Fundamentals of Atomic Force Microscopy
Part 2: Dynamic AFM Methods

Week 1, Lecture 6
Point Mass Oscillator: Interacting with Surface

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Tip-sample interaction forces can be conservative (only dependent on gap) or non-conservative (depending on whether approaching or retracting from sample).

Forces can be decomposed into an additive sum of conservative and dissipative forces.
Transient response with tip-sample interaction

\[ d = Z + q, \quad \dot{d} = \dot{q} \quad (\text{since } \dot{Z} = 0) \]
Thus \( F_{ts}(d, \dot{d}) = F_{ts}(Z + q, \dot{q}) \)

Newton's equation of motion
\[ m\ddot{q} + c\dot{q} + kq = F_{ts}(Z + q(t), \dot{q}) \]
Or dividing by \( k \)
\[ \frac{\ddot{q}}{\omega_0^2} + \frac{1}{\omega_0^2 Q} \dot{q} + q = \frac{F_{ts}(Z + q(t), \dot{q})}{k} \quad (1) \]

with \( \omega_0 = \sqrt{\frac{k}{m}} \), \( Q = \frac{m\omega_0}{c} \)
Nonlinear ordinary differential equation (autonomous)
Static and transient solution

\[
\ddot{q} + \frac{1}{\omega_0^2} \dot{q} + q = \frac{F_{ts}(Z + q(t), \dot{q})}{k} \tag{1}
\]

- **Break up solution into two parts**

\[q(t) = q^* + \dot{q}(t)\]  \hspace{1cm} (2)

where \(q^*\) is the solution to the static problem i.e.

\[q^* = \frac{F_{ts}(Z + q^*, 0)}{k} \text{ thus } d^* = Z + q^*\]  \hspace{1cm} (3)

and \(\ddot{q}^* = \dddot{q}^* = 0\) (unchanging with time)

Plug (2) and (3) into (1)

\[
\ddot{\ddot{q}} + (\ddot{q} + q^*) + \frac{1}{\omega_0^2} \dot{q} = \frac{F_{ts}(Z + q^* + \dot{q}, \ddot{q})}{k} \tag{4}
\]

If \(\ddot{q} \ll 1\), \(\dddot{q} \ll Z + q^*\) or when \(\dddot{q} \ll d^*\) then (see Appendix)

\[
\dddot{\ddot{q}} + \left(1 - \frac{1}{k} \frac{\partial F_{ts}(d, \dot{d})}{\partial d}\right) \ddot{q} + \left(\frac{1}{\omega_0 Q} - \frac{1}{k} \frac{\partial F_{ts}(d, \dot{d})}{\partial \dot{d}}\right) \dot{q} = 0 \text{ (Transient)} \tag{5}
\]

\[q^* = \frac{F_{ts}(Z + q^*, 0)}{k} \text{ (Static)}\]
The transient equation tells us how the probe would oscillate if perturbed from $q^*$.
Transient probe motion near surface

\[
\frac{\ddot{q}}{\omega_0^2} + \left( \frac{1}{\omega_0 Q} - \frac{1}{k} \frac{\partial F_{ts}(d,\dot{d})}{\partial \dot{d}} \bigg|_{d=d^*,\dot{d}=0} \right) \dot{q} + \left( 1 - \frac{1}{k} \frac{\partial F_{ts}(d,\dot{d})}{\partial d} \bigg|_{d=d^*,\dot{d}=0} \right) \ddot{q} = 0
\]

Or

\[
\ddot{q} + \frac{\omega_0}{Q} \left( 1 + \frac{Q\omega_0}{k} \left( - \frac{\partial F_{ts}(d,\dot{d})}{\partial \dot{d}} \bigg|_{d=d^*,\dot{d}=0} \right) \right) \dot{q} + \omega_0^2 \left( 1 - \frac{1}{k} \frac{\partial F_{ts}(d,\dot{d})}{\partial d} \bigg|_{d=d^*,\dot{d}=0} \right) \ddot{q} = 0
\]

\[
\omega_0'^2 = \omega_0^2 \left( 1 + \frac{1}{k} \left( - \frac{\partial F_{ts}(d,\dot{d})}{\partial d} \bigg|_{d=d^*,\dot{d}=0} \right) \right)
\]

Tip-sample force gradient, \( k_{ts} \)

\[
\frac{1}{Q'} = \frac{1}{Q} \left( 1 + \frac{Q\omega_0}{k} \left( - \frac{\partial F_{ts}(d,\dot{d})}{\partial \dot{d}} \bigg|_{d=d^*,\dot{d}=0} \right) \right)
\]

Tip-sample viscous dashpot, \( c_{ts} \)

In the limit of small transient motion, the natural frequency \( \omega_0 \) and Q factor change to \( \omega_0' \) and Q' due to tip-sample interactions.
Implications of Linearized analysis

\[ \omega_0^2 = \omega_0^2 \left( 1 + \frac{k_{ts}}{k} \right) \]

\[ \frac{1}{Q'} = \frac{1}{Q} \left( 1 + \frac{Q\omega_0}{k} c_{ts} \right) \]  \hspace{1cm} (6)

- When \( k_{ts} < 0 \) attractive gradient and natural frequency decreases from \( \omega_0 \)
- When \( k_{ts} > 0 \) repulsive gradient and natural frequency increases from \( \omega_0 \)
- \( Q' \) decreases from \( Q \) due to tip-sample dissipation
  Usually \( c_{ts} > 0 \)

Example of repulsive gradient

Example of attractive gradient