Homework No. 1 (Based on week 1 lectures)

Problem 1: Consider two different AFM cantilever probes. The first has an equivalent stiffness of 1 N/m, the second has an equivalent stiffness of 10 N/m, however both have share an identical natural frequency of 40kHz. They are both set to oscillate (without damping or forcing) in vacuum at an amplitude of 1nm. Which of the following statements is true?

(a) The equivalent mass of the second probe is the same as the first probe and kinetic energy of the second probe equals that of the first.
(b) The equivalent mass of the second probe is less than the first probe and kinetic energy of the second probe is less than that of the first.
(c) The equivalent mass of the second probe is greater than the first probe and kinetic energy of the second probe more than that of the first.

Problem 2: Consider two different AFM cantilever probes. The first has a natural frequency of 50kHz and a Q factor of 60 (typical of a force modulation cantilever), the second has a natural frequency of 300kHz and a Q factor of 300 (typical of a tapping mode cantilever). We are concerned about the settling time, i.e. the time (seconds) it takes for transient vibration to die down to say e^{-2} (~ 13%) of its original value. Which of the following statements is true?

(a) The second cantilever needs more time to settle down and undergoes more oscillation cycles than the first probe.
(b) The second cantilever needs more time to settle down but undergoes less oscillation cycles than the first probe.
(c) The second cantilever needs less time to settle down but undergoes more oscillation cycles than the first probe.
(d) The second cantilever needs less time to settle down and undergoes less oscillation cycles than the first probe.

Problem 3: Consider a magnetically excited cantilever probe. We are interested in the amplification of cantilever motion due to the resonance. In particular if one considers the cantilever amplitude when the magnetic force excites the probe at a very low frequency say $A(\omega = 0)$ and the amplitude at resonance $A(\omega = \omega_0)$ then the ratio of the two is the following:

(a) $\dfrac{A(\omega = \omega_0)}{A(\omega = 0)} = \dfrac{kA}{F_0}$

(b) $\dfrac{A(\omega = \omega_0)}{A(\omega = 0)} = Q$

(c) $\dfrac{A(\omega = \omega_0)}{A(\omega = 0)} = \sqrt{1 + Q^2}$

(d) $\dfrac{A(\omega = \omega_0)}{A(\omega = 0)} = \sqrt{4 + Q^2}$

Problem 4:
Consider a dither-piezo (acoustically) excited cantilever probe with Q=100. We are interested in comparing the amplitude of the measured signal $q_{rel}(t)$ and the absolute tip deflection $q(t)$. Which of the following is true?

(a) The measured amplitude when driving above resonance $\omega > \omega_0$ is larger than when driving below resonance $\omega < \omega_0$ but the absolute tip amplitude when driving above resonance $\omega > \omega_0$ is smaller than when driving below resonance $\omega < \omega_0$.

(b) The measured amplitude when driving above resonance $\omega > \omega_0$ is larger than when driving below resonance $\omega < \omega_0$ and the absolute tip amplitude behaves the same way with respect to the drive frequency.

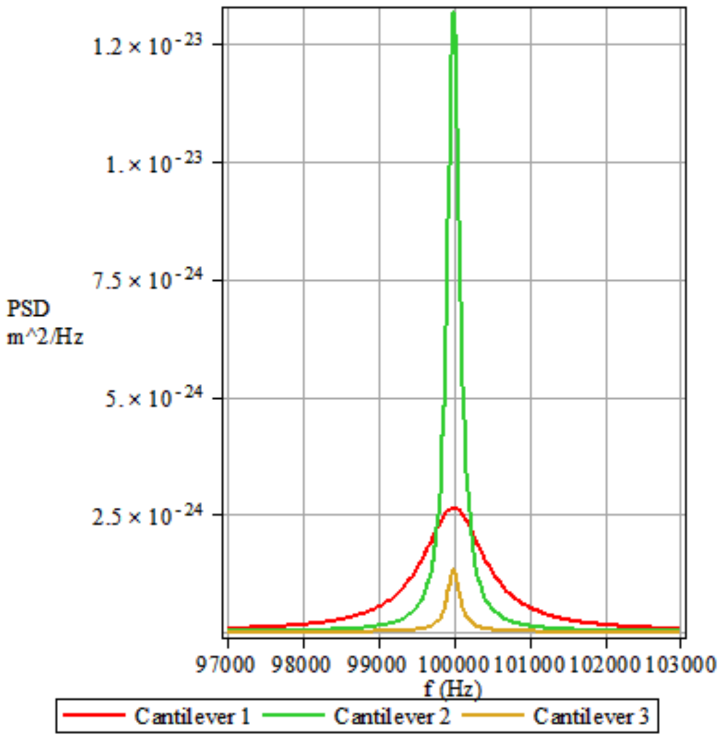
(c) The measured amplitude when driving above resonance $\omega > \omega_0$ is smaller than when driving below resonance $\omega < \omega_0$ but the absolute tip amplitude when driving above resonance $\omega > \omega_0$ is larger than when driving below resonance $\omega < \omega_0$.

(d) The measured amplitude when driving above resonance $\omega > \omega_0$ is smaller than when driving below resonance $\omega < \omega_0$ and the absolute tip amplitude behaves the same way with respect to the drive frequency.

Problem 5: The optical sensitivity (mV->nm) of three different cantilever types 1,2, and 3 with the same natural frequency (100kHz) has been already measured from static force-Z curves on a hard surface. Next their *thermal spectra in air* are measured at room temperature (absolute temperature T=300K). Recall that the Power Spectral Density (PSD) of cantilever vibration under thermal equilibrium with the surrounding gas molecules is given by

$$S_{qq}(f) = \frac{4k_BT \big/ k2\pi f_0 Q}{\left(1-\frac{f^2}{f_0^2}\right)^2 + \frac{f^2}{Q^2 f_0^2}}$$

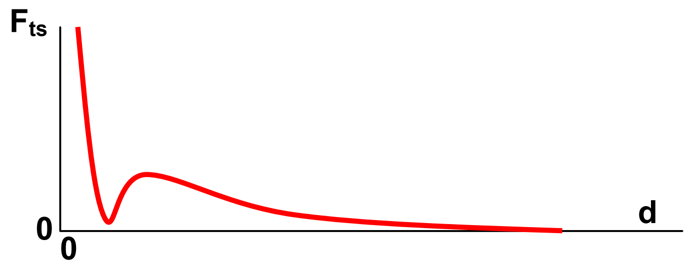
where f is the frequency content (Hz), f_0 the natural frequency, Q the quality factor, k the equivalent spring stiffness, and S_{qq} is the PSD (units m^2/Hz). The three PSD's are shown below.



These are some conclusions one could reach regarding the equivalent stiffness (k1, k2, k3) of the three levers and their Q factors (Q1, Q2, Q3). Which of them are plausible?

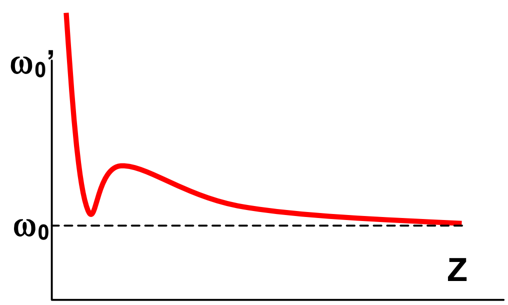
(a) k2<k1=k3, Q1<Q2=Q3
(b) k1=k2<k3, Q1<Q2=Q3
(c) k2<k1=k3, Q3<Q1<Q2
(d) k1=k2<k3, Q3<Q1<Q2

Problem 6: Consider a conservative tip-sample force given by the following curve where the interaction force is always repulsive in nature. This type of interaction often occurs in aqueous solutions where a net repulsive electrostatic force develops (discussed later in class). At small gaps the van der Waals attractive force asserts enough but is not sufficient to change the sign of the repulsive force.

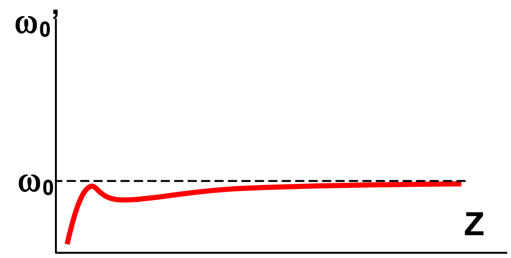


We are interested in understanding how the natural frequency of the cantilever ω_0' could change (in the limit of infinitesimal tip vibrations) as a function of Z the cantilever sample gap. Note that ω_0 is the natural frequency in the absence of tip-sample interaction. Which of the following ω_0' as a function of Z graphs is consistent with the F_{ts} *vs* d graph above? Assume that the cantilever is stiff enough for no snap-in to occur.

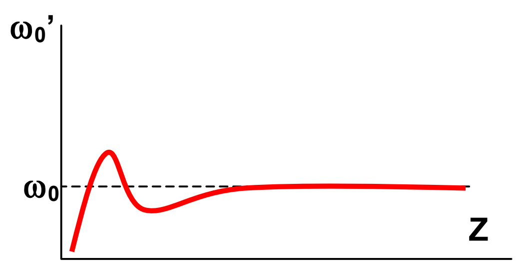
(a)



(b)



(c)



(d)