Fundamentals of Nanoelectronics

ECE495 - Session 38, Dec 2, 2009

Pauli Spin Matrices II

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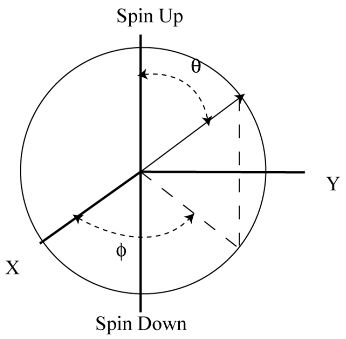
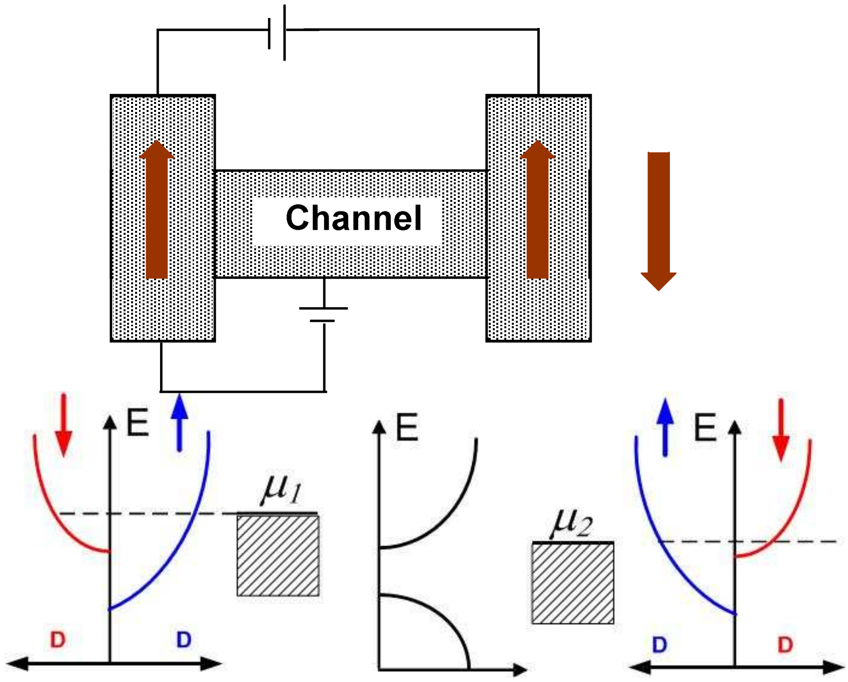
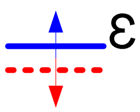
Review





$$\begin{matrix} S_x \\ S_y \\ S_z \end{matrix} \begin{Bmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{Bmatrix} \Leftrightarrow \begin{matrix} +\hat{z} \\ -\hat{z} \end{matrix} \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\varphi/2} \\ \sin\frac{\theta}{2} e^{+i\varphi/2} \end{pmatrix}$$

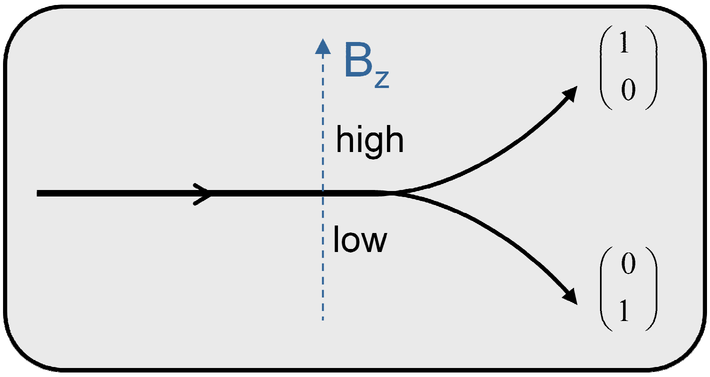


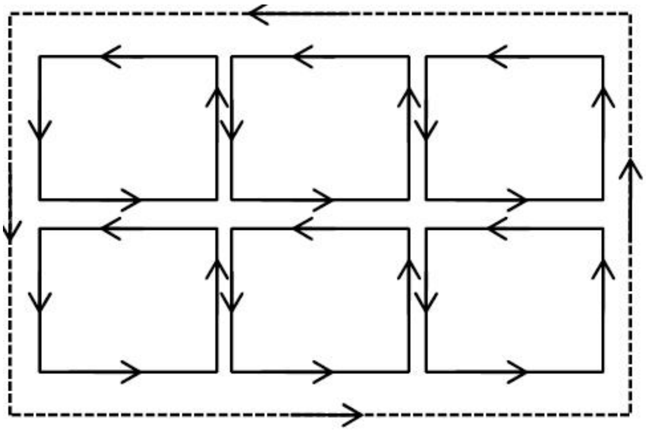
When a magnetic field is applied, say in the z direction, up and down spin levels do not remain degenerate so we add $\begin{bmatrix} \varepsilon + \mu_B B_Z & 0 \\ 0 & \varepsilon - \mu_B B_Z \end{bmatrix}$ where μ_B is the Bohr magneton constant $\frac{q\hbar}{2m} \approx 10^{-23} A - m^2$.

Stern-Gerlach Experiment

In the Stern-Gerlach experiment a beam of electrons is injected into an inhomogeneous magnetic field, due to spin interaction the beam splits after entering the magnetic field.

Example: A beam of electrons split by an inhomogeneous z magnetic field:





Bohr Magneton:

$$\mu_B = \frac{q\hbar}{2m} \approx 10^{-23} \quad A - m^2$$

$$= 10^{-3}\, A.(10^{-10}m)^2$$

$$= 10^{-3}\, A.(1\text{Å})^2$$

Pauli Spin Matrices or Rotation Matrices

For magnet $\mu_B B_z$ in z direction: $\sigma_z = \begin{matrix} & +\hat{z} & -\hat{z} \\ +\hat{z} & 1 & 0 \\ -\hat{z} & 0 & -1 \end{matrix}$

For magnet $\mu_B B_y$ in y direction: $\sigma_y = \begin{matrix} & +\hat{z} & -\hat{z} \\ +\hat{z} & 0 & -i \\ -\hat{z} & i & 0 \end{matrix}$

For magnet $\mu_B B_x$ in y direction: $\sigma_x = \begin{matrix} & +\hat{z} & -\hat{z} \\ +\hat{z} & 0 & 1 \\ -\hat{z} & 1 & 0 \end{matrix}$

$$i\hbar \frac{d}{dt}\begin{pmatrix} u \\ d \end{pmatrix} = \mu_B B_z \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow \frac{d}{dt}u = \frac{\mu_B B_z}{i\hbar}u \Rightarrow u(t) = u(0)e^{\frac{\mu_B B_z}{i\hbar}t}$$

$$and \ \Rightarrow \frac{d}{dt}d = -\frac{\mu_B B_z}{i\hbar}d \Rightarrow d(t) = d(0)e^{-\frac{\mu_B B_z}{i\hbar}t}$$

$$\Rightarrow \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} * \, e^{\frac{\mu_B B_z}{i\hbar}t} \\ * \, e^{-\frac{\mu_B B_z}{i\hbar}t} \end{pmatrix} = \begin{pmatrix} * \, e^{-i\frac{\mu_B B_z}{\hbar}t} \\ * \, e^{i\frac{\mu_B B_z}{\hbar}t} \end{pmatrix} \text{ then we assume } \frac{\varphi}{2} = \frac{\mu_B B_z}{\hbar}t$$

$$\Rightarrow \varphi(t) = 2\frac{\mu_B B_z}{\hbar}t \quad or \quad \frac{d\varphi}{dt} = 2\frac{\mu_B B_z}{\hbar}$$

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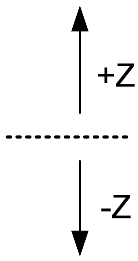
$$\frac{d}{dt}\begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix} = 2\frac{\mu_B B_z}{\hbar}\underbrace{\begin{bmatrix} 0 & -1 & 0 \\ +1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{L_z}\begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix}$$ L_z for rotate around z direction

$$\frac{dS_x}{dt} = -\sin\theta\sin\varphi\frac{2\mu_B B_z}{\hbar} \quad and \quad \frac{dS_y}{dt} = \sin\theta\cos\varphi\frac{2\mu_B B_z}{\hbar} \quad and \quad \frac{dS_z}{dt} = 0 \; becasue\ \theta\ is\ constant\ here.$$

$$L_z = \begin{bmatrix} 0 & -1 & 0 \\ +1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; and \; L_y = \begin{bmatrix} 0 & 0 & +1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \; and \; L_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & +1 & 0 \end{bmatrix}$$

One of geometrical properties is $L_x L_y - L_y L_x = L_z \equiv \sigma_x\sigma_y - \sigma_y\sigma_x = 2i\sigma_z$

For all three Pauli spin matrices eigenvalues are the same as 1 and -1. The eigenvectors are as below:



$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; and \; \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix} \; and \; \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\sigma_y = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \Rightarrow \begin{pmatrix} 1 \\ i \end{pmatrix} \; and \; \begin{pmatrix} i \\ 1 \end{pmatrix}$$