Fundamentals of Nanoelectronics

ECE495 - Session 39, Dec 4, 2009

Pauli Spin Matrices III

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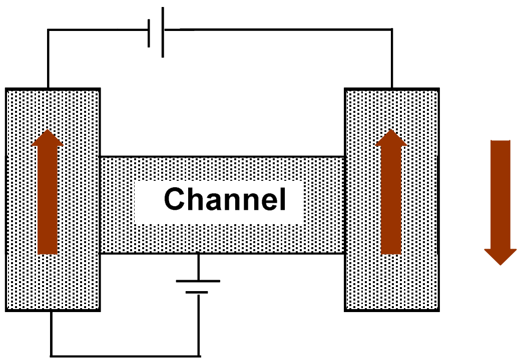
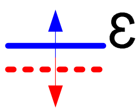
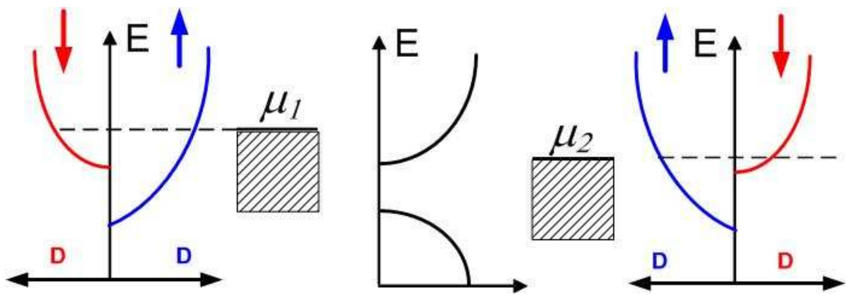
Review



$$I = \frac{q}{h}\int dE\,Trace(\Gamma_1 G \Gamma_2 G^{+})(f_1 - f_2)$$



$$\begin{bmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix}$$



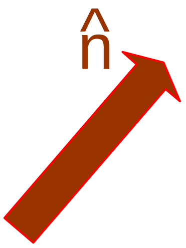
$$H_0 = \begin{bmatrix} 2t_0 & -t_0 & 0 & \cdots & \cdots & \\ -t_0 & 2t_0 & -t_0 & 0 & \cdots & \vdots \\ 0 & -t_0 & \ddots & \ddots & & \vdots \\ \vdots & 0 & \ddots & \ddots & & \vdots \\ \vdots & \vdots & & & \ddots & -t_0 \\ & \cdots & \cdots & \cdots & -t_0 & 2t_0 \end{bmatrix}$$

The proper Hamiltonian for degenerate spin levels is twice as big $\begin{bmatrix} H_0 & 0 \\ 0 & H_0 \end{bmatrix}$

$$\Sigma_1 = -\frac{i\gamma}{2} \; and \; \Gamma_1 = i[\Sigma_1 - \Sigma_1^{+}] = \gamma$$

For spin: $[\Sigma_1] = -\frac{i}{2}\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$ $and\ \Gamma_1 = i[\Sigma_1 - \Sigma_1^{+}] = i\left(-\frac{i}{2}\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} - \frac{i}{2}\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}\right) = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$

In parallel mode: $\Gamma_2 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$ and for anti-parallel: $\Gamma_2 = \begin{bmatrix} \beta & 0 \\ 0 & \alpha \end{bmatrix}$



All parameters should be in the same basis.

	$\hat{n}$	$-\hat{n}$
$\hat{n}$	α	0
$-\hat{n}$	0	β

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$$\Gamma_1 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} = aI + b\sigma_z + c\sigma_x + d\sigma_y$$

If n is in z direction, c and d would be 0:

$$\Gamma_1 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow b = \frac{\alpha - \beta}{2} \ and \ a = \frac{\alpha + \beta}{2}$$

For parallel: $\Gamma_2 = \Gamma_1 = aI + b\sigma_z$ and for anti-parallel $\Gamma_2 = aI - b\sigma_z$

$$\sigma_x\sigma_y + \sigma_y\sigma_x = 0$$

$$\sigma_x\sigma_y - \sigma_y\sigma_x = 2i\sigma_z$$

Trace will not change by basis transformation. $Trace(\sigma_z) = Trace(\sigma_y) = Trace(\sigma_z) = 0$

If n is in x direction: $\Gamma_2 = aI + b\sigma_x$

If n is in y direction: $\Gamma_2 = aI + b\sigma_y$

$$\hat{n} = \hat{x}\sin\theta\cos\varphi + \hat{y}\sin\theta\sin\varphi + \hat{z}\cos\theta$$

$$\hat{n}: \Gamma_2 = aI + b(\vec{\sigma}.\vec{n})$$

$$\vec{\sigma} = \hat{x}\sigma_x + \hat{y}\sigma_y + \hat{z}\sigma_z$$

$$\vec{\sigma}.\vec{n} = \sin\theta\cos\varphi\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \sin\theta\sin\varphi\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} + \cos\theta\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{+i\varphi} & -\cos\theta \end{bmatrix}$$