SOLUTIONS: ECE 606 Homework Week 7
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1) Consider an n-type semiconductor for which the only states in the bandgap are donor levels (i.e. \( E_T = E_D \)). Begin with the SRH formula:

\[
R = \frac{np - n_i^2}{(n+n_i)\tau_p + (p+p_i)\tau_n}
\]

where

\[
n_i = n_i e^{(E_T - E_i)/k_BT} \quad \text{and} \quad p_i = n_i e^{(E_T - E_i)/k_BT}
\]

and answer the following questions.

1a) Derive an expression for the low-injection, minority hole lifetime.

Solution:

\[
R \approx \frac{n_0 \Delta p}{(n_0 + n_i)\tau_p + (\Delta p + p_i)\tau_n}
\]

Low level injection in an N-type semiconductor means: \( n_0 + \Delta n = n_0 \) and \( p_0 + \Delta p = \Delta p \) so:

Assume a moderately doped semiconductor, so the Fermi level is located well above \( E_i \) but well below \( E_C \) and \( E_D \). In this case:

\[
n_i = n_i e^{(E_T - E_i)/k_BT} \gg n_0 = n_i e^{(E_F - E_i)/k_BT}
\]

\[
p_i = n_i e^{(E_T - E_D)/k_BT} \ll p_0 = n_i e^{(E_F - E_i)/k_BT} \quad \text{and} \quad p_0 \ll n_0
\]

so the recombination rate becomes:
\[ R = \frac{\Delta p}{(n_1/n_0) \tau_p} = \frac{\Delta p}{\tau_{\text{hole}}} \quad \tau_{\text{hole}} = (n_1/n_0) \tau_p \]

1b) Explain mathematically why donor levels are not efficient recombination centers.

**Solution:**
For donor level traps:

\[ \tau_{\text{hole}} = (n_1/n_0) \tau_p = e^{(E'_p - E_F)/k_B T} \tau_p \gg \tau_p \]

The hole minority carrier lifetime is very long, so the recombination rate is low.

1c) Explain physically why donor levels are not efficient recombination centers.

**Solution:**

Donor “traps” can efficiently capture electrons from the conduction band, but they will quickly be emitted back to the conduction band. The probability of emission to the conduction band is MUCH more efficient than the probability of capturing a hole from the valence band (which is necessary for recombination).

2) When computing SRH recombination rates, we usually focus on defects with energy levels near the middle of the bandgap. Beginning with the SRH expression, show that states near the middle of the bandgap have the largest effect on the SRH recombination rate.

**Solution:**

\[ R = \frac{np - n_i^2}{(n+n_i)\tau_p + (p+p_i)\tau_n} \quad n_i = n_f e^{(E'_i - E_F)/k_B T} \quad p_i = n_f e^{(E'_i - E_F)/k_B T} \]

Need to evaluate: \[ \frac{\partial R}{\partial E'_T} = 0 \]

\[ \frac{\partial R}{\partial E'_T} = (np - n_i^2)(-1) \left[ (n+n_i)\tau_p + (p+p_i)\tau_n \right]^{-2} \times \left( \frac{\partial n_i}{\partial E'_T} \tau_p + \frac{\partial p_i}{\partial E'_T} \tau_n \right) = 0 \]
\[
\left( \frac{\partial n_1}{\partial E'_T} \tau_p + \frac{\partial p_1}{\partial E'_T} \tau_n \right) = 0
\]

\[
\frac{\partial n_1}{\partial E'_T} = \frac{n_1}{k_B T} \quad \frac{\partial p_1}{\partial E'_T} = -\frac{p_1}{k_B T}
\]

\[
n_1 \tau_p = p_1 \tau_n \quad \frac{n_1}{p_1} = e^{(E'_i - E_i) / k_B T} = \frac{\tau_n}{\tau_p}
\]

\[
E'_i = E_i + \frac{k_B T}{2} \ln \left( \frac{\tau_n}{\tau_p} \right)
\]

If \( \tau_n \) is not too different from \( \tau_p \), then the most effective recombination centers will be located near the intrinsic level.

3) Defect states come in two flavors. A “donor-like” state is positive when empty and neutral when filled. An acceptor-like state is neutral when empty and negative when filled. These states have different cross sections for electrons and holes. For example, when empty, a donor-like state has a large cross section for electron capture because there is a Coulombic attraction (a typical number might be \(3 \times 10^{-13} \text{ cm}^2\)). When a donor like state is filled, it is neutral and has a small cross section for holes (typically the radius of the defect itself, or about \(10^{-15} \text{ cm}^2\)).

3a) Assume an N-type silicon sample with an RG center concentration of \(10^{12} \text{ cm}^{-3}\). (Assume that the RG centers are located near the middle of the bandgap, as is typically the case.) Assume room temperature, so that \(v_{th} = 10^7 \text{ cm/s}\). Compute the minority hole lifetime assuming that the defects are donor-like.

**Solution:**

Fermi level is well above the trap energy, so the traps are filled, which means neutral for donor-like traps. This will give a small cross section, because there is no Coulombic attraction for holes. So (assuming that the thermal velocity is \(10^7 \text{ cm/s}\)):

\[
\tau_p = \frac{1}{e_p N_T} = \frac{1}{10^{-15} \times 10^7 \times 10^{12}} = 10^{-4} \text{s} = 100 \mu\text{s}
\]
\[ \tau_p = 100 \, \mu s \]

3b) Repeat 3a) assuming that the defects are acceptor-like. (Assume that the RG centers are located near the middle of the bandgap, as is typically the case.)

When acceptor like traps are filled, they are negatively charged, which creates a strong attraction for holes. The cross section will be large.

\[ \tau_p = \frac{1}{c_p N_T} = \frac{1}{3 \times 10^{-13} \times 10^7 \times 10^{12}} = 3.33 \times 10^{-7} \, s = 0.33 \, \mu s \]

\[ \tau_p = 0.33 \, \mu s \]

4) The Fermi function gives the probability that a state in the conduction or valence band is occupied. One might think that the probability that a state in the forbidden gap (i.e. a trap or recombination center) would also be given by the Fermi function, but this is not quite right. Begin with eqn. (5.9a) in ASF, simplify it for equilibrium, and obtain the correct result.

Solution:

From eqn. (5.9a):

\[ r_N = c_n p_T n - e_n n_T \]

In equilibrium:

\[ r_{N0} = c_{n0} p_{T0} n_0 - e_{n0} n_{T0} = 0 \]

From eqn. (5.11a):

\[ e_{n0} = c_{n0} n_1 \]

so

\[ r_{N0} = c_{n0} p_{T0} n_0 - c_{n0} n_1 n_{T0} = 0 \]

\[ p_{T0} n_0 = n_1 n_{T0} \]
\[
\begin{align*}
(N_T - n_{T0})n_0 &= n_1 n_{T0} \\
N_T n_0 &= n_{T0} (n_1 + n_0) \\
\frac{n_{T0}}{N_T} &= f_T = \frac{n_0}{n_1 + n_0} = \frac{1}{1 + n_1/n_0} \\
n_1/n_0 &= \frac{n_1 e^{(E_T - E_i)/k_B T}}{n_0 e^{(E_T - E_i)/k_B T}} = e^{(E_T - E_i)/k_B T} \\
\frac{1}{1 + e^{(E_T - E_i)/k_B T}} \leq f_T(E_T^*) &= \frac{1}{1 + e^{(E_T - E_i)/k_B T}} \\
E_T^* &= E_T \pm k_B T \ln g_T \text{ (plus sign for acceptor-like defects and minus sign for donor-like defects)} \\
\text{For the plus sign (acceptor-like states):} & \quad \frac{1}{1 + g_T e^{(E_T - E_i)/k_B T}} \\
\text{and for the minus sign (donor-like states):} & \quad \frac{1}{1 + \frac{1}{g_T} e^{(E_T - E_i)/k_B T}} \\
\text{These are almost Fermi functions – except for the trap degeneracy factor (like the degeneracy factors we saw for donors and acceptors).}
\end{align*}
\]

Problems 5) – 13): The following problems concern the Minority Carrier Diffusion equation for electrons as follows:

\[
\frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} + \frac{\Delta n}{\tau_n} + G_L
\]

For all the following problems, assume silicon at room temperature, uniformly doped with \( N_A = 10^{17} \text{ cm}^{-3} \), \( \mu_n = 300 \text{ cm}^2/\text{V sec} \), \( \tau_n = 10^{-6} \text{ s} \). From these numbers, we find:
D_n \frac{k_B T}{q} \mu_n = 7.8 \text{ cm}^2/\text{s} \quad L_n = \sqrt{D_n \tau_n} = 27.9 \mu\text{m}

Unless otherwise stated, these parameters apply to all of the problems below.

5) The sample is uniformly illuminated with light, resulting in an optical generation rate \( G_L = 10^{30} \text{ cm}^{-3} \text{ sec}^{-1} \). Find the steady-state excess minority carrier concentration and the QFL’s \( F_n \) and \( F_p \). Assume spatially uniform conditions, and approach the problem as follows.

5a) Simplify the Minority Carrier Diffusion Equation for this problem.

Solution:

Begin with:

\[
\frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L
\]

Simplify for steady-state:

\[
0 = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L
\]

Simplify for spatially uniform conditions:

\[
0 = 0 - \frac{\Delta n}{\tau_n} + G_L
\]

So the simplified MDE equation is:

\[
- \frac{\Delta n}{\tau_n} + G_L = 0
\]

5b) Specify the initial and boundary conditions, as appropriate for this problem.

Solution:

Since there is no time dependence, there is no initial condition. Since there is no spatial dependence, there are no boundary conditions.

5c) Solve the problem.

Solution:

In this case the solution is trivial:

\[
\Delta n = G_L \tau_n = 10^{30} \times 10^{-6} = 10^{14} \text{ cm}^3
\]
Now compute the QFLs:

Since we are doped p-type and in low level injection:

\[ p \approx p_0 = N_A = n_i e^{(E_i - F_p)/k_B T} \]

\[ F_p = E_i - k_B T \ln \left( \frac{N_A}{n_i} \right) = E_i - 0.026 \ln \left( \frac{10^{17}}{10^{10}} \right) = E_i - 0.41 \text{ eV} \]

\[ n \approx \Delta n >> n_0 = n_i e^{(F_n - E_i)/k_B T} \]

\[ F_n = E_i + k_B T \ln \left( \frac{\Delta n}{n_i} \right) = E_i + 0.026 \ln \left( \frac{10^{14}}{10^{10}} \right) = E_i + 0.24 \text{ eV} \]

5d) Provide a sketch of the solution, and explain it in words.

**Solution:**
The excess carrier density is just constant, independent of position. So are the QFL’s, but they split because we are not in equilibrium.

![Diagram of QFLs](image)

The hole QFL is essentially where the equilibrium Fermi level was, because the hole concentration is virtually unchanged. But the electron QFL is much closer to the conduction band because there are orders of magnitude more electrons.

6) The sample has been uniformly illuminated with light for a long time. The optical generation rate is \( G_L = 10^{20} \text{ cm}^{-3} \text{ sec}^{-1} \). At \( t = 0 \), the light is switched off. Find the excess minority carrier concentration and the QFL’s vs. time. Assume spatially uniform conditions, and approach the problem as follows.

6a) Simplify the Minority Carrier Diffusion Equation for this problem.

**Solution:**
Begin with:

\[ \frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L \]
Simplify for spatially uniform conditions with no generation:

$$\frac{d\Delta n}{dt} = -\frac{\Delta n}{\tau_n}$$

6b) Specify the initial and boundary conditions, as appropriate for this problem.

**Solution:**
Because there is no spatial dependence, there is no need to specify boundary condition. The initial condition is (from prob. 5):

$$\Delta n(t = 0) = 10^{14} \text{ cm}^{-3}$$

6c) Solve the problem.

**Solution:**

$$\frac{d\Delta n}{dt} = -\frac{\Delta n}{\tau_n}$$

The solution is: $$\Delta n(t) = Ae^{-t/\tau_n}$$

Now use the initial condition:

$$\Delta n(0) = 10^{14} = A$$

$$\Delta n(t) = 10^{14} e^{-t/\tau_n}$$

6d) Provide a sketch of the solution, and explain it in words.

**Solution:**

For the electron QFL: $$F_n(t) = E_i + k_B T \ln \left( \frac{\Delta n(t) + n_0}{n_i} \right)$$
Initially, $\Delta n(t) >> n_0$ and $\Delta n(t) = \Delta n(0)e^{-t/\tau_n}$, so $F_n(t)$ initially drops linearly with time towards $E_F$.

7) The sample is uniformly illuminated with light, resulting in an optical generation rate $G_L = 10^{20}$ cm$^{-3}$ sec$^{-1}$. The minority carrier lifetime is 1 μsec, except for a thin layer (10 nm wide near $x = 0$ where the lifetime is 0.1 nsec. Find the steady state excess minority carrier concentration and QFL’s vs. position. You may assume that the sample extends to $x = +\infty$. **HINT:** treat the thin layer at the surface as a boundary condition – do not try to resolve $\Delta n(x)$ inside this thin layer.

Approach the problem as follows.

7a) Simplify the Minority Carrier Diffusion Equation for this problem.

**Solution:**

Begin with: 

$$\frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L$$

Simplify for steady-state conditions: 

$$0 = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L$$

The simplified MDE equation is:

$$D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L = 0 \quad \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L^2_n} + \frac{G_L}{D_n} = 0 \quad L_n = \sqrt{D_n \tau_n}$$

$$\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L^2_n} + \frac{G_L}{D_n} = 0$$

where $L_n = \sqrt{D_n \tau_n}$ is the minority carrier “diffusion length.”

7b) Specify the initial and boundary conditions, as appropriate for this problem.

**Solution:**

Since this is a steady-state problem, there is no initial condition. As $x \to \infty$, we have a uniform semiconductor with a uniform generation rate. In a uniform semiconductor under illumination, $\Delta n = G_L \tau_n$, so

$$\Delta n(x \to \infty) = G_L \tau_n$$
At the surface, the total number of e-h pairs recombining per cm² per second is

\[ R_S = \frac{\Delta n(0)}{\tau_S} \Delta x = \frac{\Delta x}{\tau_S} \Delta n(0) = S_F \Delta n(0) \text{ cm}^{-2}\text{ s}^{-1} \]

where \( S_F = \frac{\Delta x}{\tau_S} \text{ cm/s} \) is the “front surface recombination velocity.”

\[ S_F = \frac{\Delta x}{\tau_S} = \frac{10^{-6}}{10^{-10}} = 10^4 \text{ cm/s} \]

In steady-state, carriers must diffuse to the surface at the same rate that there are recombining there so that the excess minority carrier concentration at the surface stays constant with time.

\[ +D_n \frac{d\Delta n}{dx} \bigg|_{x=0} = R_S = S_F \Delta n(0) \]

**Note:** We usually specify surface recombination by just giving the surface recombination velocity – not the lifetime and thickness of the thin layer at the surface.

7c) Solve the problem.

**Solution:**

\[ \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n^2} + \frac{G_L}{D_n} = 0 \]

Solve the homogeneous problem first \( G_L = 0 \).

\[ \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n^2} = 0 \text{ solution is } \Delta n(x) = Ae^{-x/L_n} + Be^{+x/L_n} \]

Now solve for a particular solution by letting \( x \to \infty \) where everything is uniform:

\[ -\frac{\Delta n}{L_n^2} + \frac{G_L}{D_n} = 0 \text{ The solution is: } \Delta n = -\frac{L_n^2}{D_n} G_L = G_L \tau_n \]
Add the two solutions: \( \Delta n(x) = A e^{-x/L_n} + B e^{+x/L_n} + G_L \tau_n \)

To satisfy the first boundary condition in 7b): \( B = 0 \). Now consider the second:

\[
+D_n \frac{d \Delta n}{dx} \bigg|_{x=0} = -\frac{D_n}{L_n} A = S_F \Delta n(0) = S_F (A + G_L \tau_n)
\]

\[
A = - \frac{S_F G_L \tau_n}{D_n/\tau_n + S_F} = - \frac{G_L \tau_n}{1 + \left(D_n/\tau_n\right)/S_F}
\]

\[
\Delta n(x) = G_L \tau_n \left[ 1 - \frac{e^{-x/L_n}}{1 + \left(D_n/\tau_n\right)/S_F} \right]
\]

Check some limits.

i) \( S_F = 0 \) cm/s, which implies that there is no recombination at the surface. Then we find: \( \Delta n(x) = G_L \tau_n \), which make sense, since we have spatial uniformity.

ii) \( S_F \to \infty \). Strong recombination at the surface should force \( \Delta n(x = 0) = 0 \), but in the bulk we should still have \( \Delta n(x) = G_L \tau_n \). The transition from 0 to a finite value on the bulk should take a diffusion length or two.

From the solution:

\[
\Delta n(x) = G_L \tau_n \left[ 1 - \frac{e^{-x/L_n}}{1 + \left(D_n/\tau_n\right)/S_F} \right]
\]

For \( S_F \to \infty \), we find

\[
\Delta n(x) \to G_L \tau_n \left[ 1 - e^{-x/L_n} \right]
\]

which behaves as expected.
7d) Provide a sketch of the solution, and explain it in words.

**Solution:**

Concentration is $G_L \tau_n$ in the bulk, but less at the surface, because of surface recombination. The transition from the surface to the bulk takes place over a distance that is roughly the diffusion length.

The hole QFL is constant and almost exactly where the equilibrium Fermi level was, because we are in low level injection (the hole concentration is very, very near its equilibrium value). But the electron QFL is much closer to the conduction band edge. It moves away from $E_C$ near the surface, because surface recombination reduces $\Delta n(x)$ near the surface. The variation with position is linear, because $\Delta n(x)$ varies exponentially with position.
8) The sample is uniformly illuminated with light, resulting in an optical generation rate
\( G_L = 10^{24} \text{ cm}^{-3} \text{ sec}^{-1} \), but all of the photons are absorbed in a thin layer (10 nm wide
near \( x = 0 \)). Find the steady state excess minority carrier concentration and QFL’s vs.
position. You may assume that the sample extends to \( x = +\infty \). **HINT:** treat the thin
layer at the surface as a boundary condition – do not try to resolve \( \Delta n(x) \) inside this
thin layer. Approach the problem as follows.

8a) Simplify the Minority Carrier Diffusion Equation for this problem.

**Solution:**

Begin with:

\[
\frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L
\]

Simplify for steady-state:

\[
0 = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L
\]

Let’s treat the generation in a thin surface layer as a boundary condition, \( G_L = 0 \)

The simplified MDE equation is:

\[
D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} = 0
\]

where \( L_n = \sqrt{D_n \tau_n} \) is the minority carrier diffusion length.

8b) Specify the initial and boundary conditions, as appropriate for this problem.

**Solution:**

Since this is a steady-state problem, there is no initial condition. As \( x \to \infty \), we
expect all of the minority carriers to have recombined, so:

\[
\Delta n(x \to \infty) = 0
\]

At the surface, the total number of e-h pairs generation per cm² per second is
\( G_S = G_L \Delta x = 10^{24} \times 10^{-6} = 10^{18} \text{ cm}^{-2} \text{ s}^{-1} \). In steady-state, these must diffuse away at
the same rate that they are generated, so
8c) Solve the problem.

**Solution:**

\[-D_n \frac{d\Delta n}{dx} \bigg|_{x=0} = G_S\]

\[
\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n^2} = 0 \text{ solutions is } \Delta n(x) = Ae^{-x/L_n} + Be^{x/L_n}
\]

To satisfy the first boundary condition in 8b): \( B = 0 \). Now consider the second:

\[-D_n \frac{d\Delta n}{dx} \bigg|_{x=0} \rightarrow +\frac{D_n}{L_n} A = G_S \rightarrow A = \frac{G_S}{\left(D_n/L_n\right)} = \frac{10^{18}}{7.8/27.9 \times 10^{-4}} = 3.6 \times 10^{14} \text{ cm}^{-3}
\]

\[
\Delta n(x) = \frac{G_S}{\left(D_n/L_n\right)} e^{-x/L_n} = \left(3.6 \times 10^{14}\right)e^{-x/L_n}
\]

8d) Provide a sketch of the solution, and explain it in words.

**Solution:**

Electrons are generated at the surface and diffuse into the bulk, so the concentration is high at the surface and approaches zero several diffusion lengths into the bulk.
Deep in the bulk, there are no excess carriers so \( F_n = F_p = E_F \). The electron QFL must get closer to the conduction band near the surface, because the excess electron concentration is larger there. The variation is linear with position because \( \Delta n(x) \) varies exponentially with position.

9) The sample is uniformly illuminated with light, resulting in an optical generation rate \( G_L = 10^{24} \text{ cm}^{-3} \text{ sec}^{-1} \), but all of the photons are absorbed in a thin layer (10 nm wide near \( x = 0 \)). Find the steady state excess minority carrier concentration and QFL's vs. position. **Assume that the semiconductor is only 5 μm long.** You may also assume that there is an “ideal ohmic contact” at \( x = L = 5 \mu m \), which enforces equilibrium conditions at all times. Make reasonable approximations, and approach the problem as follows. **HINT:** treat the thin layer at the surface as a boundary condition – do not try to resolve \( \Delta n(x) \) inside this thin layer.

9a) **Simplify the Minority Carrier Diffusion Equation for this problem.**

**Solution:**

Begin with:

\[
\frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L
\]

Simplify for steady-state:

\[
0 = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L
\]

Let’s treat the generation in a thin surface layer as a boundary condition, so \( G_L = 0 \);

the simplified MDE equation is:
\[
\begin{align*}
D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} = 0 \\
\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{D_n \tau_n} = 0 \\
\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n^2} = 0 \\
L_n = \sqrt{\frac{D_n \tau_n}{\Delta n}}
\end{align*}
\]

Since the sample is much thinner than a diffusion length, we can ignore recombination, so

\[
\frac{d^2 \Delta n}{dx^2} = 0.
\]

9b) Specify the initial and boundary conditions, as appropriate for this problem.

Since this is a steady-state problem, there is no initial condition. At \( x = L \), we expect all of the minority carriers to have recombined, so:

\[
\Delta n(x = L) = 0
\]

At the surface, the total number of e-h pairs generation per cm² per second is \( G_s = G_i \Delta x = 10^{21}10^{-6} = 10^{18} \text{ cm}^2 \text{s}^{-1} \). In steady-state, these must diffuse away at the same rate that they are generated, so

\[
-D_n \frac{d\Delta n}{dx} \bigg|_{x=0} = G_s
\]

9c) Solve the problem.

**Solution:**

\[
\frac{d^2 \Delta n}{dx^2} = 0 \text{ solutions is } \Delta n(x) = Ax + B
\]

To satisfy the first boundary condition in 9b): \( \Delta n(L) = AL + B = 0 \).

\( B = -AL \)

Now consider the second:

\[
-D_n \frac{d\Delta n}{dx} \bigg|_{x=0} \rightarrow -\frac{D_n}{L} A = G_s \rightarrow A = -\frac{G_s}{D_n/L} = -\frac{10^{18}}{7.8/5 \times 10^{-4}} = -6.4 \times 10^{13} \text{ cm}^{-3}
\]
\[ \Delta n(x) = \frac{G_s}{(D_n/L)(L-x)} = (6.4 \times 10^{13})(L-x) \]

9d) Provide a sketch of the solution, and explain it in words.

Concentration increases towards surface, because generation occurs there. Is zero at \( x = L \) because of the boundary condition there. Variation is linear with position because there is no recombination.

Electron QFL(x) follows from \( n(x) = \Delta n(x) = n_i e^{-(E_n(x)-E_f)/k_BT} \).

10) The sample is in the dark, but the excess carrier concentration at \( x = 0 \) is held constant at \( \Delta n(0) = 10^{12} \text{ cm}^{-3} \). Find the steady state excess minority carrier concentration and QFL's vs. position. You may assume that the sample extends to \( x = +\infty \). Make reasonable approximations, and approach the problem as follows.

10a) Simplify the Minority Carrier Diffusion Equation for this problem.

Solution:

Begin with: \[ \frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L \]

Simplify for steady-state: \[ 0 = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L \]

No generation: \( G_L = 0 \);
the simplified MDE equation is:

\[ D_n \frac{d^2 \Delta n}{dx^2} \frac{\Delta n}{\tau_n} = 0 \quad \frac{d^2 \Delta n}{dx^2} \frac{\Delta n}{D_n \tau_n} = 0 \quad \frac{d^2 \Delta n}{dx^2} \frac{\Delta n}{L_n^2} = 0 \quad L_n = \sqrt{D_n \tau_n} \]

where \( L_n = \sqrt{D_n \tau_n} \) is the minority carrier diffusion length.

10b) Specify the initial and boundary conditions, as appropriate for this problem.

**Solution:**

Since this is a steady-state problem, there is no initial condition. As \( x \to \infty \), we expect all of the minority carriers to have recombined, so:

\[ \Delta n(x \to \infty) = 0 \]

At the surface, the excess electron concentration is held constant, so

\[ \Delta n(x = 0) = 10^{12} \text{ cm}^{-3} \]

10c) Solve the problem.

**Solution:**

\[ \frac{d^2 \Delta n}{dx^2} \frac{\Delta n}{L_n^2} = 0 \]

solutions is \( \Delta n(x) = Ae^{-x/L_n} + Be^{+x/L_n} \)

To satisfy the first boundary condition in 10b): \( B = 0 \). Now consider the second:

\[ \Delta n(0) = 10^{12} \text{ cm}^{-3} \]

\[ \Delta n(x) = \Delta n(0) e^{-x/L_n} = \left(10^{12}\right) e^{-x/L_n} \]

10d) Provide a sketch of the solution, and explain it in words.
Solution:

Looks just like the solutions for prob. 8). Only difference is that instead of creating $\Delta n(0)$ by generation at the surface, we just specify $\Delta n(0)$ directly.

11) The sample is in the dark, and the excess carrier concentration at $x = 0$ is held constant at $\Delta n(0) = 10^{12} \text{cm}^{-3}$. Find the steady state excess minority carrier concentration and QFL’s vs. position. **Assume that the semiconductor is only 5 \( \mu \text{m} \) long.** You may also assume that there is an “ideal ohmic contact” at $x = L = 5 \mu \text{m}$, which enforces equilibrium conditions at all times. Make reasonable approximations, and approach the problem as follows.

11a) Simplify the Minority Carrier Diffusion Equation for this problem.

**Solution:**

Begin with: \[
\frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L.
\]

Simplify for steady-state: \[
0 = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L.
\]

Generation is zero for this problem: $G_L = 0$;

the simplified MDE equation is:

\[
D_n \frac{d^2 \Delta n}{dx^2} \frac{\Delta n}{\tau_n} = 0 \quad \frac{d^2 \Delta n}{dx^2} \frac{\Delta n}{L_n^2} = 0 \quad L_n = \sqrt{D_n \tau_n}
\]

Since the sample is much thinner than a diffusion length, we can ignore recombination, so

\[
\frac{d^2 \Delta n}{dx^2} = 0.
\]

11b) Specify the initial and boundary conditions, as appropriate for this problem.

**Solution:**

Since this is a steady-state problem, there is no initial condition. As $x \to \infty$, we expect all of the minority carriers to have recombined, so:
\[ \Delta n(x = L) = 0 \]

At the surface:

\[ \Delta n(0) = 10^{12} \text{ cm}^{-2} \]

11c) Solve the problem.

**Solution:**

\[
\frac{d^2 \Delta n}{dx^2} = 0 \quad \text{the solution is } \Delta n(x) = Ax + B
\]

To satisfy the first boundary condition in 9b): \( \Delta n(L) = AL + B = 0 \).

\[ B = -AL \]

Now consider the second boundary condition:

\[ \Delta n(0) = B = 10^{12} \text{ cm}^{-3} \]

\[ \Delta n(x) = \Delta n(0)(L-x) = (10^{12})(L-x) \]

11d) Provide a sketch of the solution, and explain it in words.

**Solution:**

Just like problem 9)

12) The sample is in the dark, and the excess carrier concentration at \( x = 0 \) is held constant at \( \Delta n(0) = 10^{12} \text{ cm}^{-3} \). Find the steady state excess minority carrier concentration and QFL’s vs. position. **Assume that the semiconductor is 30 \( \mu \text{m} \) long.** You may also assume that there is an “ideal ohmic contact” at \( x = L = 30 \mu \text{m} \), which enforces equilibrium conditions at all times. Make reasonable approximations, and approach the problem as follows.

12a) Simplify the Minority Carrier Diffusion Equation for this problem.
Solution:
Begin with: \[ \frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L \]

Simplify for steady-state and no generation:

\[ D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} = 0 \]
\[ \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n^2} = 0 \]

where \( L_n = \sqrt{D_n \tau_n} \) is the minority carrier diffusion length.

12b) Specify the initial and boundary conditions, as appropriate for this problem.

Solution:
Steady state, so no initial conditions are necessary. The boundary conditions are:

\[ \Delta n(0) = 10^{12} \text{ cm}^{-3} \]
\[ \Delta n(30 \mu m) = 0 \]

12c) Solve the problem.

Solution:

\[ \Delta n(x) = Ae^{-x/L_n} + Be^{x/L_n} \]

Because the region is about one diffusion length long, we need to retain both solutions.

\[ \Delta n(0) = A + B \]
\[ \Delta n(L = 30 \mu m) = Ae^{-L/L_n} + Be^{L/L_n} = 0 \]

Solve for \( A \) and \( B \) to find:
\[ A = \frac{-\Delta n(0)e^{+L/L_n}}{e^{-L/L_n} - e^{+L/L_n}} \]

\[ B = \frac{\Delta n(0)e^{-L/L_n}}{e^{-L/L_n} - e^{+L/L_n}} \]

So the solution is:

\[ \Delta n(x) = \frac{\Delta n(0)}{e^{-L/L_n} - e^{+L/L_n}} \left[ -e^{-(x-L)/L_n} + e^{+(x-L)/L_n} \right] \]

\[ \Delta n(x) = \Delta n(0) \frac{\sinh\left[ (x - L) / L_n \right]}{\sinh(L / L_n)} \]

12d) Provide a sketch of the solution, and explain it in words.

**Solution:**

The short base result is linear, but in this case, the slope in a little steeper initially and a little shallower at the end. Since the diffusion current is proportional to the slope, this means that inflow greater than outflow. This occurs because some of the electrons that flow in, recombine in the structure, so the same number cannot flow out.

13) Consider a sample that extends from \(-5 \leq x \leq 200 \, \mu m\). The sample is illuminated with light, resulting in an optical generation rate \( G_L = 10^{24} \, \text{cm}^{-3} \, \text{sec}^{-1} \), but all of the photons are absorbed in a very thin layer (10 nm wide centered about \( x = 0 \)). Assume
that half of the carriers generated in this region diffuse to the left and half to the right. You may also assume that there are “ideal ohmic contacts”, which enforce equilibrium conditions at all times located at \( x = -5 \, \mu m \) and at \( x = 200 \, \mu m \). Find the steady state excess minority carrier concentration and QFL’s vs. position. Make reasonable approximations, and approach the problem as follows.

13a) Simplify the Minority Carrier Diffusion Equation for this problem.

\[
\frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L
\]

Steady-state, no generation (treat generation near \( x = 0 \) as a boundary condition).

For \( x > 0 \):

\[
\frac{d^2 \Delta n}{dx^2} = 0
\]

For \( x < 0 \), we could also solve this equation, but the region is thin for \( x < 0 \), so we can ignore recombination and solve:

\[
\frac{d^2 \Delta n}{dx^2} = 0
\]

13b) Specify the initial and boundary conditions, as appropriate for this problem.

**Solution:**

Steady-state, so no initial condition. At the boundaries:

\[
\Delta n(x = -5 \, \mu m) = 0 \\
\Delta n(x = 200 \, \mu m) = 0
\]

At \( x = 0^- \) and \( x = 0^+ \) we can assume that one-half of the carriers generated in the thin region diffuse to the left and one-half to the right.
\[
\frac{D_n}{dx} \frac{d\Delta n}{dx} \bigg|_{x=0^-} = \frac{G_s}{2} = \frac{G_s}{2}
\]

\[
-\frac{D_n}{dx} \frac{d\Delta n}{dx} \bigg|_{x=0^+} = \frac{G_s}{2}
\]

13c) Solve the problem.

**Solution:**

For \( x < 0 \), the solutions is: \( \Delta n(x) = Ax + B \)

At \( x = -5 \) micrometers: \( \Delta n(-5 \times 10^{-4}) = 0 = A(-5 \times 10^{-4}) + B \quad B = (5 \times 10^{-4})A \)

At \( x = 0^-\):

\[
\left. \frac{D_n}{dx} \frac{d\Delta n}{dx} \right|_{x=0^-} = D_n A = \frac{G_s}{2} \quad A = \frac{G_s}{2D_n}
\]

\[
\Delta n(x) = \frac{G_s}{2D_n} (x + 5 \, \mu m)
\]

For \( x > 0^+ \), the solution is:

\[
\Delta n(x) = Ae^{-x/L_n} + Be^{+x/L_n} \quad \Delta n(x) = Ae^{-x/L_n}
\]

Because for \( x > 0 \), we can treat the region as being infinitely long.

\[
\left. -\frac{D_n}{dx} \frac{d\Delta n}{dx} \right|_{x=0^+} = \frac{D_n}{L_n} A = \frac{G_s}{2} \quad A = \frac{G_s}{2(D_n/L_n)}
\]

\[
\Delta n(x) = \frac{G_s}{2(D_n/L_n)} e^{-x/L_n}
\]

Note that \( \Delta n(0^-) = \frac{G_s}{2D_n} (5 \, \mu m) = \frac{G_s}{2(D_n/5 \, \mu m)} \) and

Note that \( \Delta n(0^+) = \frac{G_s}{2(D_n/L_n)} = \frac{G_s}{2(D_n/27.9 \, \mu m)} \)
so

\[
\frac{\Delta n(0^+)}{\Delta n(0^-)} = \frac{27.9}{5} = 5.6
\]

13d) Provide a sketch of the solution, and explain it in words.

Carriers are generated at \( x = 0 \) and diffuse away to both sides. They diffuse slower for \( x > 0 \), so the carrier density for \( x > 0 \) is higher than for \( x < 0 \).