

# ***Network for Computational Nanotechnology (NCN)***

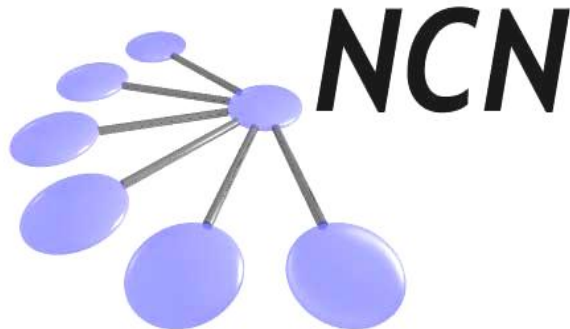
*UC Berkeley, Univ. of Illinois, Norfolk State, Northwestern, Purdue, UTEP*

## **Thermoelectric effects in ultra-scaled semiconductor devices Role of electronic and lattice properties**

[Abhijeet Paul](#)

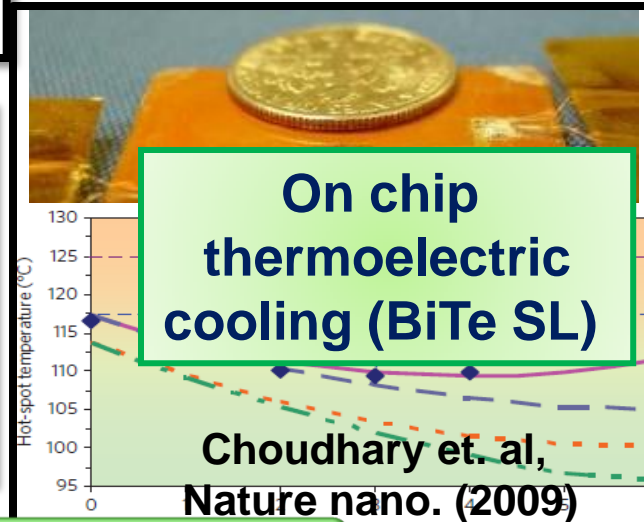
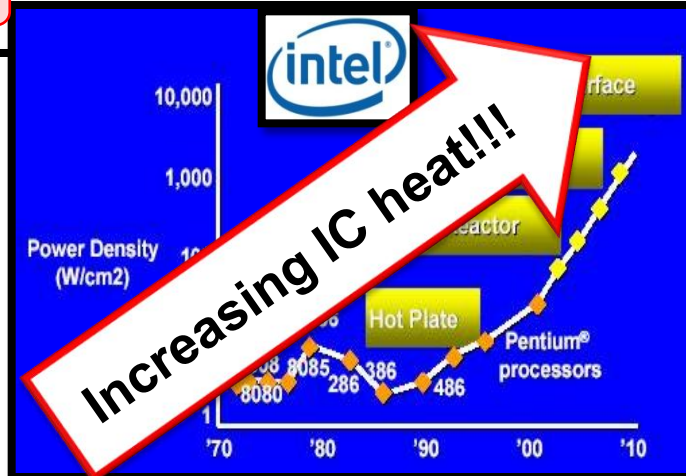
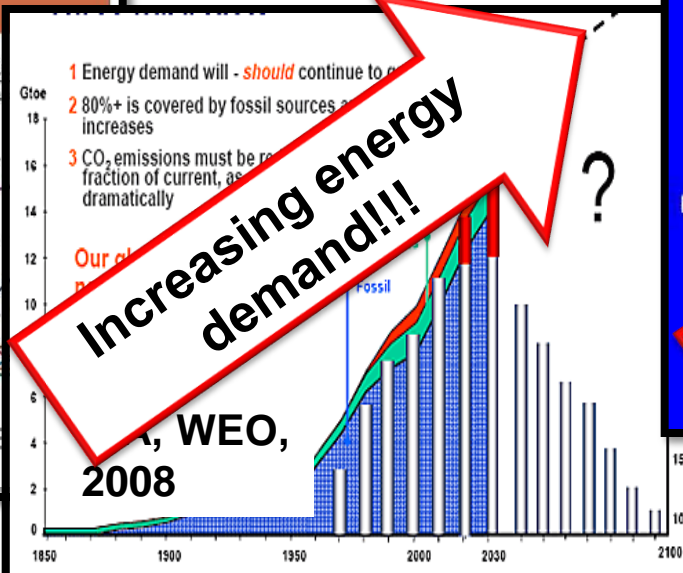
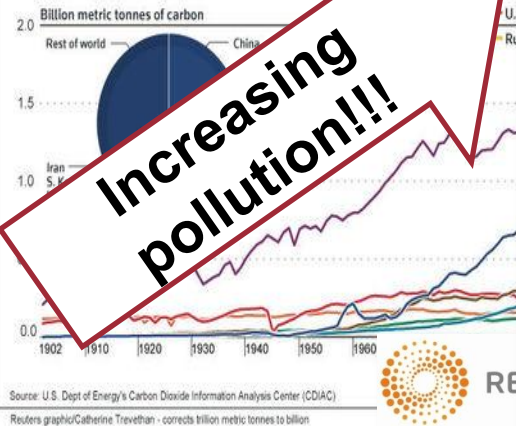
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## Nasty Problems

### World carbon emission



www.tellure

## Green Solutions from thermoelectricity !!!

Electronic structure in nanostructures?

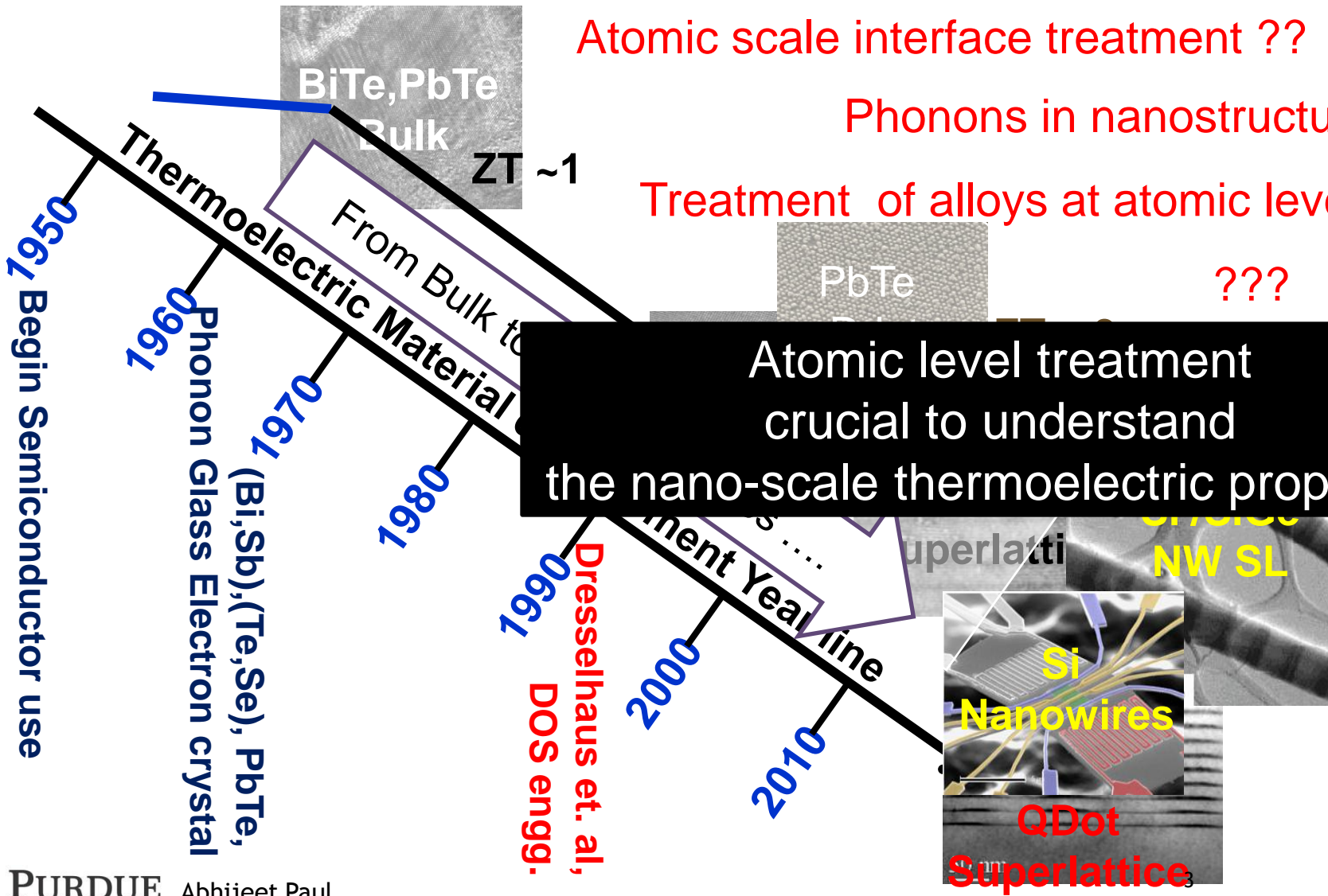
Atomic scale interface treatment ??

Phonons in nanostructures ??

Treatment of alloys at atomic level ??

???

Atomic level treatment  
crucial to understand  
the nano-scale thermoelectric properties



- Introduction to Thermoelectricity
  - Basics
  - Material Development
  - Research vectors
- Approach for thermoelectric (TE) analysis.
- Research milestones
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Coefficient of Performance

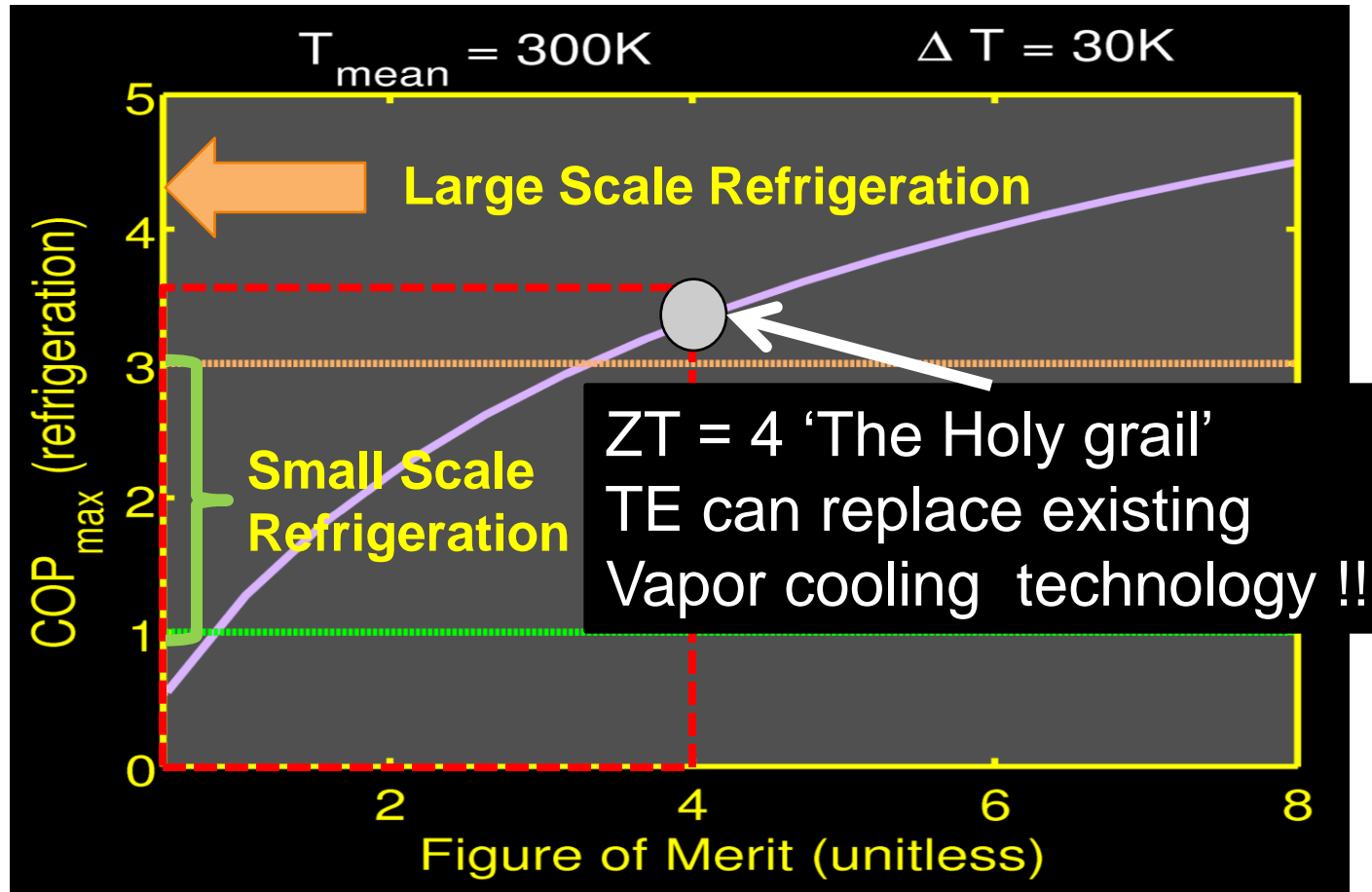
=

$$\left[ \frac{T_{hot}}{T_{hot} - T_{cold}} \right] \cdot \left[ \frac{\sqrt{1 + ZT} - T_{hot}/T_{cold}}{\sqrt{1 + ZT} + 1} \right]$$



Heat energy removed from cold side

Heat energy added to hot side





Generation of potential difference due to applied temperature difference → **'Seebeck Coefficient'**.

$$S = \frac{\Delta V}{\Delta T}$$

**Measure of thermoelectric power generation (High)**

Generation of temperature difference due to applied potential difference → **'Peltier Coefficient'**

$$\Pi = T \frac{\Delta V}{\Delta T}$$

**Measure of thermoelectric cooling (High)**

$$ZT = \frac{GS^2T}{\kappa_l + \kappa_e}$$

**'Thermoelectric Figure of Merit'**  
unitless quantity obtained at temp 'T'. Defined by Ioffe in 1949.

Ability of material to conduct electricity → **'Electrical Conductance'**

$$G = \frac{\Delta I}{\Delta V}$$

**Measure of charge flow (High)**

Ability of material to conduct heat energy → **'Thermal Conductance'**

$$\kappa = \frac{1}{T} \frac{\Delta Q}{\Delta d}$$

**Measure of heat flow (Low)**  
Both electrons ( $\kappa_e$ ) and lattice ( $\kappa_l$ ) carry heat.

Large COP → High ZT → large G ↑ large S ↑ and small  $\kappa$  ↓ desired !!!

# Material of choice for thermoelectricity

TE Parameters → ----- Materials ↓		Electrical Conductivity (G) ↑	Seebeck Coefficient (S) ↑	Thermal Conductivity (κ) ↓
Metals	✗	Very High ~ $10^7$ S/m ↑	Low ~ $10 \mu\text{V/K}$ ↓	High ~ $10^2$ W/m-K ↑
Insulators	✗	Extremely low (~ $10^{-10}$ S/m) ↓	High ↑	Low ~ $10^{-2}$ - $10^{-4}$ W/m-K ↓
Semiconductors	✓	Moderate $10^{-3}$ S/m ↑	High ~ $120 \mu\text{V/K}$ ↑	Low ~ $10$ W/m-K ↓

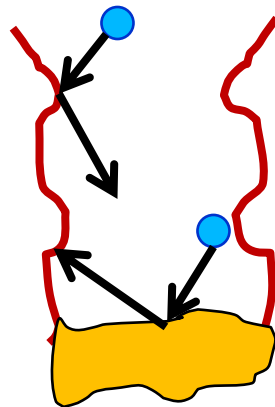
**Semiconductors most suitable TE material.  
Allow separate control of G (electrons) and κ (phonons).**

1990s →

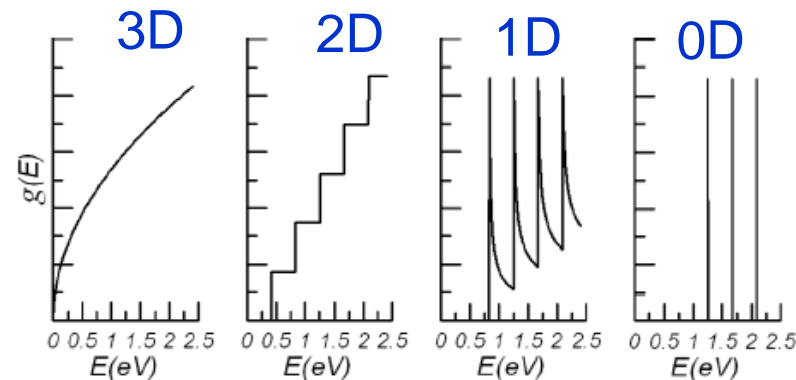
Enhance Power factor ( $S^2G$ ) by electronic structure modification.

**Nanostructures** provide DOS modification.

$$ZT = \frac{TGS^2}{K_e + K_l}$$



Phonon scattering



DOS engineering

1960s →

Reduce thermal conductivity by phonon scattering.

**Nanostructures and alloys** increase phonon scattering.

**Nanostructures allow tuning of  $G$ ,  $S$  and  $\kappa$  !!!**



Promising  
TE Materials

Making research  
Economically viable

Potential  
Markets [1]

Crucial R&D vectors

- Thin Films
- Nano-particles
- Super Lattices
- Nano-composites
- Nanowires
- Quantum Dots

Research

- Fabrication of nanostructures.
- Robust thermoelectric characterization
- Higher reliability
- Better structural stability.
- Efficient thermoelectric modules.
- Bulk and low cost production.
- Better simulation and analysis tools.

- Consumer (35%)
- Automobile (14%)
- Telecom (16%)
- Medical and Bio (12%)
- Industry (9%)
- Semicon. Process (8%)
- Defense & space (6%)

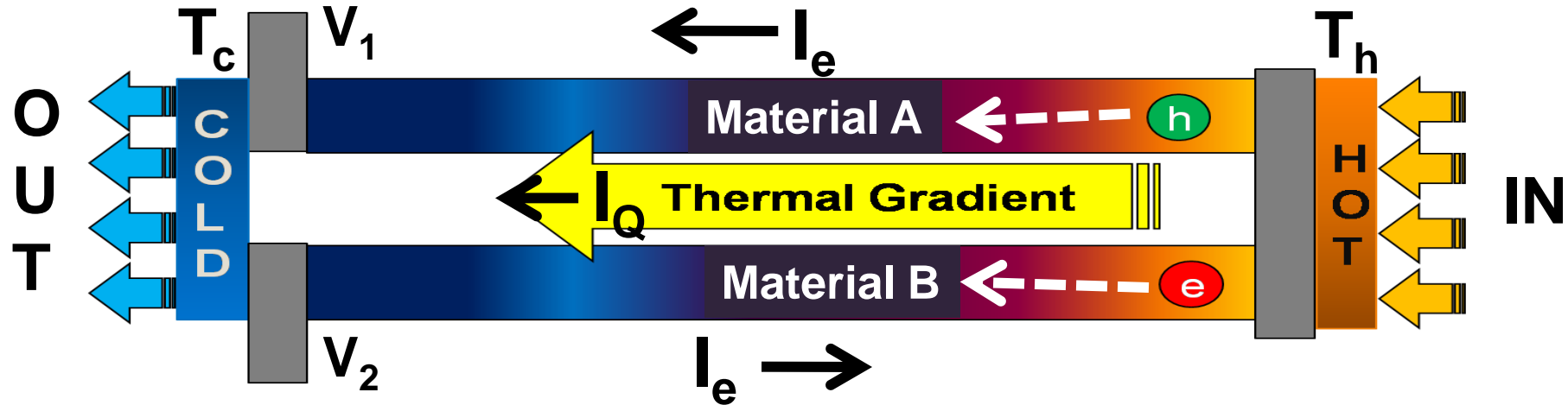
Economy \$\$\$

Computer simulation an integral  
part to develop better TE  
materials and modules

[1] Hachiuma & Fukuda  
ECT, 2007

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# How to analyze thermoelectric properties of materials ?



Steady-state linear thermoelectric (Onsager's) relations [1,2]

Electric current

$$I_e = G.\Delta V - (GS).\Delta T$$

Heat current

$$I_Q = (GST)\Delta V + (\kappa - S^2GT)\Delta T$$

Landauer's Formula can be used to evaluate the transport parameters

$$|\Delta V| < \frac{k_B \bar{T}}{q}$$

$$\Delta T \ll \bar{T}$$

$$\Delta V = V_1 - V_2, \Delta T = T_h - T_c, \bar{T} = T_h + T_c / 2, \kappa = \kappa_e + \kappa_l$$

[1] L. Onsager, Phys. Rev. 37 405 (1931).

[2] G. D. Mahan, Many-body Physics.

$G, S$  (Electronic)

$\kappa_e$

= Pre - factor  $\times f(L_m^{e/l})$

$\kappa_l$  (Lattice)

Landauer's Integral

Under zero current condition

$$G \propto L_0^e$$

$$S \propto L_1^e / L_0^e$$

$$\kappa_l \propto L_1^l$$

Landauer's approach  $\rightarrow$   
A suitable approach to calculate  
thermoelectric transport parameters in nanostructures.

$$L_m^l = \int_0^{\omega_{\max}} \omega^m \left[ \frac{\langle \lambda_{ph}(\omega) \rangle}{L} \right] \left( \frac{\partial F_{BE}(\omega)}{\partial T} \right) M(\omega) d\omega$$

Phonon Integral

Phonons need

- No Fermi Level
- Bose Einstein distribution (bosons!!)
- $M(\omega) \rightarrow$  Phonon dispersion.

Accurate electronic & phonon dispersions must !!!.

Both need

- No. of modes,  $M(E)$ .
- Mean free path ( $\lambda$ ).

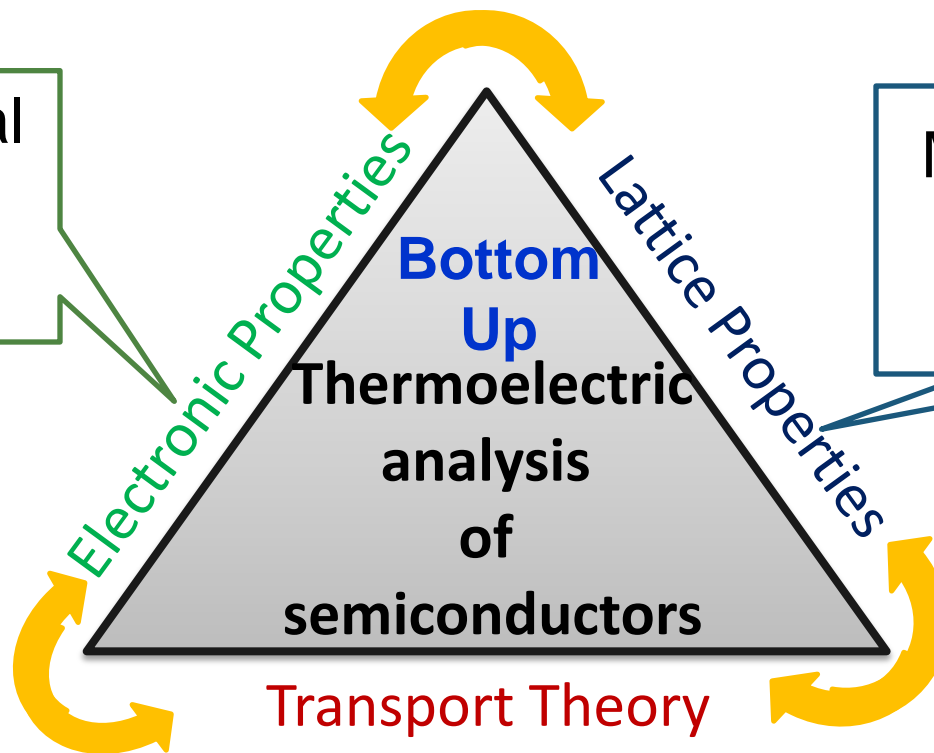
Electrons need

- Moment calculation near Fermi Level
- Fermi Dirac distribution (fermions!!)
- $M(E) \rightarrow$  Electronic bandstructure.

$$L_m^e = \int_{-\infty}^{E_{\text{top}}} \left( \frac{E - E_f}{k_B T} \right)^m \left[ \frac{\langle \lambda_{el}(E) \rangle}{L} \right] \left( -\frac{\partial F_{FD}(E)}{\partial E} \right) M(E) dE$$

Electron Integral

Semi-empirical  
Tight-Binding  
(TB) method.



Modified Valence  
Force Field  
(MVFF) method.

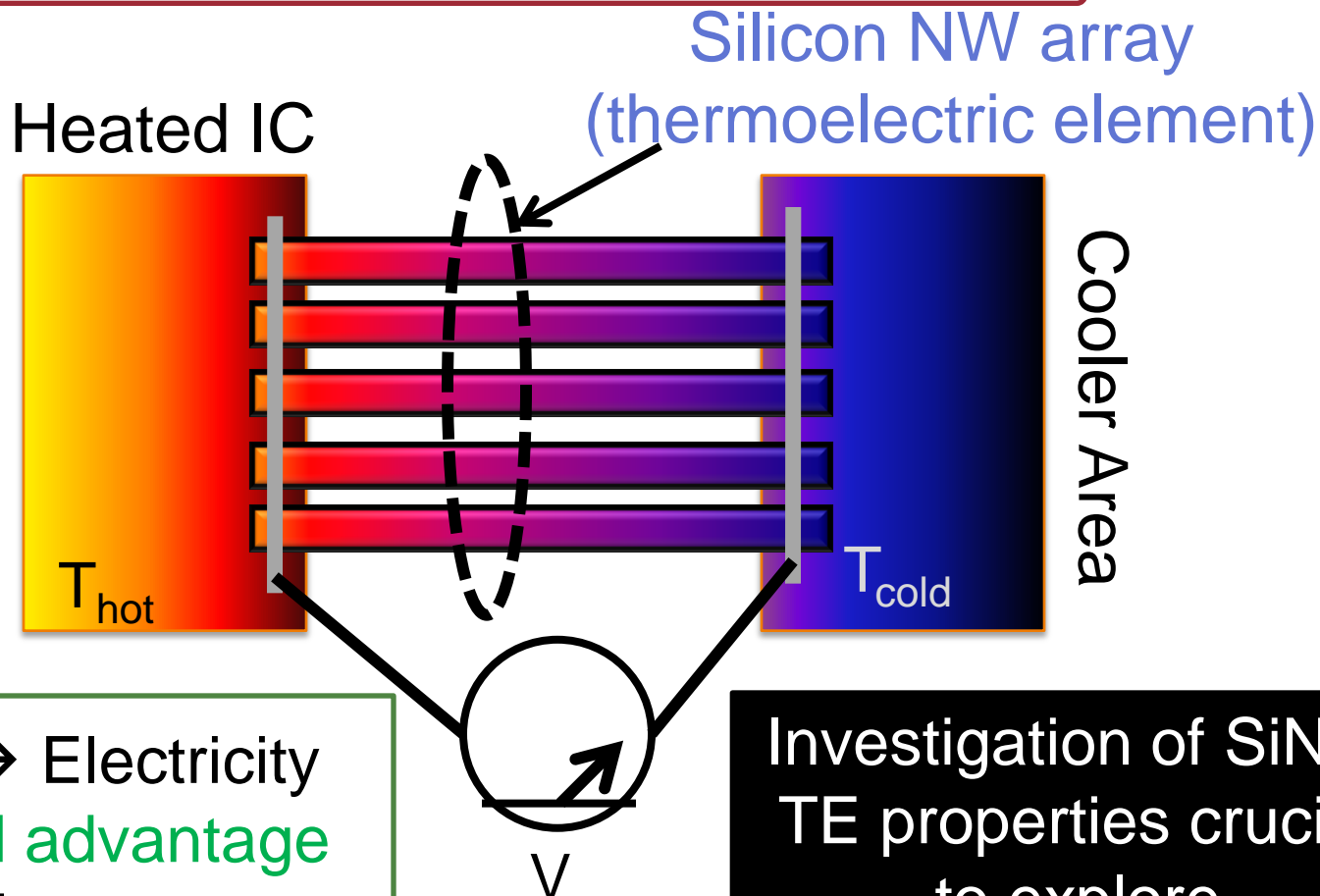
Landauer's approach and  
Green's function method

Three ingredients for TE analysis in nanostructures



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How to cool the heating ICs ??



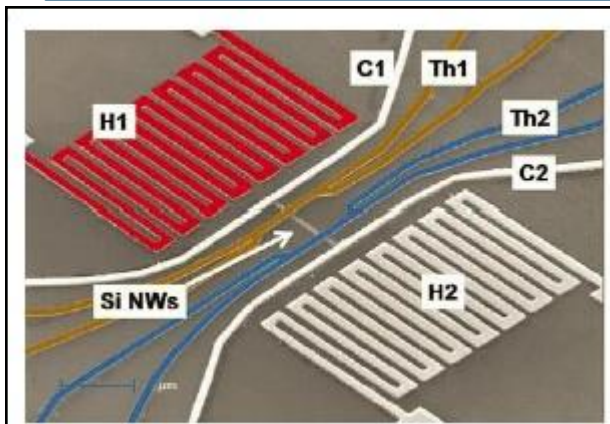
Waste heat  $\rightarrow$  Electricity

Two pronged advantage

- Cool the chip.
- Obtain electricity

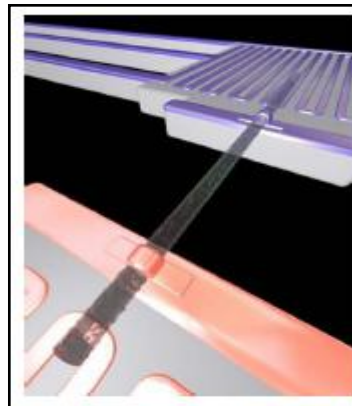
Investigation of SiNW  
TE properties crucial  
to explore  
more ideas !!!

## High ZT p-type SiNW waste heat conversion



**ZT ~1 @ 200K**

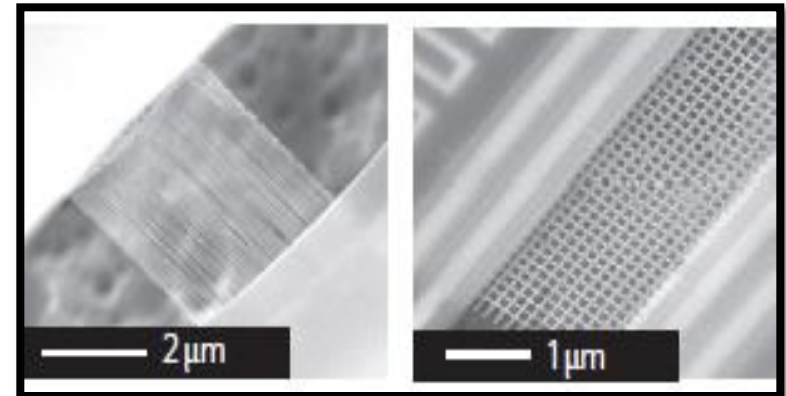
Caltech, Nature,  
451,168, 2008



**ZT ~0.6 @ 300K**

Berkeley, Nature,  
451,163, 2008

## Thermal conductance reduction Silicon phonon mesh



**$\kappa \sim 1.9 \text{ W/m-K}$**

Caltech, Nature nano.2010,  
doi:10.1038/nnano.2010.149

100 fold rise in SiNW ZT  
compared to  
Bulk Si ZT (0.01 @ 300K)!!!

100 fold reduction in  
Si nanomesh  $\kappa$  compared to  
Bulk Si ( $\sim 148 \text{ W/m-K}$  @ 300K)!!!

**Nanostructuring (SiNW) turns 'lousy bulk Si' to better TE material !!**

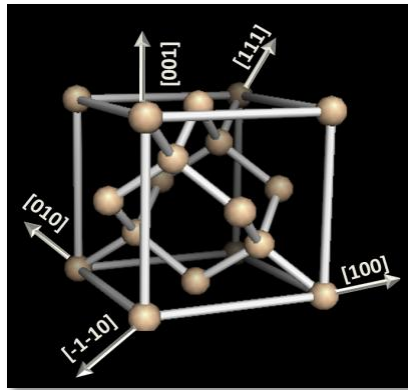
1. Atomistic confinement effects on the Seebeck coefficient ( $S$ ) in SiNWs.
2. Atomistic and uniaxial strain effect on thermoelectric power-factor ( $S^2G$ ) of SiNWs.

## DEVICE DETAILS:

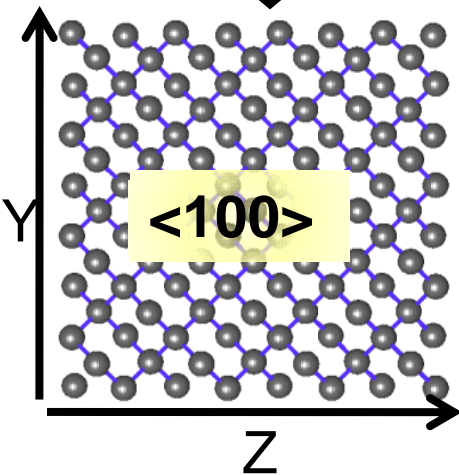
- Rectangular SiNW  $\rightarrow$  [100], [110] and [111] channels.
- Width ( $W$ ) and height ( $H$ ) varied from 2 to 14nm.

Electronic structure using  
Atomistic Tight Binding  
method.

$S$  and  $G$  calculation using  
Landauer's approach.

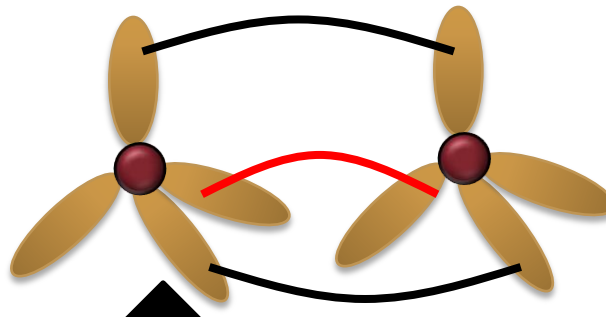


Zinc blende  
unitcell



Nano-structure

Assemble  
TB Hamiltonian  
and  
obtain eigen energies



Atomic Orbital  
Interactions



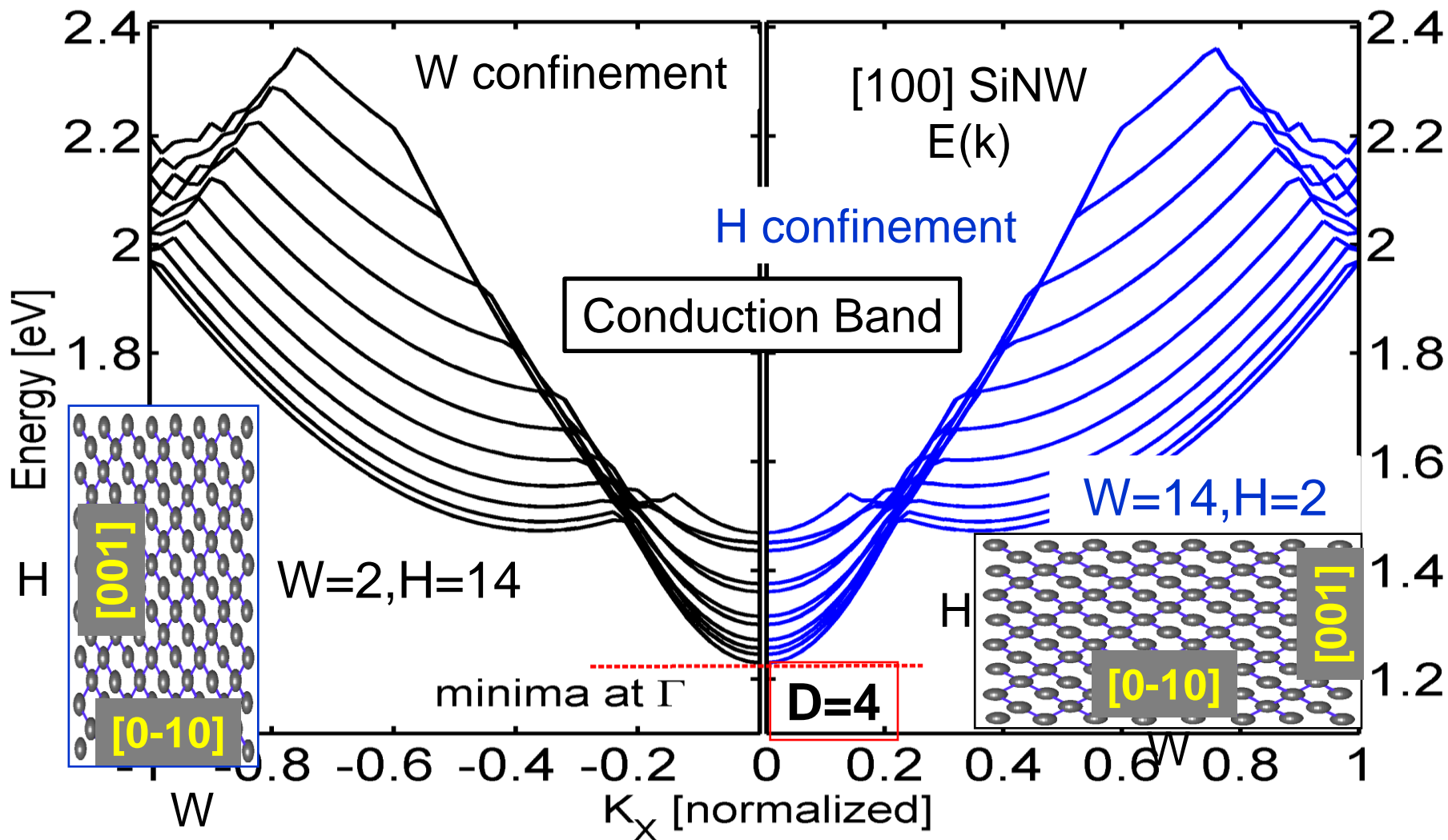
## ADVANTAGES

- ✓ Appropriate for treating atomic level disorder.
- ✓ Strain treatment at atomic level.
- ✓ Structural, material and potential variation at atomic level treated.

10 band nearest neighbor  
 $sp^3d^5s^*$  model with spin  
orbit coupling.

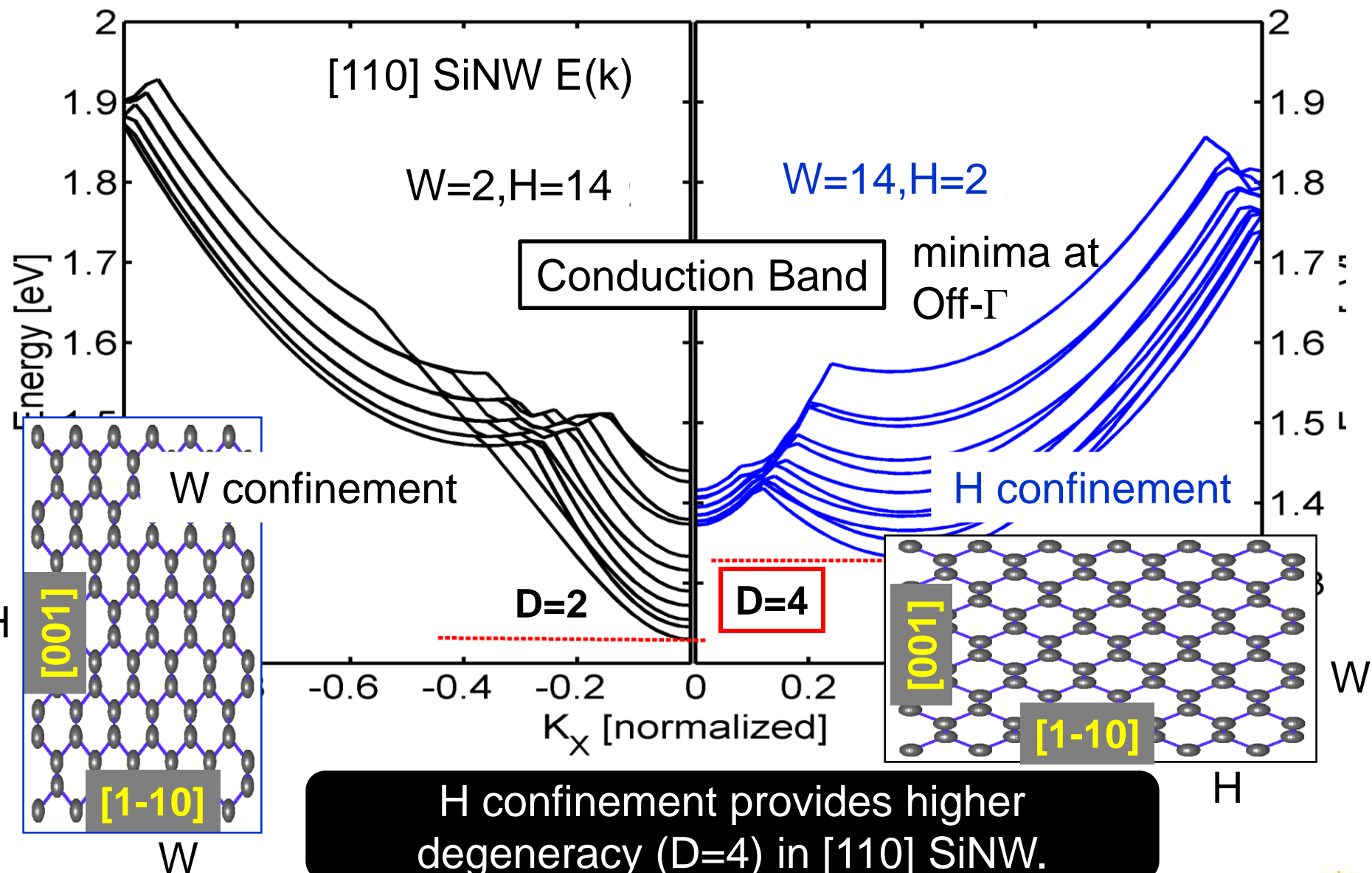
Electronic structure  
calculation in SiNWs  
using Tight Binding [1] (TB)

[1] Klimeck et. al  
CMES, 3,  
No. 5 (2002);

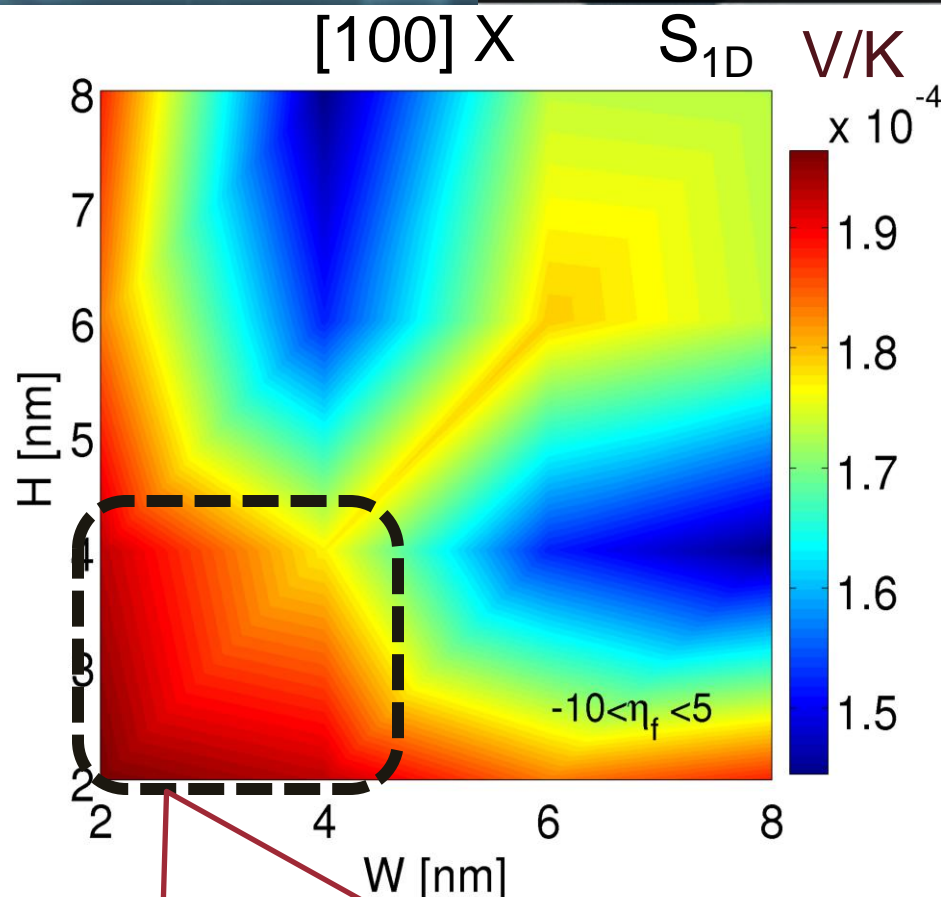


H and W confinement symmetric for  
[100] oriented wires

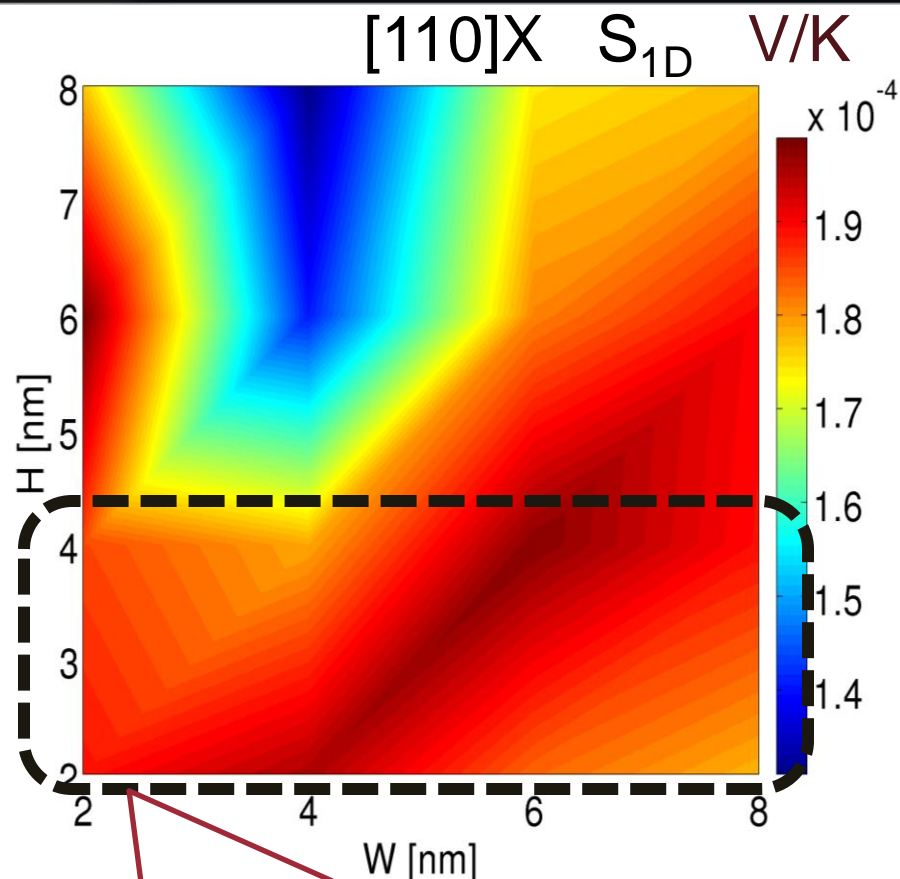




# Tuning S by confinement

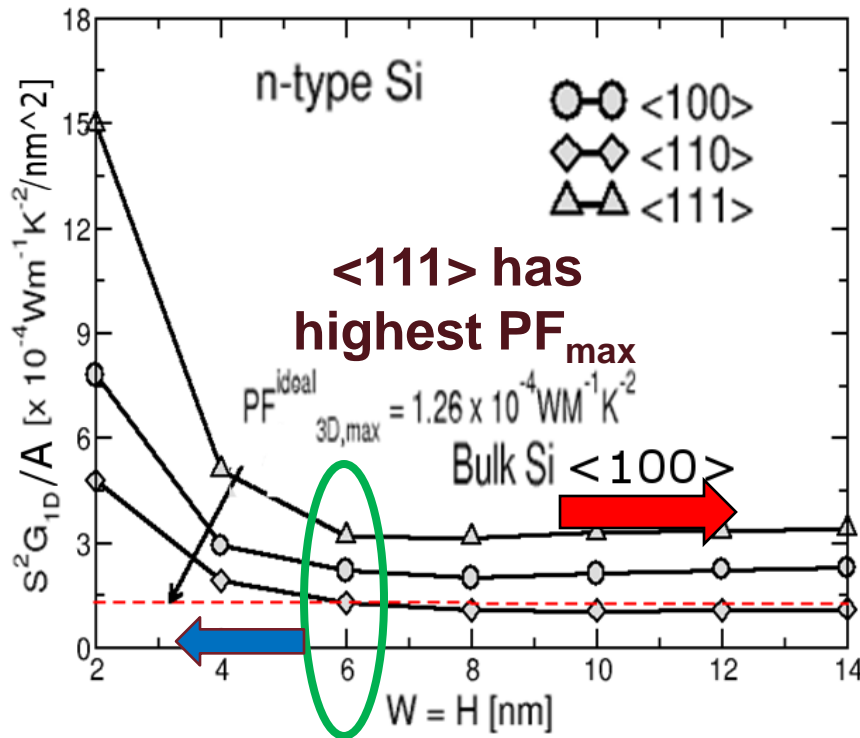


Both H and W confined  
for high S

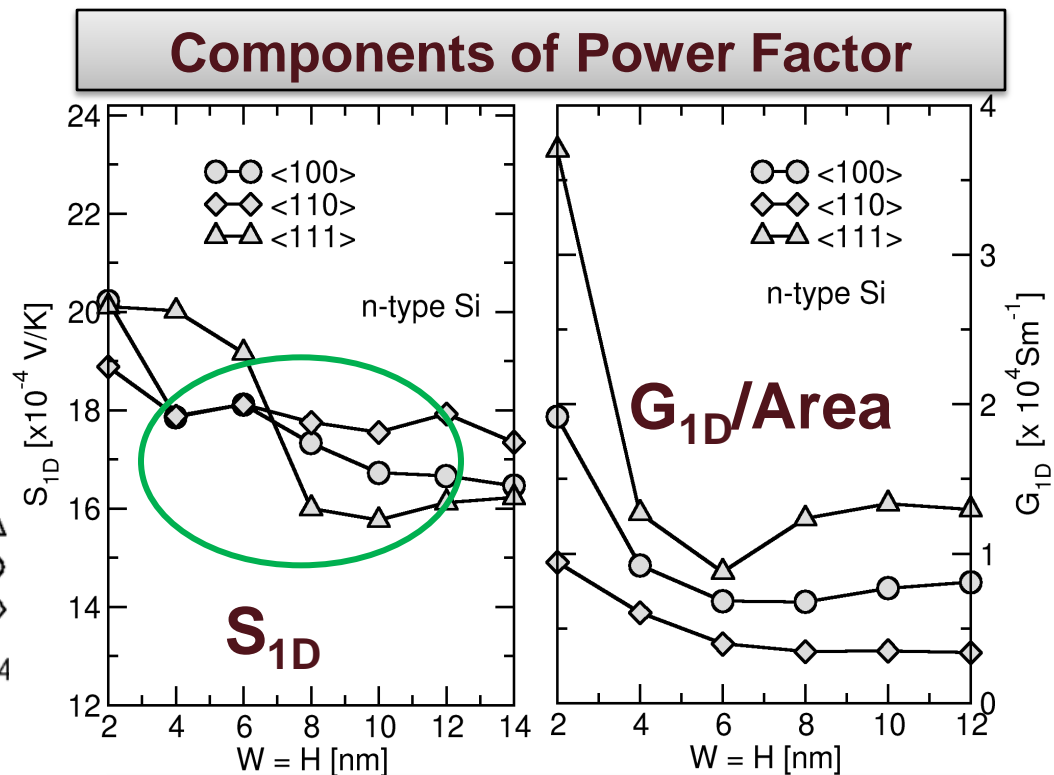


Only H confinement  
increases S

Geometrical confinement a nice way to tune 'S' in SiNWs.

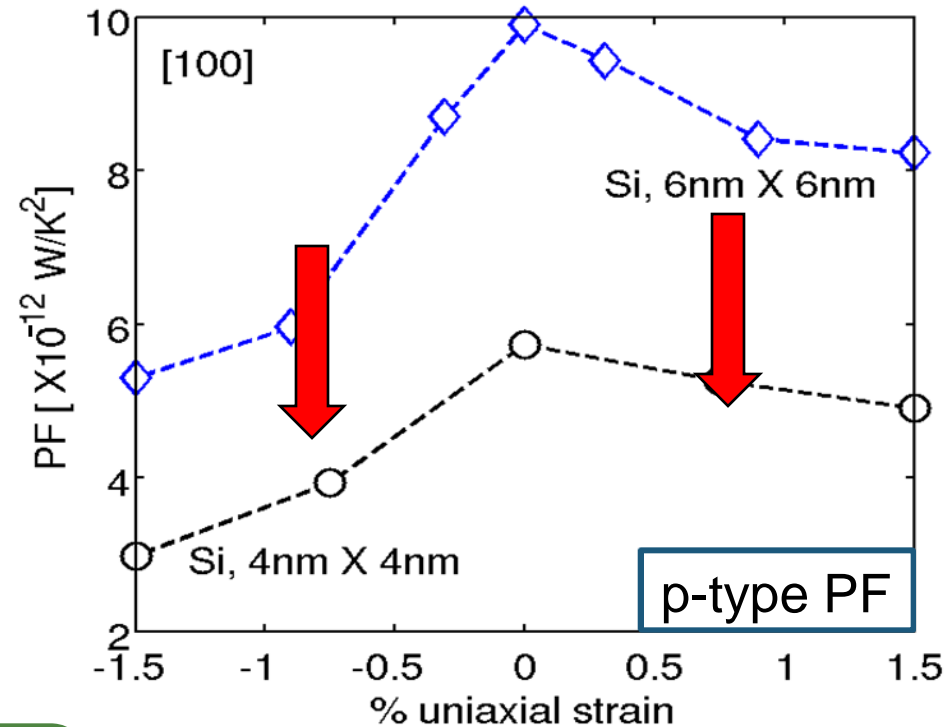
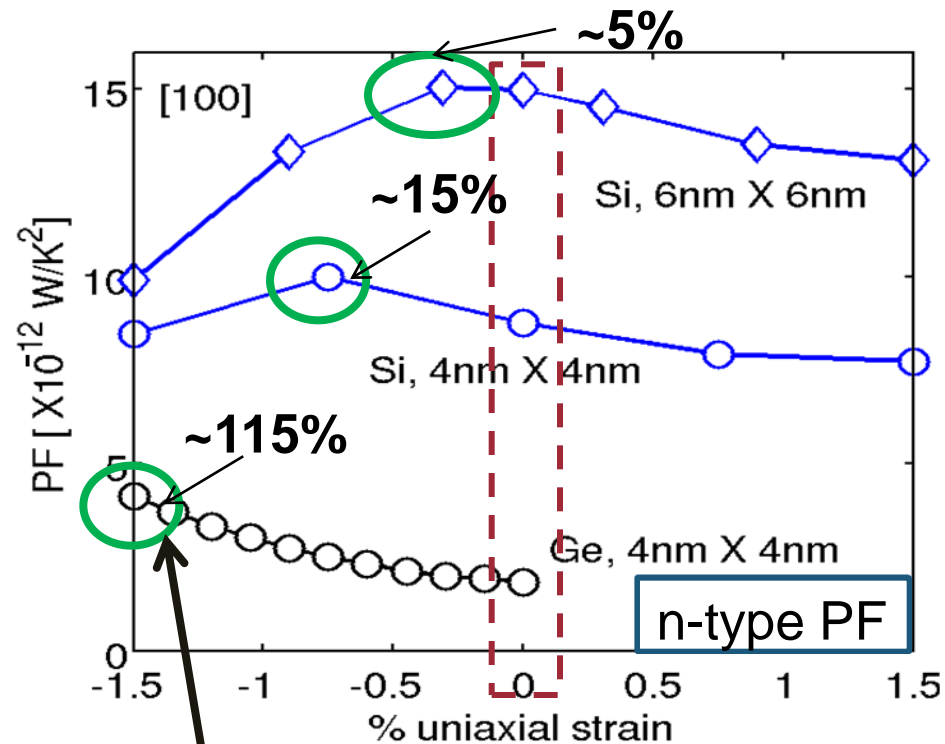


- PF/Area improved for SiNW with  $W/H < 6\text{nm}$ .
- $PF_{\max}$  saturates in larger SiNW.



- Seebeck Coefficient is almost constant at  $PF_{\max}$ .
- $G$  per area shows a saturation with  $\langle 111 \rangle$  showing highest  $G/\text{area}$  value.

**$\langle 111 \rangle$  shows maximum PF  
 $W/H < 6\text{nm}$  improves PF**



GeNW  
better due  
higher DOS  
L-valley.

Compressive strain  
→ inc. DOS near Fermi  
level → Improves PF.

Compressive/Tensile strain  
split VB → dec. DOS near  
Fermi level → degrades PF.

**Compressive uniaxial strain improves n-type ballistic PF.**

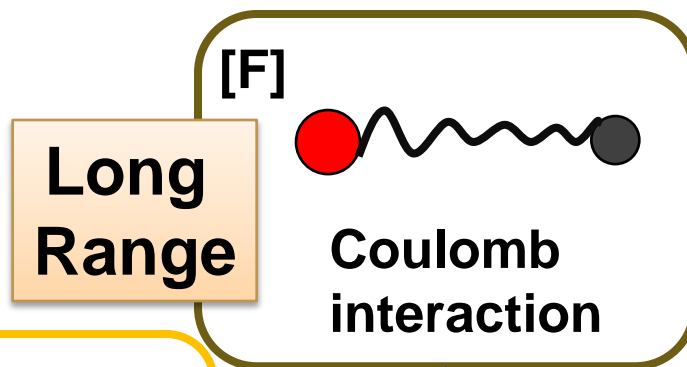
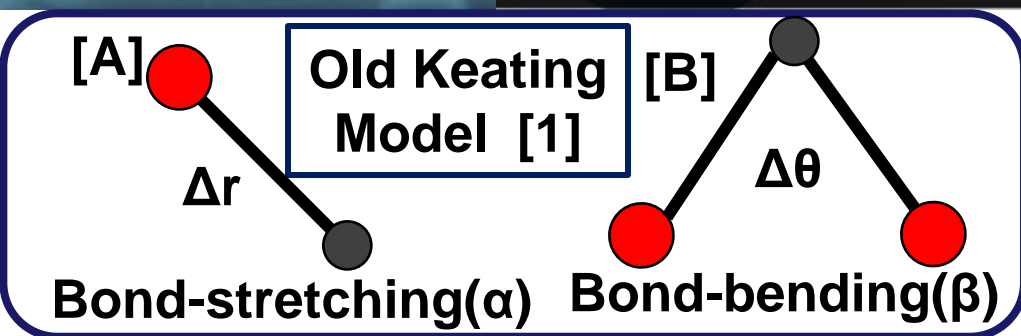
- ✓ Atomistic approach shows:
  - Width and height confinement → not equivalent at atomic scale.
  - Crystal transport orientation crucial.
- ✓ Confinement direction important → design high S devices.
- ✓ SiNWs with  $W \text{ \& } H < 6\text{nm}$  → improvement in Ballistic PF.
- ✓  $\langle 111 \rangle$  orientated SiNW → best ballistic PF.
- ✓ Uniaxial Compressive strain → improves n-type PF.

1. Phonon dispersion in bulk Si using Modified VFF.
2. Phonon dispersion in calculation in SiNWs.
3. Effect of phonon dispersion on SiNW lattice thermal properties.

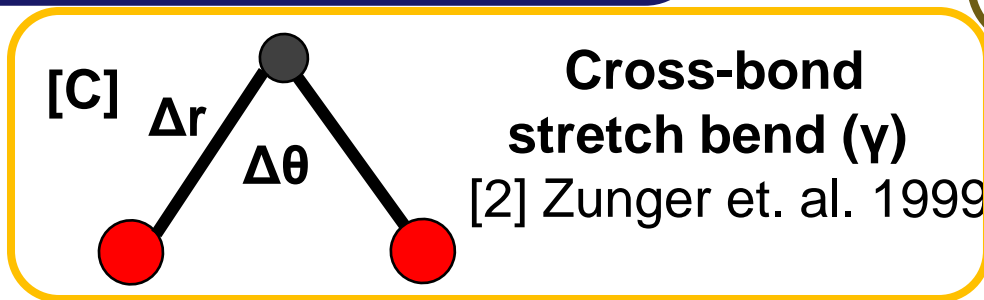
## SiNW DETAILS:

- Rectangular SiNW  $\rightarrow$  [100] channels
- Width (W) and height (H) varied from 2 to 6nm.



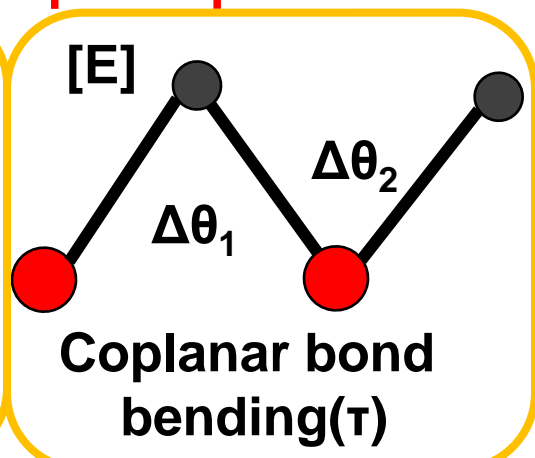
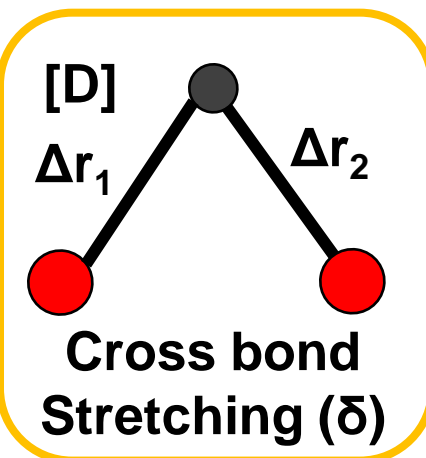


**Short Range**



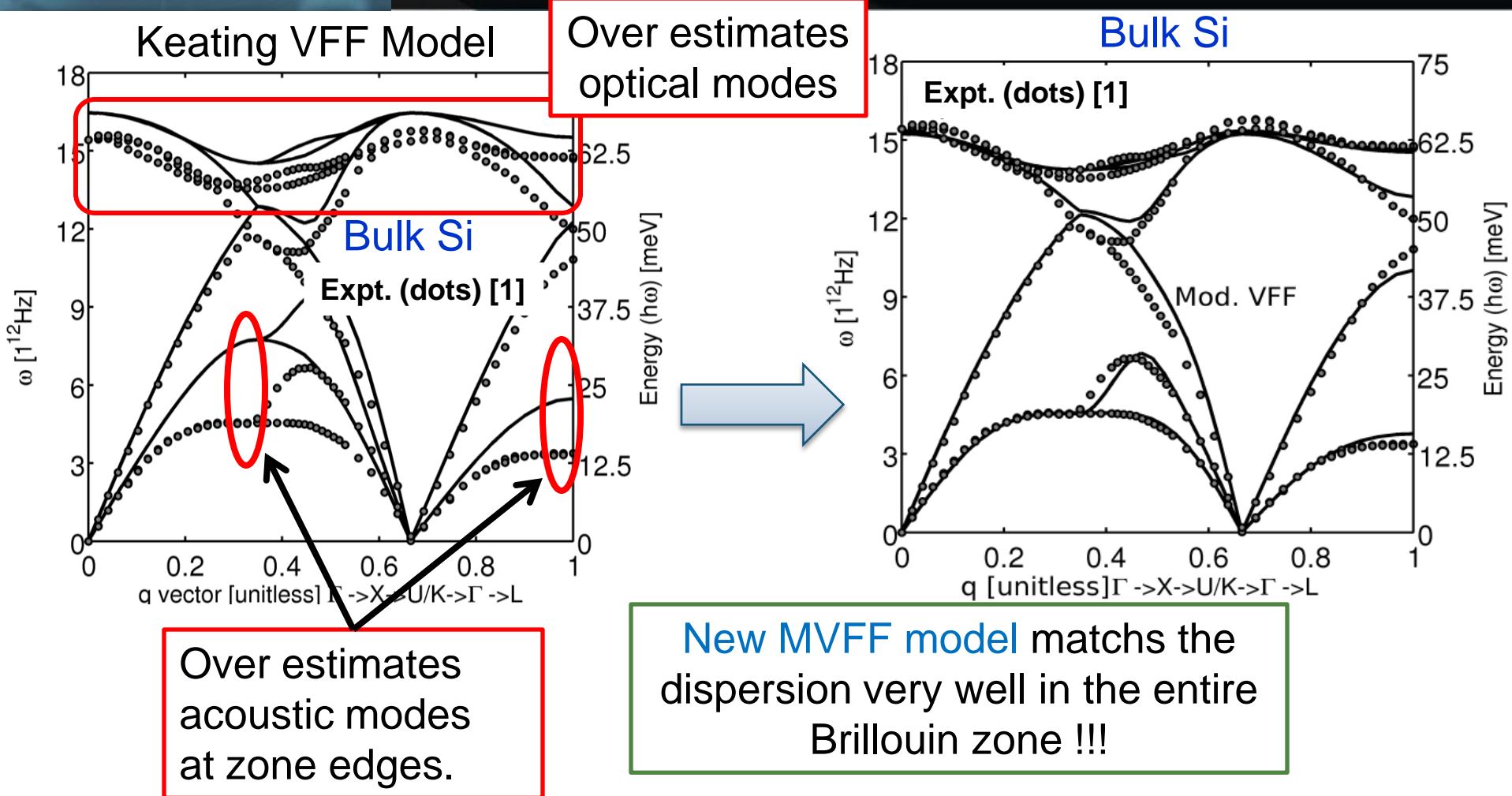
**Imp. For polar materials [2]**

**Imp. for polar materials [2]**



**New combination of Interactions:  
Modified Valence Force Field  
Calculate phonons in zinc-blende materials.**

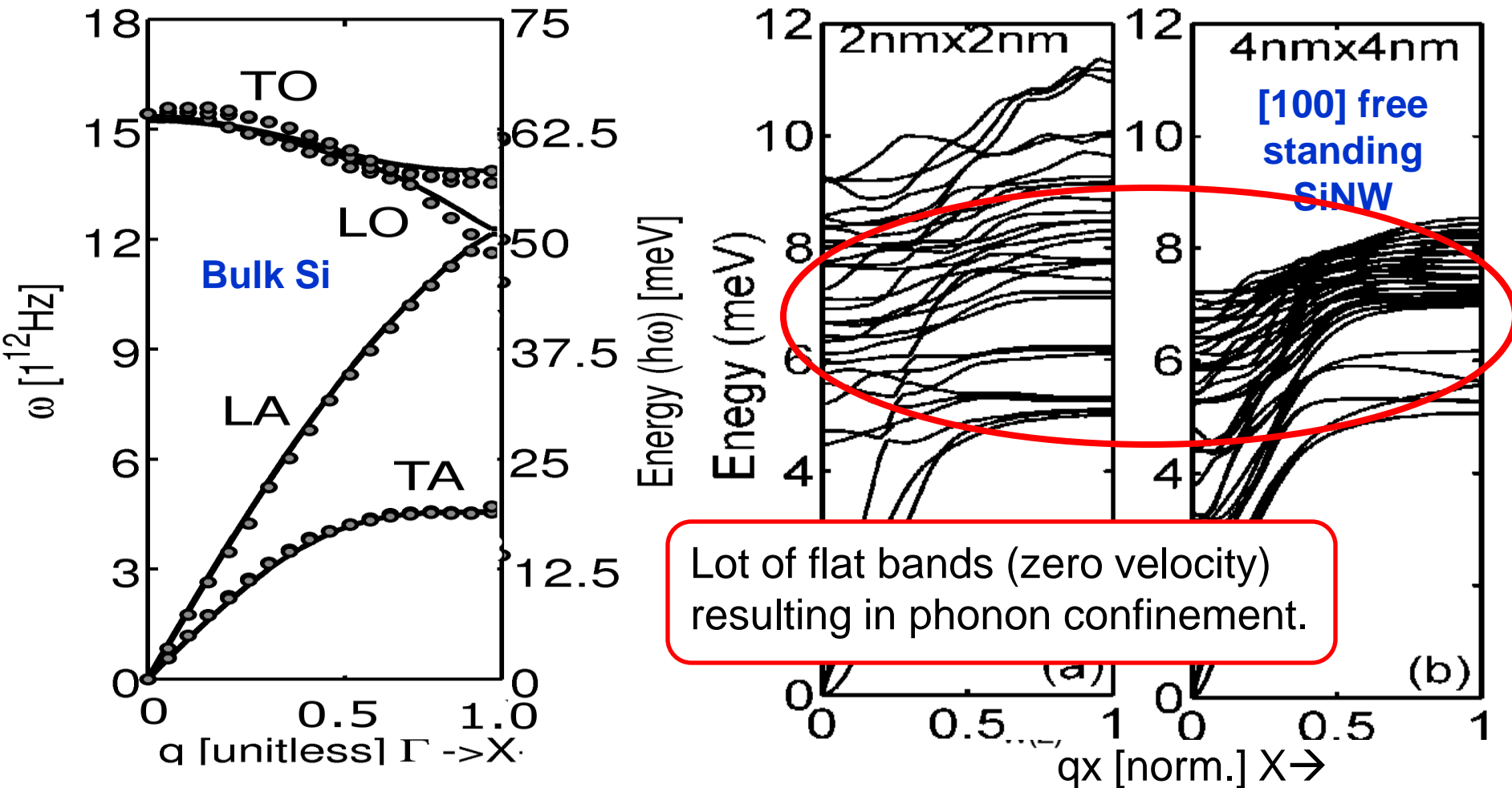
**Imp. for non-polar materials**



Expt. Data, inelastic neutron scattering (80K and 300K).

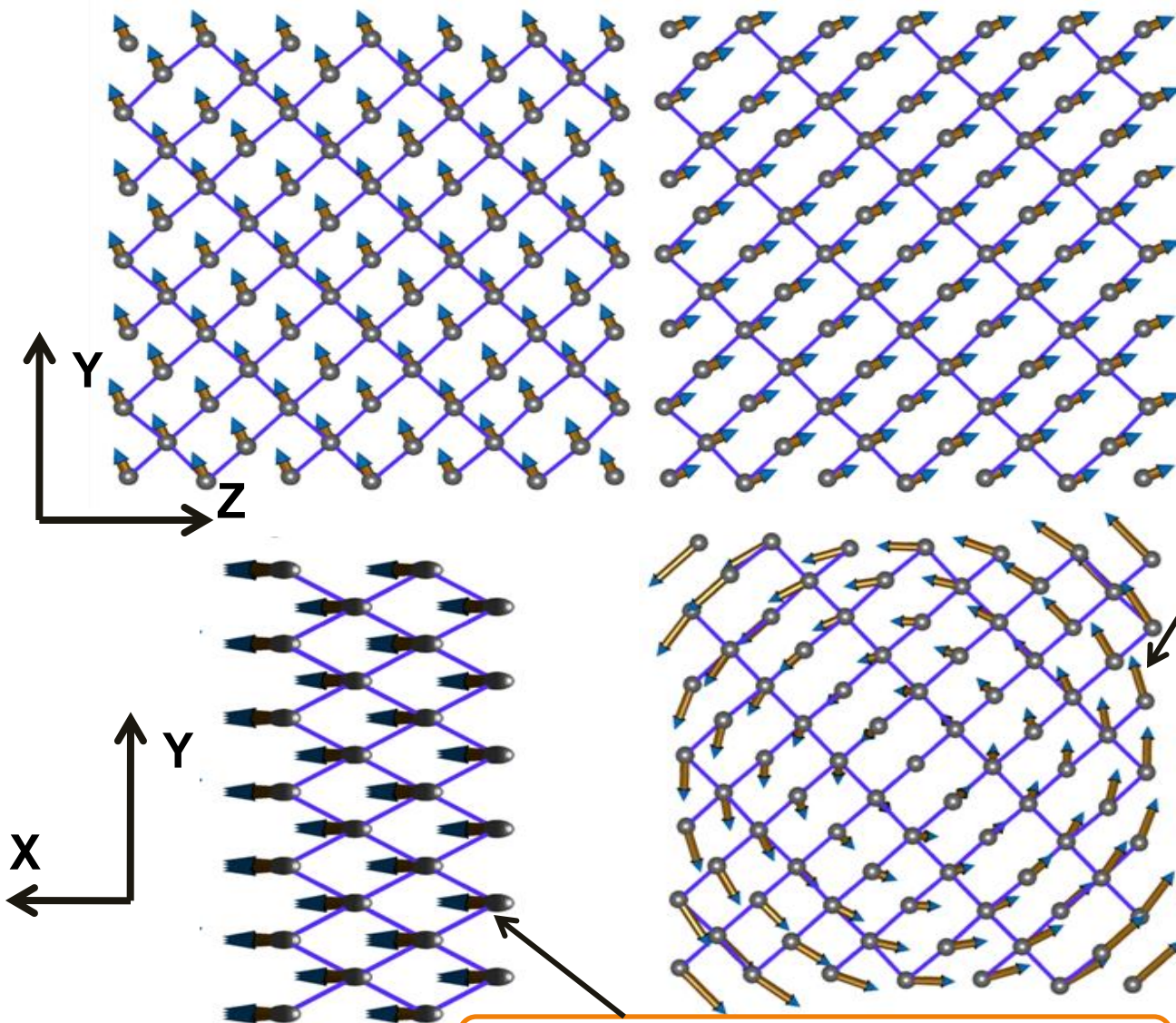
Accurate phonon model crucial for correct calculation of phonon dispersion in nanostructures.

# Phonon dispersion in free-standing nanowires



Strong phonon confinement responsible for different lattice properties in SiNWs compared to bulk.

# Vibrational modes of free-standing [100] SiNWs



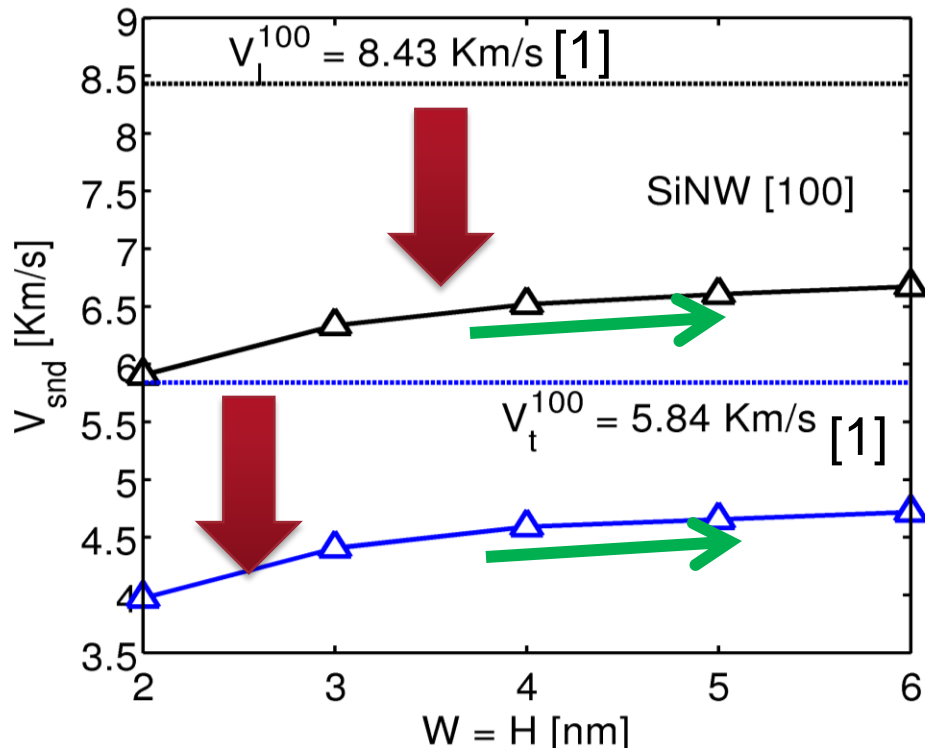
Flexural modes  
(1,2)  
Bends the wire  
along the axis.

Torsional modes  
(4)  
Rotates the wire  
along the axis.

Longitudinal modes (3)

New vibrational  
modes appear in  
free-standing  
nanowires.





$$V_{\text{snd}} = V_{\text{grp}} = \left. \frac{\partial \omega}{\partial q} \right|_{q \rightarrow 0}$$

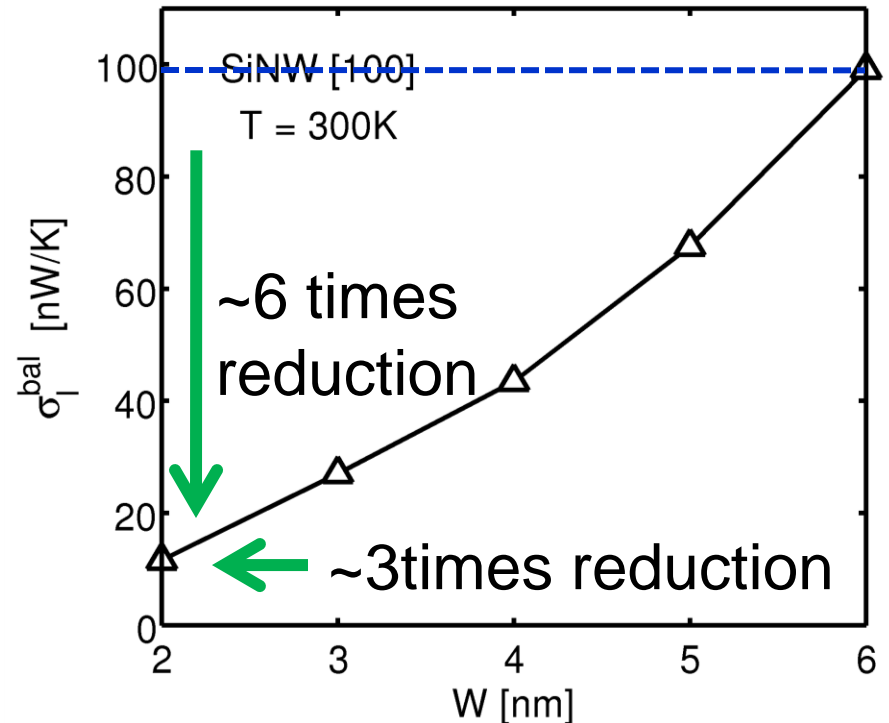
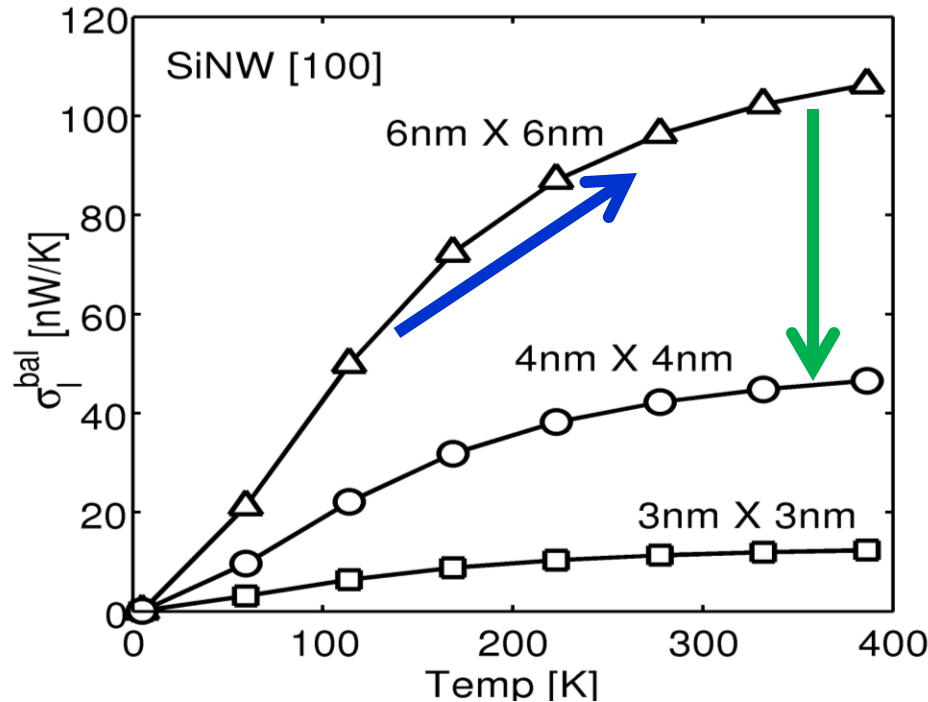
Reduced sound velocity results in lesser dissipation of heat.

A result of phonon confinement.

- ❑ Both longitudinal and transverse sound velocity is less in SiNW.
- ❑ Phonon confinement results in flatter dispersions and hence smaller sound velocity.
- ❑ With increasing W/H  $V_{\text{snd}}$  move towards bulk values.

[1] [www.ioffe.ru/SVA/NSM/Semicond/Si/mechanic.html#Acoustic](http://www.ioffe.ru/SVA/NSM/Semicond/Si/mechanic.html#Acoustic)

# Ballistic lattice thermal conductance ( $\sigma_l^{\text{bal}}$ ) in [100] SiNW



- Higher temperature  $\rightarrow$  more phonon population  $\rightarrow$  inc. in thermal conductance.
- Thermal conductance drops with decreasing cross-section size.
- ~6 fold reduction in thermal conductance for ~3fold increase in width (from 6nm to 2nm).

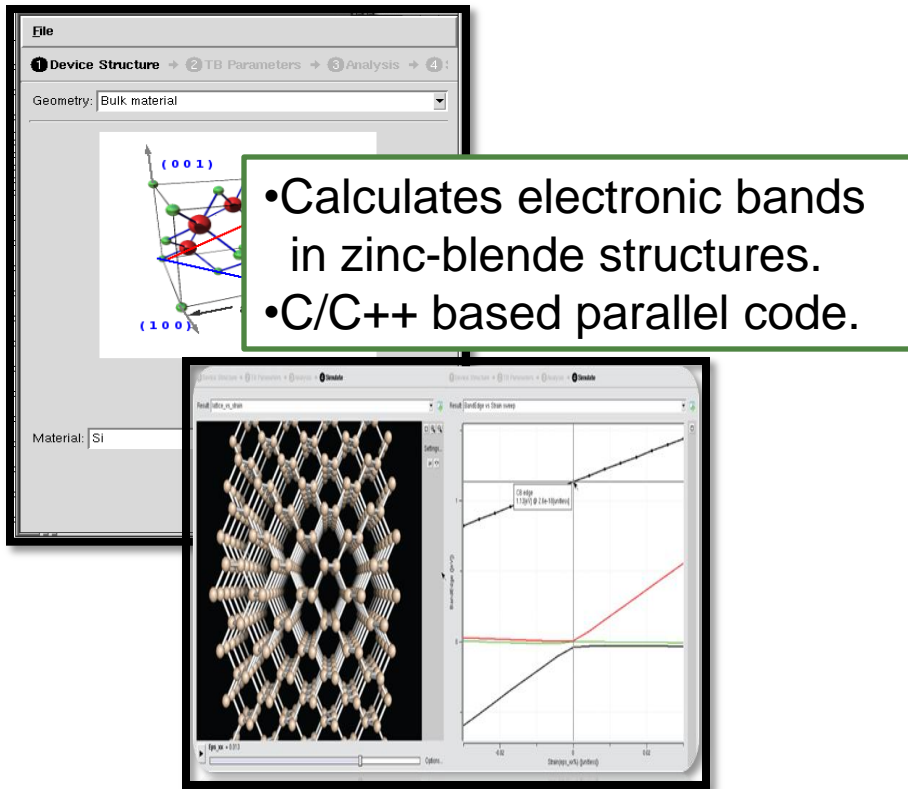
Reduction in ballistic  $\sigma_l$  due to decreasing modes with cross-section size reduction.



- ✓ A **new generalized model** for phonon dispersion in zinc-blende semiconductors.
- ✓ Model **benchmarked with expt. data**.
- ✓ Free standing SiNW show:
  - **Very different phonon dispersion compared to bulk Si.**
  - **New flexural and torsional modes**
  - **Strong phonon confinement.**
- ✓ Phonon confinement results in:
  - **Reduction of both longitudinal and transverse sound velocity.**
  - **Reduction of thermal conductance in small SiNWs.**

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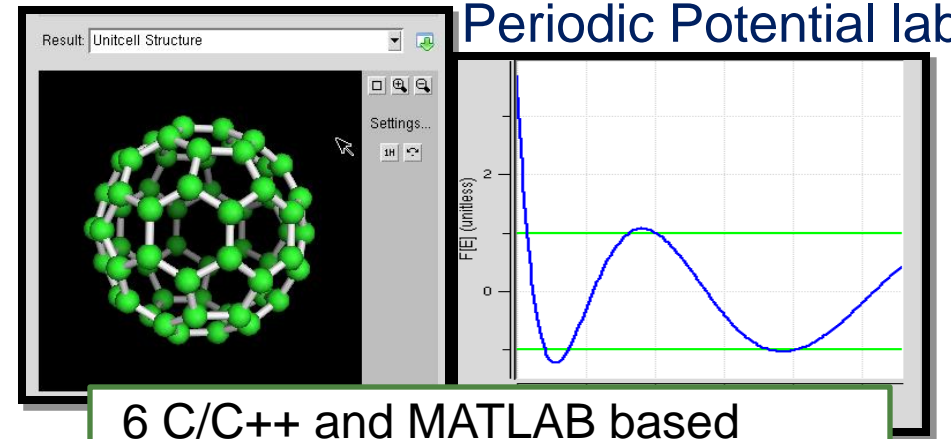
## BandStructure Lab (Research Tool)



- Most popular tool on nanoHUB.
- Over 3K users.
- Till now ran 34503 simulations.
- Has been cited 28 times in research.

## Semiconductor Educational Tools

### Crystal Viewer Tool



- Users (last 12 months) = 887
- Simulations (last 12 months) ~3K

Enabled dissemination of device physics knowledge globally.

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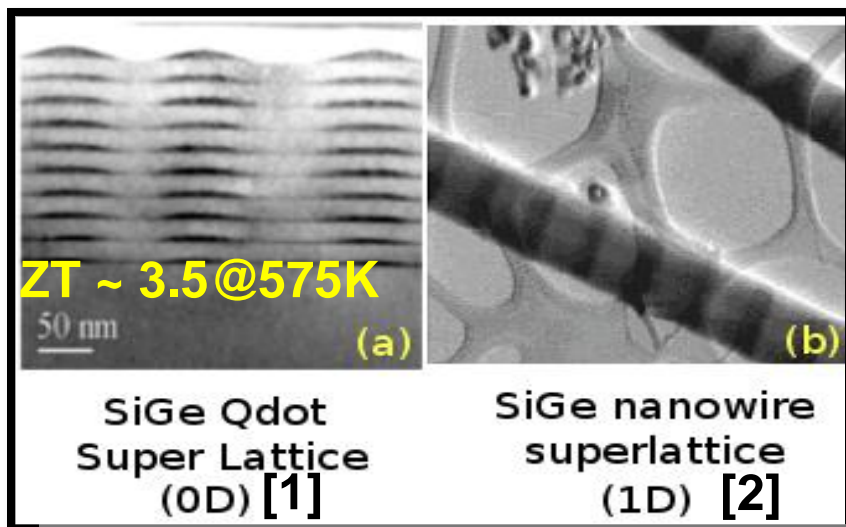
## Ge/Si(001) nanodots

Allows precise thermal conductivity ( $\kappa$ ) control.  
 $\kappa \rightarrow 0.9 \text{ W/m-K}$   
 (>100 fold reduction!!!)

Nature mat., 2010,  
 doi:10.1038/NMAT2752

## Advantages of using SiGe:

- ✓ Advanced CMOS fabrication  
 $\rightarrow$  high quality SiGe structure.
- ✓ Easy integration with Si  $\rightarrow$  better heat recovery at chip level.
- ✓ Monolithic growth on Si  $\rightarrow$  higher energy conversion by thermal resistance reduction.
- ✓ In/cross plane tailoring  $\rightarrow$  optimize TE properties.



SiGe structures provide high ZT.

Nanoscale SiGe structures  
 will need atomic level  
 understanding!!!

[1] Harman et. al, Science, 80, 2002

[2] Wu et. al, Nano. Lett., 2, 2002.

2011

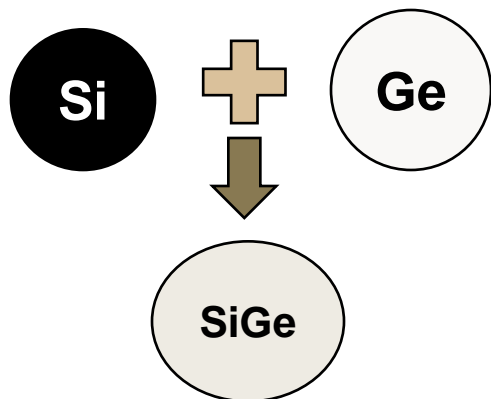
## TE and thermal analysis SiGe nano-structures

- Calculation of  $E(k)$  in SiGe alloys.
- Transmission calculation in SiGe nanowires.
- Lattice property calculations in Si-Ge structures.
- Thermal transport in SiGe superlattices (1D).

Sept 2010

Some initial results are presented for  
the future directions





“Virtual Atom”

[1] Bond-length modification.

$$a_{SiGe} = xa_{Si} + (1 - x)a_{Ge}$$

[2] On-site TB parameter modification.

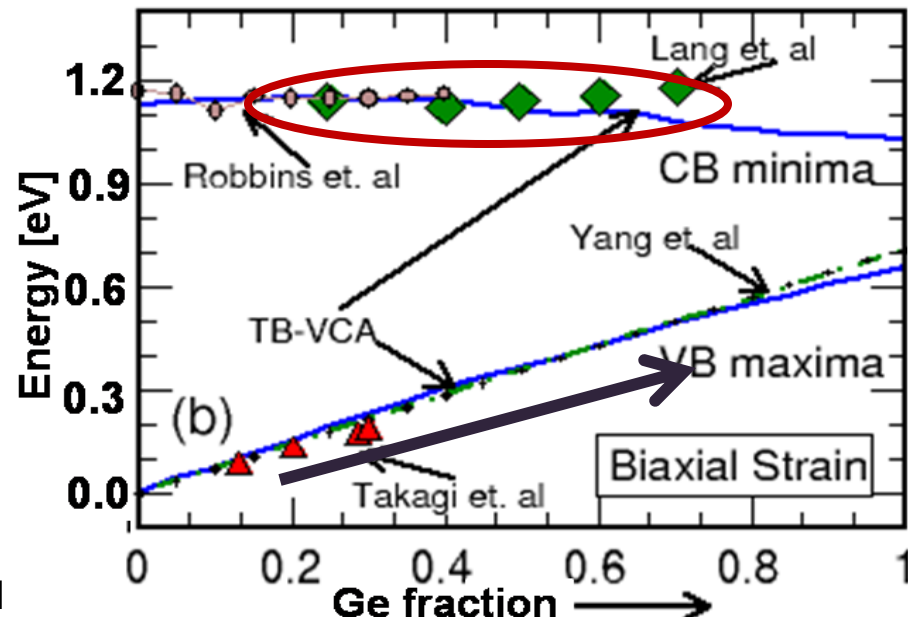
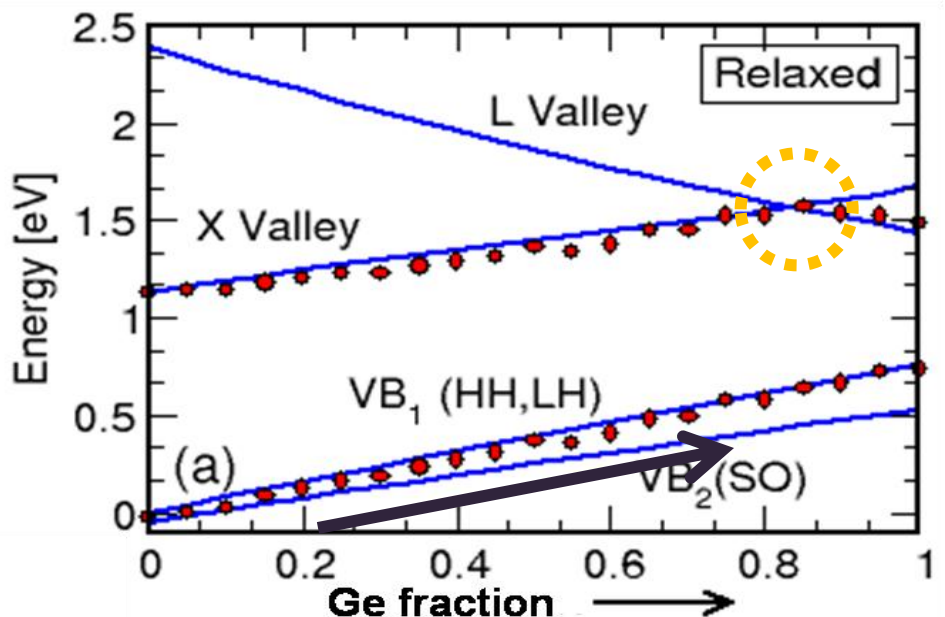
$$E_{A,B}^{strain} = x(E_A + \Delta_A) + (1 - x)(E_B + \Delta_B)$$

[3] Modification of coupling parameters

$$V_{\sigma_1\sigma_2}^{SiGe, strain} = x(V_{\sigma_1\sigma_2}^{Si} \left[ \frac{a_{Si}}{a_{SiGe}} \right]^{\eta_{Si}}) + (1 - x)(V_{\sigma_1\sigma_2}^{Ge} \left[ \frac{a_{Ge}}{a_{SiGe}} \right]^{\eta_{Ge}})$$

39

Tight-Binding based Virtual Crystal Approximation **→** **TB-VCA**

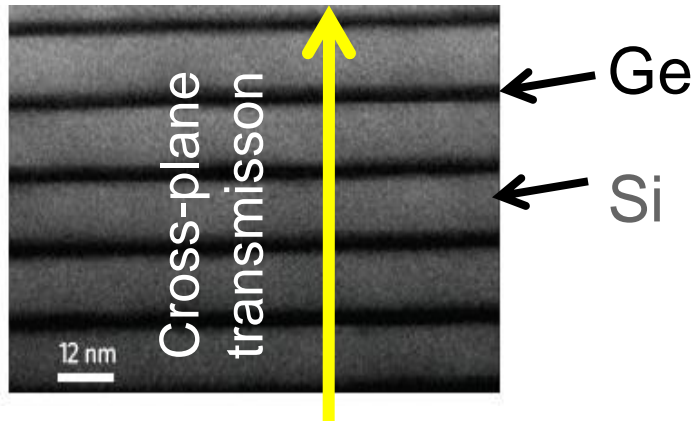


- **Cross-over at 85% Ge** for relaxed SiGe Conduction band (CB) captured.
- Valance Band Edge → **equal amount of change in relaxed and strained SiGe.**
- **CB edge is almost constant** for all Ge% for strained SiGe Bulk.

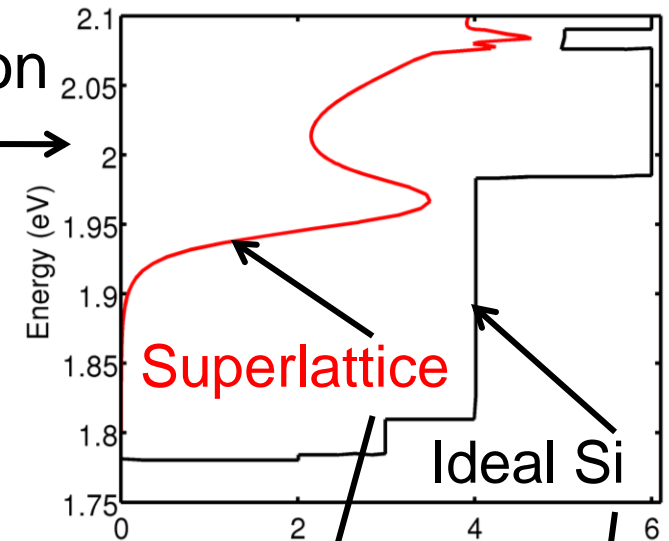
First benchmark of experimental SiGe bandedges using TB-VCA.

Work Published in IEEE EDL , 31, 2010. doi: 10.1109/LED.2010.2040577

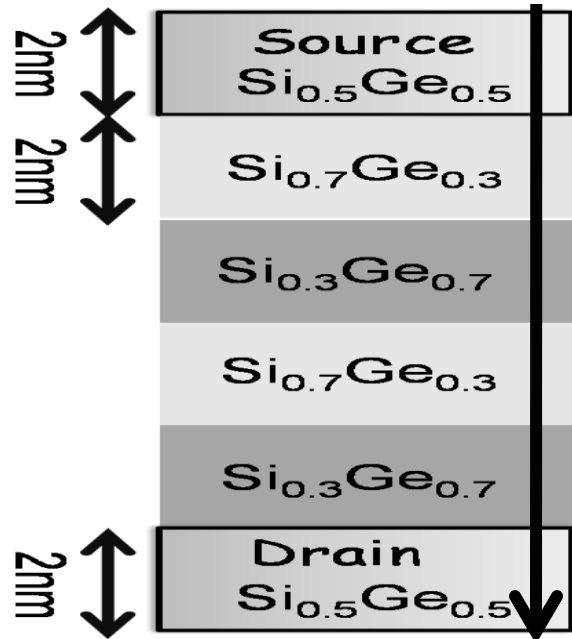
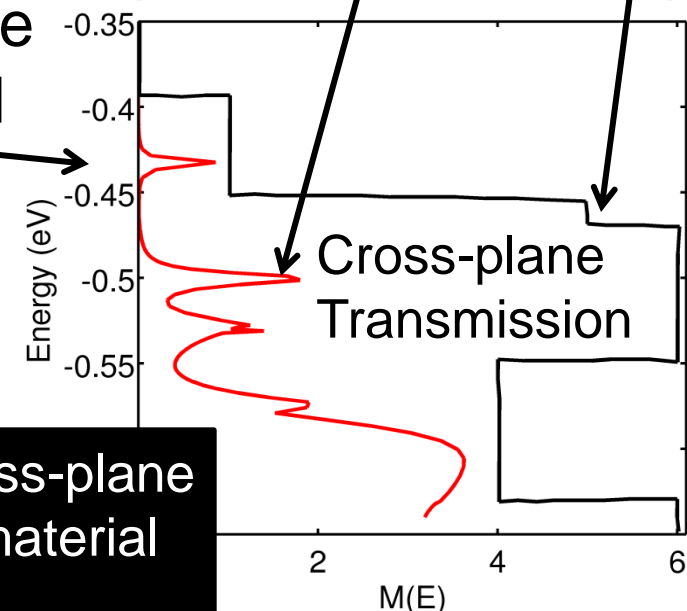
# Transport in SiGe superlattice: Transmission results\*



Conduction  
Band



Valence  
Band



**Simulated  
SiGe  
Nanowire  
Superlattice**  
Radius =  
3nm

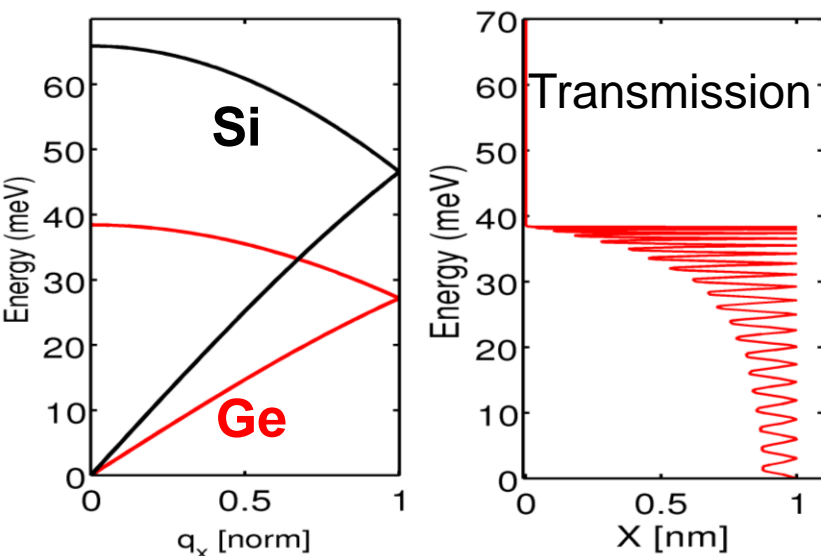
**Strong reduction in cross-plane  
transmission due to material  
mismatch.**

# Thermal transport in SiGe superlattices: Phonon NEGF\*

How does heat flow in  
nano-structures ?

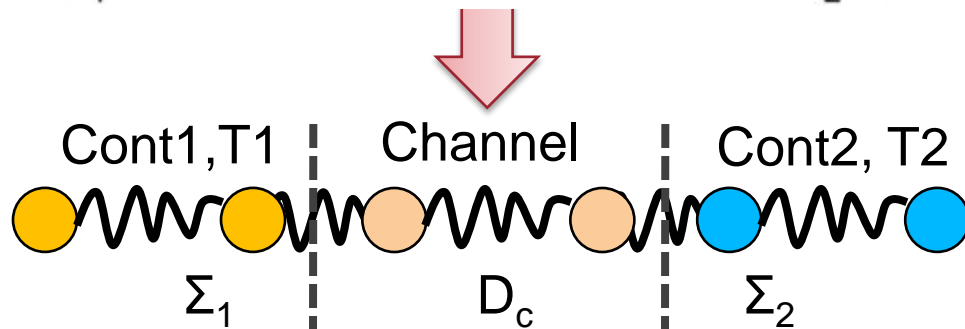
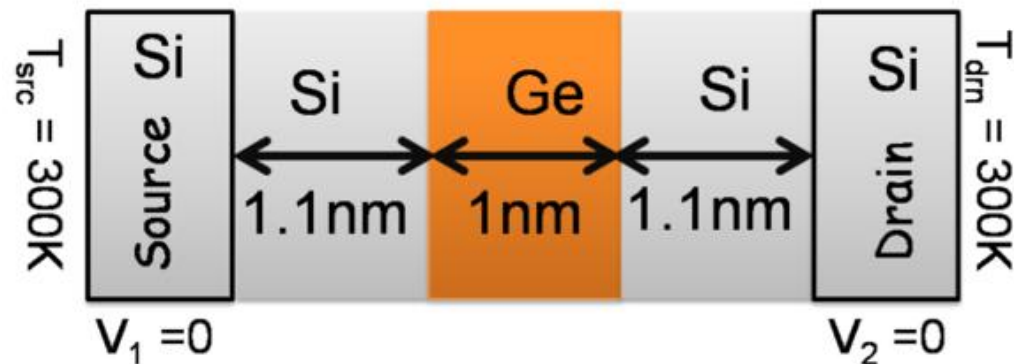
## APPROACH

Coherent phonon picture  
within NEGF\* approach.



- Ge blocks the phonons.
- Resonant states appear.

Simulated Nano-scale  
Si-Ge-Si device  $\lambda_{ph} = 2-3\text{nm}$



1D Spring Model representation  
of the device

$D_c$  = channel dynamical matrix

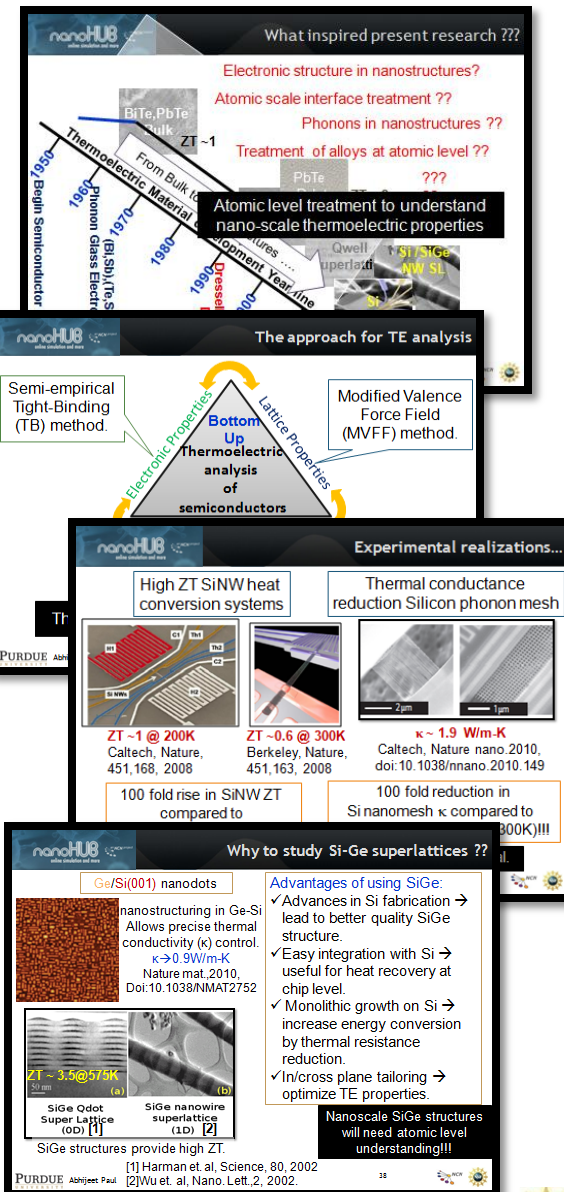
Work in progress for calculating energy  
density, phonon local temperature, etc.

- **How to handle alloy scattering in VCA for nanostructures?**
  - Use of bulk scattering potential not adequate in nanostructures.
  - Use of random alloy method can provide solution.
  - Work going on in this direction with Saumitra Mehrotra.
- **Transmission in SiGe super lattices:**
  - What happens to inplane transmission?
  - What happens at other composition and widths ?
  - Work in progress with Lang Zeng.
- **Nanoscale thermal transport:**
  - Is boundary condition (BC) with temperatures correct?
  - What is ‘**temperature**’ in non eqb. nanoscale systems?
  - Need BCs in terms of energy fluxes.
  - Work in progress with Dr. Tillmann Kubis and Dr. Mathieu Luisier.

- Introduction to Thermoelectricity
  - Basics
  - Material Development
  - Research vectors
- Approach for thermoelectric (TE) analysis.
- Research milestones
  - Results for Silicon nanowires
  - Scientific Outreach
- Future Proposal
  - Investigation of SiGe nanowire superlattices as TE material.
- Summary



- The **current developments, challenges and opportunities** in thermoelectricity introduced.
- Thermoelectric analysis in semiconductor nanostructures:
  - **Electronic structure and new lattice dynamics model with transport.**
- Electronic and lattice effects on SiNWs TE properties:
  - **Tuning Seebeck coefficient by geometry confinement.**
  - **Uniaxial strain improves n-type ballistic PF.**
  - **Reduction in ballistic thermal conductance due to phonon confinement.**
- Future research direction:
  - Analysis of **thermoelectric and thermal effects in SiGe nanowire superlattices.**



- **Overall guidance and direction**
  - Prof. Gerhard Klimeck and Prof. Mark Lundstrom, Purdue University, USA.
  - Prof. Leonid Rokhinson, Purdue University, USA (PhD committee member).
- **Theory and Code development**
  - Dr. Mathieu Luisier, Purdue University, USA (OMEN/OMEN-BSLAB development).
  - Prof. Timothy Boykin, University of Alabama Huntsville, USA (PhD committee member, TB and solid state phys. theory)
  - Dr. Neophytos Neophytou, TU Wien, Austria (Initial MATLAB codes)
- **Discussions and work**
  - Saumitra Mehrotra, Parijat Sengupta, Sunhee Lee, Lang Zeng, Dr. Tillmann Kubis, Raseong Kim and Changwook Jeong, Purdue University, USA
- **Experimental Collaborators**
  - Dr. Giuseppe Tettamanzi, TU Delft, Netherlands, Shweta Deora, IIT Bombay, India, Dr. Subash Rustagi, IME, Singapore.
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  - Junzhe Geng, Victoria Savikhin and Mohammad Zulkifli, Purdue University, USA
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