ECE606: Solid State Devices
Lecture 14: Bulk Recombination

Muhammad Ashraful Alam
alam@purdue.edu
Outline

1) Derivation of SRH formula
2) Application of SRH formula for special cases
3) Direct and Auger recombination
4) Conclusion

Ref. ADF, Chapter 5, pp. 141-154
Sub-processes of SRH Recombination

(1)+(3): one electron reduced from Conduction-band & one-hole reduced from valence-band

(2)+(4): one hole created in valence band and one electron created in conduction band
SRH Recombination

Physical picture

(1) 
(2) 
(3) 
(4)

Equivalent picture

(1) 
(2) 
(3) 
(4)

(1)+(3): one electron reduced from C-band &
          one-hole reduced from valence-band

(2)+(4): one hole created in valence band &
          one electron created in conduction band
Changes in electron and hole Densities

\[
\frac{\partial n}{\partial t} \bigg|_{1,2} = -c_n n p_T + e_n n_T (1 - f_c)
\]

\[
\frac{\partial p}{\partial t} \bigg|_{3,4} = -c_p p n_T + e_p p_T f_v
\]
Detailed Balance in \textit{Equilibrium}

\[
\frac{\partial n}{\partial t}_{1,2} = -c_n n_0 p_{T0} + e_n n_{T0}
\]

\[
0 = -c_n n_0 p_{T0} + e_n n_{T0}
\]

\[
e_n = c_n \frac{n_0 p_{T0}}{n_{T0}} \equiv c_n n_1
\]

\[
0 = -c_n \left( n_0 p_{T0} - n_{T0} n_1 \right)
\]

\[
\frac{\partial p}{\partial t}_{3,4} = -c_p p n_T + e_p p_T
\]

\[
0 = -c_p p_0 n_{T0} + p_{T0} e_p
\]

\[
e_p \equiv \frac{c_p p_0 n_{T0}}{p_{T0}} = c_p p_1
\]

\[
0 = -c_p \left( p_0 n_{T0} - p_{T0} p_1 \right)
\]
Expressions for \((n_1)\) and \((p_1)\)

\[ n_1 = \frac{n_0 p_{T0}}{n_{T0}} \]

\[ p_1 = \frac{p_0 n_{T0}}{p_{T0}} \]

\[ n_1 p_1 = \frac{n_0 p_{T0}}{n_{T0}} \times \frac{p_0 n_{T0}}{p_{T0}} = n_0 p_0 = n_i^2 \]
Expressions for \((n_1)\) and \((p_1)\)

\[
n_{T0} = N_T \left(1 - f_{00}\right) = \frac{N_T}{1 + g_D e^{\beta(E_T - E_F)}}
\]

\[
n_1 = n_0 \frac{p_{T0}}{n_{T0}} = n_0 \frac{\left(N_T f_{00}\right)}{N_T \left(1 - f_{00}\right)}
\]

\[
n_1 = n_i e^{\beta(E_F - E_i)} \left[1 + g_D e^{\beta(E_T - E_F)} - 1\right]
\]

\[
= n_i g_D e^{\beta(E_T - E_i)}
\]

\[
p_1 n_1 = n_i^2
\]

\[
p_1 = \frac{n_i^2}{n_1}
\]

\[
= n_i g_D^{-1} e^{\beta(E_i - E_T)}
\]
Dynamics of Trap Population

\[
\frac{\partial n_T}{\partial t} = -\frac{\partial n}{\partial t}_{1,2} + \frac{\partial p}{\partial t}_{3,4}
\]

\[
= c_n n p_T - e_n n_T - c_p p n_T + e_p p_T
\]

\[
= c_n \left( np_T - n_T n_1 \right) - c_p \left( p n_T - p_T p_1 \right)
\]
**Steady-state** Trap Population

\[
\frac{\partial n_T}{\partial t} = 0 = c_n (np_T - n_T n_1) - c_p (p n_T - p_T p_1)
\]

\[
n_T = \frac{c_n N_T n + c_p N_T p_1}{c_n (n + n_1) + c_p (p + p_1)} = c_n (np_T - n_T n_1)
\]
Net Rate of Recombination-Generation

\[ R = -\frac{dp}{dt} = c_p \left( p n_T - p_T p_1 \right) \]

\[ = \frac{np - n_i^2}{\left( \frac{1}{c_p N_T} \right) (n + n_1) + \left( \frac{1}{c_n N_T} \right) (p + p_1)} \]
Outline

1) Derivation of SRH formula
2) **Application of SRH formula for special cases**
3) Direct and Auger recombination
4) Conclusion
Case 1: Low-level Injection in p-type

\[ R = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)} \]

\[ = \frac{(n_0 + \Delta n)(p_0 + \Delta p) - n_i^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta p + p_1)} \]

\[ = \frac{\Delta n(n_0 + p_0) + \Delta n^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta p + p_1)} \]

\[ = \frac{\Delta n(p_0)}{\tau_n(p_0)} \approx \frac{\Delta n}{\tau_n} \quad \Delta n^2 \approx 0 \]

\[ p_0 \gg \Delta n \gg n_0 \]
Case 2: High-level Injection

\[ R = \frac{np - n_i^2}{\tau_p (n + n_1) + \tau_n (p + p_1)} \]

\[ = \frac{(n_0 + \Delta n)(p_0 + \Delta p) - n_i^2}{\tau_p (n_0 + \Delta n + n_1) + \tau_n (p_0 + \Delta p + p_1)} \]

\[ = \frac{\Delta n (n_0 + p_0) + \Delta n^2}{\tau_p (n_0 + \Delta n + n_1) + \tau_n (p_0 + \Delta n + p_1)} \]

\[ = \frac{\Delta n^2}{(\tau_n + \tau_p) \Delta n} = \frac{\Delta n}{(\tau_n + \tau_p)} \]

\[ \Delta n \gg p_0 \gg n_0 \]

e.g. organic solar cells

Alam ECE-606 S09
High/Low Level Injection ...

\[ R_{\text{high}} = \frac{\Delta n}{(\tau_n + \tau_p)} \quad \Delta n \gg p_0 \gg n_0 \]

\[ R_{\text{low}} = \frac{\Delta n}{\tau_p} \quad p_0 \gg \Delta n \gg n_0 \]

which one is larger and why?
Case 3: Generation in Depletion Region

\[ R = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)} \]

\[ = -\frac{n_i^2}{\tau_p(n_1) + \tau_n(p_1)} \]

\[ n \ll n_1 \quad p \ll p_1 \]
Outline

1) Derivation of SRH formula
2) Application of SRH formula for special cases
3) Direct and Auger recombination
4) Conclusion
Band-to-band Recombination

\[ R = B \left( np - n_i^2 \right) \]

Direct recombination at low-level injection

\[ n_0 \ll \left( \Delta n = \Delta p \right) \ll p_0 \]

\[ R = B \left[ \left( n_0 + \Delta n \right) \left( p_0 + \Delta p \right) - n_i^2 \right] \approx B p_0 \times \Delta n \]

Direct generation in depletion region

\[ n, p \sim 0 \]

\[ R = B \left( np - n_i^2 \right) \approx -B n_i^2 \]
Auger Recombination

2 electron & 1 hole

\[ R = c_n \left( n^2 p - n_i^2 n \right) + c_p \left( np^2 - n_i^2 p \right) \]

\[ c_n, c_p \sim 10^{-29} \text{ cm}^6/\text{sec} \]

Auger recombination at low-level injection

\[ n_0 \ll \left( \Delta n = \Delta p \right) \ll \left( p_0 = N_A \right) \]

\[ R \approx c_p N_A^2 \Delta n = \frac{\Delta n}{\tau_{\text{auger}}} \quad \tau_{\text{auger}} = \frac{1}{c_p N_A^2} \]
Effective Carrier Lifetime

\[ \Delta n(t) = \Delta n(t = 0) e^{-\frac{t}{\tau_{\text{eff}}}} \]

\[ \tau_{\text{eff}} = \left( c_n N_T + BN_D + c_{n,\text{auger}} N_D^2 \right)^{-1} \]

\[ R = R_{\text{SRH}} + R_{\text{direct}} + R_{\text{Auger}} \]

\[ = \Delta n \left( \frac{1}{\tau_{\text{SRH}}} + \frac{1}{\tau_{\text{direct}}} + \frac{1}{\tau_{\text{Auger}}} \right) \]

\[ = \Delta n \left( c_n N_T + BN_D + c_{n,\text{auger}} N_D^2 \right) \]

Alam ECE-606 S09
Effective Carrier Lifetime with all Processes

\[ \tau_{\text{eff}} \approx c_{n,\text{auger}} N_D^{-2} \]

\[ \tau_{\text{eff}} = \left( c_n N_T + B N_D + c_{n,\text{auger}} N_D^2 \right)^{-1} \]

SRH is an important recombination mechanism in important semiconductors like Si and Ge.

SRH formula is complicated, therefore simplification for special cases are often desired.

Direct band-to-band and Auger recombination can also be described with simple phenomenological formula.

These expressions for recombination events have been widely validated by measurements.