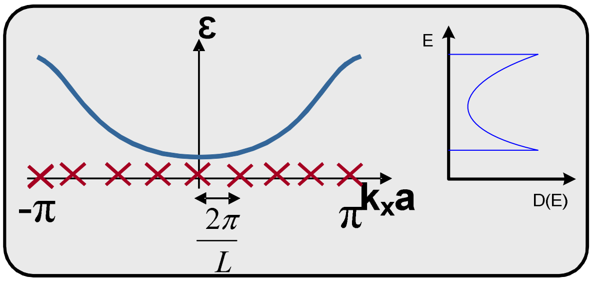
$$\Rightarrow I = \frac{q}{L}\int dE\,\frac{D(E)}{2}v(E)(f_1(E) - f_2(E))$$

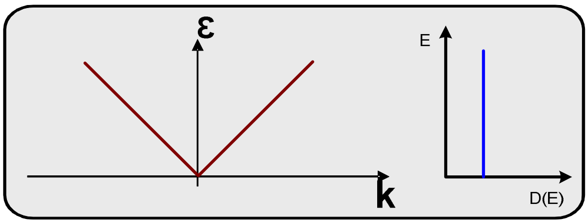
D(E) is divided by 2 because of half side

$$I = \frac{q}{h}\int dE\,\underbrace{\frac{hD(E)v(E)}{2L}}_{M(E)}(f_1(E) - f_2(E))$$

M(E) -i.e. number of modes- is a dimensionless number. $I = \frac{q}{h}\int dE M(E)(f_1 - f_2)$

Density of states for large 1-D devices





$$D(E) = \sum_{\vec{k}} \delta(E - \varepsilon(\vec{k}))$$

If length of device is big we can change summation to integral.

$$D(E) = 2\int_0^\infty \frac{dkL}{2\pi}\delta\left(E - \varepsilon(\vec{k})\right)$$

$$= 2\frac{L}{2\pi}\int_0^\infty d\varepsilon\,\delta\left(E - \varepsilon(\vec{k})\right)\frac{dk}{d\varepsilon} = \frac{L}{\pi\hbar v(E)}$$

If $\varepsilon(\vec{k}) = \frac{\hbar^2 k^2}{2m} \Rightarrow v = \frac{1}{\hbar}\frac{d\varepsilon}{dk} = \frac{\hbar k}{m} \sim\sqrt{\varepsilon}$

Two Dimensional Density of State

$$D(E) = \sum_{\vec{k}} \delta(E - \varepsilon(\vec{k})) = \iint \frac{dk_x dk_y}{\frac{2\pi}{L}\frac{2\pi}{W}}\delta\left(E - \varepsilon(\vec{k})\right) = \frac{WL}{4\pi^2}\iint \delta\left(E - \varepsilon(\vec{k})\right)\underbrace{dk_x dk_y}_{kdkd\theta}$$

$$= \frac{WL}{4\pi^2}\int_0^{2\pi} d\theta \int_0^\infty \delta\left(E - \varepsilon(\vec{k})\right) kdk$$

$$= \frac{WL}{2\pi}\int_0^\infty \delta\left(E - \varepsilon(\vec{k})\right) kdk = \frac{WL}{2\pi}\int_0^\infty \delta\left(E - \varepsilon(\vec{k})\right) kdk$$

If $\varepsilon(\vec{k}) = \frac{\hbar^2 k^2}{2m} \Rightarrow d\varepsilon = \frac{\hbar^2}{m}kdk$ then $D(E) = \frac{mLW}{2\pi\hbar^2}$