EE-606: Solid State Devices
Lecture 8: Density of States

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Outline

1) Calculation of density of states

2) Density of states for specific materials

3) Characterization of Effective Mass

4) Conclusions

Reference: Vol. 6, Ch. 3 (pages 88-96)
A single band has total of $N$-states

Only a fraction of states are occupied

*How many states are occupied upto $E$?*

Or equivalently...

*How many states per unit energy? (DOS)*
Density of States in 1-D Semiconductors

States between $E_1 + \Delta E$ & $E_1 = 2 \times \frac{\Delta k}{\delta k}$

$= 2 \times \frac{\Delta k}{2\pi/Na}$

States/unit energy @ $E_1 = \frac{Na \Delta k}{\pi \Delta E}$
1D-DOS

States/unit energy @\( E \) = \( \frac{Na \Delta k}{\pi \Delta E} \)

\( E - E_0 = \frac{\hbar^2 k^2}{2m^*} \Rightarrow k = \sqrt{\frac{2m^* (E - E_0)}{\hbar^2}} \)

\( \frac{dk}{dE} = \frac{m^*}{\sqrt{2\hbar^2 (E - E_0)}} \)

States/unit energy @\( E \) = \( \frac{L}{\pi} \sqrt{\frac{m^*}{2\hbar^2 (E - E_0)}} \)

States/unit energy/unit length @\( E \)

\( \equiv \text{DOS} = \frac{1}{\pi} \sqrt{\frac{m^*}{2\hbar^2 (E - E_0)}} \)
1D-DOS

\[ \Delta k = \frac{2\pi}{Na} \]

\[ \frac{\pi}{a} \]

\[ DOS = \frac{1}{\pi} \sqrt{\frac{m^*}{2\hbar^2 (E - E_0)}} \]

Conservation of DOS
Density of States in 2D Semiconductors

Show that 2D DOS is a constant independent of energy!
Density of States in 3D Semiconductors

States between $E_1 + \Delta E$ & $E_1$

$$\frac{4}{3} \pi (k + dk)^3 - \frac{4}{3} \pi k^3 = \frac{V}{2\pi^2} k^2 \Delta k$$

States/unit energy @ $E = \frac{V}{2\pi^2} k^2 \frac{\Delta k}{dE}$

$$E - E_0 = \frac{\hbar^2 k^2}{2m^*} \Rightarrow k = \sqrt{\frac{2m^*(E - E_0)}{\hbar^2}} \Rightarrow \frac{dk}{dE} = \sqrt{2\hbar^2 \frac{m^*}{E - E_0}}$$

States/unit energy/unit volume @ $E_1$

$$DOS = \frac{m^*}{2\pi^2 \hbar^3} \sqrt{2m^*(E - E_0)}$$
$$DOS = \frac{m^*_3 \sqrt{2m^*_3(E - E_0)}}{\pi^2 \hbar^3}$$
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Density of States of GaAs: Conduction/Valence Bands

\[ g_c(E) = \frac{m_n^*\sqrt{2m_n^*(E - E_c)}}{2\pi^2\hbar^3} \]

\[ g_v(E) = \begin{cases} 
\frac{m_{hh}^*\sqrt{2m_{hh}^*(E - E_v)}}{2\pi^2\hbar^3} \\
\frac{m_{lh}^*\sqrt{2m_{lh}^*(E - E_v)}}{2\pi^2\hbar^3}
\end{cases} \]
Four valleys inside BZ for Germanium
Ellipsoidal Bands and DOS Effective Mass

\[ E - E_C = \frac{\hbar^2 k_1^2}{2m_l^*} + \frac{\hbar^2 k_2^2}{2m_l^*} + \frac{\hbar^2 k_3^2}{2m_l^*} \]

\[ 1 = \frac{k_1^2}{\frac{2m_l^*(E - E_C)}{\hbar^2}} + \frac{k_2^2}{\frac{2m_l^*(E - E_C)}{\hbar^2}} + \frac{k_3^2}{\frac{2m_l^*(E - E_C)}{\hbar^2}} \]

\[ \mathcal{V}_k = N_{el} \left( \frac{4}{3} \pi \alpha \beta^2 \right) \equiv \frac{4}{3} \pi k_{eff}^3 \]

\[ N_{el} \frac{4}{3} \pi \sqrt{\frac{2m_l^*(E - E_C)}{\hbar^2}} \sqrt{\frac{2m_l^*(E - E_C)}{\hbar^2}} \sqrt{\frac{2m_l^*(E - E_C)}{\hbar^2}} = \frac{4}{3} \pi \left[ \sqrt{\frac{2m_{eff}^*(E - E_C)}{\hbar^2}} \right]^3 \]

\[ m_{eff}^* = N_{el}^{2/3} \left( m_l^* m_t^* \right)^{1/3} \]

\[ k_1 \quad k_2 \quad k_3 \]

const. \( E \)
DOS Effective Mass for Conduction Band

\[ m^*_{\text{eff}} = 4^{2/3} \left( m_l m_t^* \right)^{1/3} \]

\[ g_c (E) = \frac{m^*_{\text{eff}} \sqrt{2m^*_{\text{eff}} (E - E_c)}}{2\pi^2\hbar^3} \]

\[ m^*_{\text{eff}} = 6^{2/3} \left( m_l m_t^* \right)^{1/3} \]

\[ g_c (E) = \frac{m^*_{\text{eff}} \sqrt{2m^*_{\text{eff}} (E - E_c)}}{2\pi^2\hbar^3} \]
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Measurement of Effective Mass

\[ \nu_0 = 24 \text{ GHz} \] (fixed)

\[ m^* = \frac{qB_{con}}{2\pi \nu_0} \]
Motion in Real Space and Phase Space

Energy = constant.

Liquid He temperature …
Derive the Cyclotron Formula

$$m^* = \frac{qB_0}{2\pi v_0}$$

For an particle in (x-y) plane with B-field in z-direction, the Lorentz force is ...

$$\frac{m^* v^2}{r_0} = qv \times B_z = qvB_z$$

$$v = \frac{qB_0r_0}{m^*}$$

$$\tau = \frac{2\pi r_0}{v} = \frac{2\pi m^*}{qB_0}$$

$$v_0 \equiv \frac{1}{\tau} = \frac{qB_0}{2\pi m^*}$$
Effective mass in Ge

4 angles between B field and the ellipsoids ...
Recall the HW1
Derivation for the Cyclotron Formula

Show that
\[
\frac{1}{m_c^2} = \frac{\cos^2 \theta}{m_t^2} + \frac{\sin^2 \theta}{m_l m_t}
\]

Given three \( m_c \) and three \( \theta \), we will find \( m_t \) and \( m_l \)

The Lorentz force on electrons in a B-field

\[
F = q\nu \times B = \left[ M \right] \frac{d\nu}{dt}
\]

In other words,

\[
F_x = q\left( v_y B_z - v_z B_y \right) = m_t^* \frac{d\nu_x}{dt}
\]
\[
F_y = q\left( v_z B_x - v_x B_z \right) = m_t^* \frac{d\nu_y}{dt}
\]
\[
F_z = q\left( v_x B_y - v_y B_x \right) = m_l^* \frac{d\nu_z}{dt}
\]
Let \((B)\) make an angle \((\theta)\) with longitudinal axis of the ellipsoid (ellipsoids oriented along \(k_z\))

\[
B_x = B_0 \cos(\theta), \quad B_y = 0, \quad B_z = B_0 \sin(\theta),
\]

Differentiate \((v_y)\) and use other equations to find ...

\[
\frac{d^2 v_y}{dt^2} + v_y \omega^2 = 0 \quad \text{with} \quad \omega^2 \equiv \left[ \omega_l \omega_l \sin^2 \theta + \omega_l^2 \cos^2 \theta \right]
\]

\[
\omega_0 = \frac{qB_0}{m_c}, \quad \omega_l = \frac{qB_0}{m_t}, \quad \omega_i = \frac{qB_0}{m_l}
\]

so that ...

\[
\frac{1}{(m_c^*)^2} = \frac{\sin^2 \theta}{m_t m_l} + \frac{\cos^2 \theta}{m_t^2}
\]
Measurement of Effective Mass

\[ B = [0.61, 0.61, 0.5] \]

Three peaks \( B_1, B_2, B_3 \)

Three masses \( m_{c1}, m_{c2}, m_{c3} \)

Three unique angles: 7, 65, 73

\[
\frac{1}{m_c^2} = \frac{\cos^2 \theta}{m_t^2} + \frac{\sin^2 \theta}{m_t^2}
\]

Known \( \theta \) and \( m_c \) allows calculation of \( m_t \) and \( m_l \).
HW. Which peaks relate to valence band? Why are there two valence band peaks?
Conclusions

1) Measurement of Effective mass and band gaps define the energy-band of a material.

2) Only a fraction of the available states are occupied. The number of available states change with energy. DOS captures this variation.

3) DOS is an important and useful characteristic of a material that should be understood carefully.