EE-606: Solid State Devices
Lecture 6: Energy Bands (continued)

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Outline

1) Properties of electronic bands
2) E-k diagram and constant energy surfaces
3) Conclusions

Reference: Vol. 6, Ch. 3 (pages 63-70)
Electron and Hole fluxes: Filled/Empty Bands

\[ J_3 = -\frac{q}{L} \sum_{i(filled)} \nu_i = 0 \]

\[ J_2 = -\frac{q}{L} \sum_{i(filled)} \nu_i = -\frac{q}{L} \sum_{k=0}^{k_{max}} \nu_i - \frac{q}{L} \sum_{-k_{min}}^{0} -|\nu_i| = 0 \]

Filled and empty bands carry no current!
Electron and Hole Fluxes: Partially Filled Bands

\[ J_3 = -\frac{q}{L} \sum_{i(filled)} v_i \neq 0 \]

\[ J_2 = -\frac{q}{L} \sum_{i(filled)} v_i = -\frac{q}{L} \sum_{all} v_i + \frac{q}{L} \sum_{i(\text{empty})} |v_i| \]

-ve charge moving with –ve mass

+ve charge moving with +ve mass
what good is effective mass?

\[ \frac{1}{m^*} = \frac{1}{\hbar^2} \frac{d^2E}{dk^2} \]
Outline

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Solution Space: Brillouin Zone

The diagram illustrates the energy bands (E) as a function of the wave vector (k) in the context of a periodic lattice (K-lattice). The Brillouin Zone is indicated by the green shaded region, which represents the first Brillouin Zone. The real space (x) and reciprocal space (k) are shown with the periodicity (p) marked. The energy bands are shown at various k values: -4π/p, -2π/p, 2π/p, 4π/p. The arrows indicate the direction of the wave vector in the reciprocal space.
General rules for Brillouin Zone

1) Define reciprocal lattice with the following vectors ….

\[ k_x = 2\pi \frac{b \times c}{a \cdot b \times c} \quad k_y = 2\pi \frac{c \times a}{a \cdot b \times c} \quad k_z = 2\pi \frac{a \times b}{a \cdot b \times c} \]

2) Use Wigner Seitz algorithm to find the unit cell in the wave-vector (reciprocal) space.
Wigner-Seitz Method for Reciprocal Space

Primitive cell in real space

\[
k_x = 2\pi \frac{b \times \hat{z}}{|a \cdot b \times \hat{z}|} \quad k_y = 2\pi \frac{\hat{z} \times a}{|a \cdot b \times \hat{z}|}
\]
Brillouin Zone for One-dimensional Solids

Real-space

$\pi \quad \pi \quad \pi \quad \pi$ $k_x$

$\frac{2\pi}{p} \quad \frac{\pi}{p} \quad \frac{\pi}{p} \quad \frac{2\pi}{p}$

Replacing $(a+b)$ by $p$ ...

$\text{E-k diagram}$

$-\frac{\pi}{p} \quad 0 \quad \frac{\pi}{p}$ $k_x$
E-k diagram in 2D solids

Real-space

1st B-Z

E-k diagram
Constant Energy-surface in 2D

1st B-Z

E-k diagram

Const. Energy Surface
Conclusion

1) Electrons can only sit in specific energy bands. Effective masses and band gaps summarize information about possible electronic states.

2) Effective mass is not a fundamental concept. There are systems for which effective mass cannot be defined.

3) Of all the possible bands, only a few contribute to conduction. These are often called conduction and valence bands.

4) For 2D/3D systems, energy-bands are often difficult to visualize. E-k diagrams along specific direction and constant energy surfaces for specific bands summarize such information.

5) Most of the practical problems can only be analyzed by numerical solution.