ECE 606 Homework Week 5
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Some of the problems below are taken/adapted from Chapter 4 in Advanced Semiconductor Fundamentals, 2nd Ed. By R.F. Pierret.

1) The conduction band minima in GaP occurs at the boundary of the first Brillouin zone along [100] directions. The constant energy surfaces are ellipsoidal with \( m^*_i = 1.12m_0 \) and \( m^*_r = 0.22m_0 \). Determine the density-of-states effective mass for electrons in GaP.

2) Assuming that the Si valence bands are spherical with \( m^*_m = 0.537m_0 \) and \( m^*_l = 0.153m_0 \), determine the fraction of holes that are in the heavy hole band. You should assume non-degenerate conditions at \( T = 300 \) K. Also assume that the effective masses quoted above, which were measured by cyclotron resonance at 4K, can be used at room temperature.

3) In a Si MOSFET, the source and drain regions are heavily doped. Assuming that the dopants are fully ionized and that \( N_D = 10^{20} \) cm\(^{-3} \), compute the location of the Fermi level. Compare the answer you get assuming Maxwell-Boltzmann statistics with the answer from Fermi-Dirac statistics. You may assume \( T = 300\)K and that the effective density of states is

\[
N_C = \frac{1}{4} \left( \frac{2m^*_D k_B T}{\pi \hbar^2} \right)^{3/2} = 3.23\times10^{19} \text{ cm}^{-3}
\]

You will need to compute an inverse Fermi-Dirac integral. A FD integral tool is available on nanoHUB.org at: nanohub.org/resources/fdical

You can also find an iPhone app that does Fermi-Dirac integrals.

4) The distribution of electrons in the conduction band, \( n(E) \), is peaked at an energy, \( \hat{n} \) that is very near the bottom of the conduction band, \( E_C \). Assume Maxwell-Boltzmann carrier statistics and derive an expression for \( \hat{n} \) for the following cases:

4a) 1D semiconductor with parabolic energy bands
4b) 2D semiconductor with parabolic energy bands
5) Sometimes we need to know the average kinetic energy for electrons in the conduction band (or for holes in the valence band). Answer the follow questions:

5a) Derive a general expressions for the average kinetic energy per electron in the conduction band.
5b) Assume spherical energy bands and evaluate the general expression for a **1D** semiconductor. Simplify your answer for Maxwell-Boltzmann carrier statistics.
5c) Assume spherical energy bands and evaluate the general expression for a **2D** semiconductor. Simplify your answer for Maxwell-Boltzmann carrier statistics.
5d) Assume spherical energy bands and evaluate the general expression for a **3D** semiconductor. Simplify your answer for Maxwell-Boltzmann carrier statistics.

6) Assume Si at $T = 300K$, doped with arsenic at $N_D = 10^{16} \text{ cm}^{-3}$. Make reasonable assumptions and answer the following questions.

6a) Compute the density of electrons in the conduction band.
6b) Compute the location of the Fermi-level.
6c) Compute the fraction of the dopants that are ionized.
6d) Compute the density of holes in the valence band.

7) Assume Si at $T = 640K$, doped with arsenic at $N_D = 10^{16} \text{ cm}^{-3}$. Make reasonable assumptions and answer the following questions.

7a) Compute the density of electrons in the conduction band.
7b) Compute the location of the Fermi-level.
7c) Compute the fraction of the dopants that are ionized.
7d) Compute the density of holes in the valence band.

8) Assume Si at $T = 77K$, doped with arsenic at $N_D = 10^{16} \text{ cm}^{-3}$. Make reasonable assumptions and answer the following questions.

8a) Compute the density of electrons in the conduction band.
8b) Compute the location of the Fermi-level.
8c) Compute the fraction of the dopants that are ionized.
8d) Compute the density of holes in the valence band.
9) Assume Si at $T = 300K$, doped with arsenic at $N_D = 10^{18}$ cm$^{-3}$. Make reasonable assumptions and answer the following questions.
9a) Compute the density of electrons in the conduction band.
9b) Compute the location of the Fermi-level.
9c) Compute the fraction of the dopants that are ionized.
9d) Compute the density of holes in the valence band.

10) Assume Si at $T = 300K$, doped with arsenic at $N_D = 10^{20}$ cm$^{-3}$. Make reasonable assumptions and answer the following questions. (In this case, you should assume that “heavy doping effects” cause the dopants to be fully ionized and the bandgap to shrink by 0.1 eV.)
10a) Compute the density of electrons in the conduction band.
10b) Compute the location of the Fermi-level.
10c) Compute the fraction of the dopants that are ionized.
10d) Compute the density of holes in the valence band.

11) For the energy band sketched below, provide sketches of the following:
11a) the carrier densities, $n(x)$, and $p(x)$ vs. position,
11b) the electrostatic potential, $\psi(x)$, vs. position.
11c) the electric field $E$ vs. position.
11d) the space charge density, $\rho(x)$ vs. position.