Fundamentals of Nanoelectronics

ECE495 - Session 37, Nov 30, 2009

Pauli Spine Matrices I

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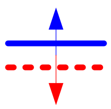
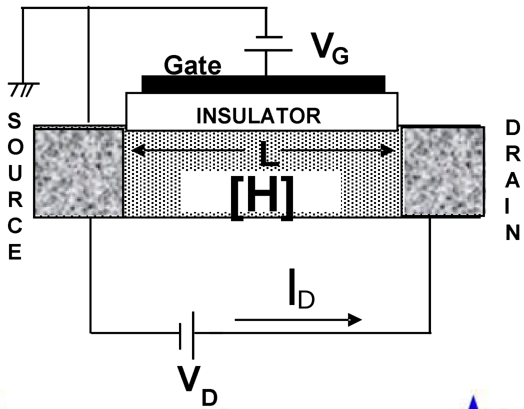
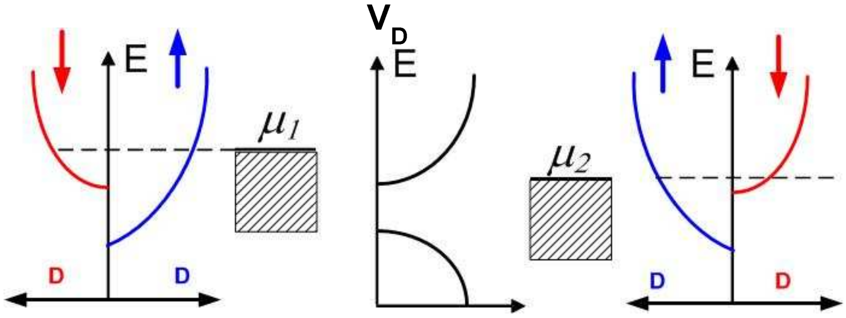
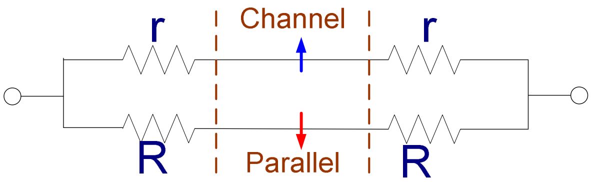
<u>Review</u>

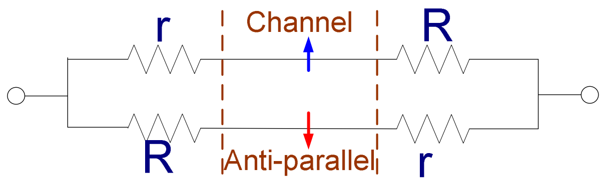
Spin-valve



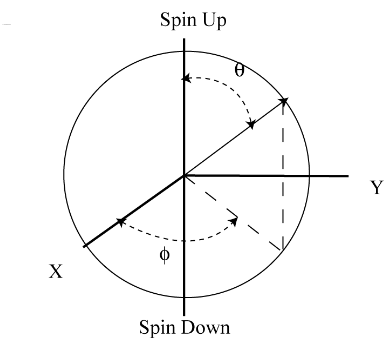








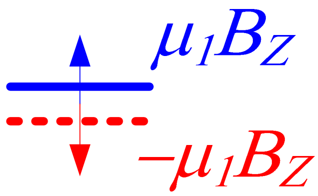
For spin we use ***spinor*** instead of vector. A spinor is a set of two complex numbers:



$$\underset{Vector}{\begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}} \Leftrightarrow \underset{Spinor}{\begin{pmatrix} \cos\frac{\theta}{2}e^{-i\varphi/2} \\ \sin\frac{\theta}{2}e^{+i\varphi/2} \end{pmatrix}} \equiv \underset{Spinor}{\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\varphi} \end{pmatrix}}$$

Recall, when deriving current through a small structure, we first obtain a Hamiltonian [H]. Usually, however, the eigenenergies of [H] represent two degenerate spin levels.

The proper Hamiltonian for degenerate spin levels is twice as big: $\begin{bmatrix} H & O \\ O & H \end{bmatrix}$ with no coupling between the spin up and spin down portions. If we assume [H]= [ε] then for two spins proper Hamiltonian would be as $H = \begin{bmatrix} \varepsilon & O \\ O & \varepsilon \end{bmatrix}$.



$$\begin{array}{c} \\ +\hat{z} \\ -\hat{z} \end{array} \begin{array}{c} \begin{array}{cc} +\hat{z} & \quad -\hat{z} \end{array} \\ \begin{bmatrix} \mu_B B_z & 0 \\ 0 & -\mu_B B_z \end{bmatrix} \end{array}$$

We assumed ε=0.

If magnet is in x direction for channel: $\begin{array}{c} \\ +\hat{x} \\ -\hat{x} \end{array} \begin{array}{c} \begin{array}{cc} +\hat{x} & \quad -\hat{x} \end{array} \\ \begin{bmatrix} \mu_B B_x & 0 \\ 0 & -\mu_B B_x \end{bmatrix} \end{array}$

To achieve this matrix in z direction basis:

$$\begin{bmatrix} \mu_B B_x & 0 \\ 0 & -\mu_B B_x \end{bmatrix} = \mu_B B_x \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\hat{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \; then\; \theta = \frac{\pi}{2} \; and \; \varphi = 0 \Rightarrow \begin{array}{c} +\hat{z} \\ -\hat{z} \end{array} \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\varphi/2} \\ \sin\frac{\theta}{2} e^{+i\varphi/2} \end{pmatrix} \Rightarrow \begin{array}{c} \\ +\hat{z} \\ -\hat{z} \end{array} \begin{array}{c} \begin{array}{cc} +\hat{x} & \quad -\hat{x} \end{array} \\ \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \end{array}$$

$$1/\sqrt{2} \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}}_{U} \mu_B B_x \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} 1/\sqrt{2} \underbrace{\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}}_{U^+} = \mu_B B_x \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

If magnet is in x direction for channel: $\begin{array}{c} \\ +\hat{y} \\ -\hat{y} \end{array} \begin{array}{c} \begin{array}{cc} +\hat{y} & \quad -\hat{y} \end{array} \\ \begin{bmatrix} \mu_B B_y & 0 \\ 0 & -\mu_B B_y \end{bmatrix} \end{array}$

To achieve this matrix in z direction basis:

$$\begin{bmatrix} \mu_B B_y & 0 \\ 0 & -\mu_B B_y \end{bmatrix} = \mu_B B_y \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\hat{y} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \; then\; \theta = \frac{\pi}{2} \; and \; \varphi = \frac{\pi}{2} \Rightarrow \begin{array}{c} +\hat{z} \\ -\hat{z} \end{array} \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} e^{i\varphi} \end{pmatrix} \Rightarrow \begin{array}{c} \\ +\hat{z} \\ -\hat{z} \end{array} \begin{array}{c} \begin{array}{cc} +\hat{y} & \quad -\hat{y} \end{array} \\ \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \end{array}$$

$$\underbrace{\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}}_{U} \mu_B B_y \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}}_{U^+} = \mu_B B_y \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

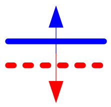
Pauli Spin Matrices

For magnet $\mu_B B_z$ in z direction: $\sigma_z = \begin{matrix} & +\hat{z} & -\hat{z} \\ +\hat{z} & 1 & 0 \\ -\hat{z} & 0 & -1 \end{matrix}$, i.e. $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

For magnet $\mu_B B_y$ in y direction: $\sigma_y = \begin{matrix} & +\hat{z} & -\hat{z} \\ +\hat{z} & 0 & -i \\ -\hat{z} & i & 0 \end{matrix}$, i.e. $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

For magnet $\mu_B B_x$ in y direction: $\sigma_x = \begin{matrix} & +\hat{z} & -\hat{z} \\ +\hat{z} & 0 & 1 \\ -\hat{z} & 1 & 0 \end{matrix}$, i.e. $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

And For arbitrary direction: $\mu_B \begin{bmatrix} B_z & B_x - iB_y \\ B_x + iB_y & -B_z \end{bmatrix}$



With magnet $\Gamma_1 = \begin{bmatrix} \gamma_\uparrow & 0 \\ 0 & \gamma_\downarrow \end{bmatrix}$

Without magnet $\Gamma_1 = \begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix} = \gamma \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$