ECE606: Solid State Devices
Lecture 16: Carrier Transport

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Outline

1) Overview
2) Drift Current
3) Physics of Mobility
4) High field effects
5) Conclusion

REF: Advanced Device Fundamentals, Pages 175-192
Current Flow Through Semiconductors

\[ I = G \times V \]

\[ = q \times n \times v \times A \]

Depends on chemical composition, crystal structure, temperature, doping, etc.

**Quantum Mechanics + Equilibrium Statistical Mechanics**

\[ \Rightarrow \text{Encapsulated into concepts of effective masses and occupation factors (Ch. 1-4)} \]

**Transport with scattering, non-equilibrium Statistical Mechanics**

\[ \Rightarrow \text{Encapsulated into drift-diffusion equation with recombination-generation (Ch. 5 & 6)} \]
Non-equilibrium Systems

Chapter 5

**vs.**

Chapter 6

vs.

$I$

$V$
Summary of Transport Equations ...

\[ \nabla \cdot D = q \left( p - n + N_D^+ - N_A^- \right) \]

\[ \frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot \mathbf{J}_n - r_N + g_N \]

\[ \mathbf{J}_n = q n \mu_n E + q D_N \nabla n \]

\[ \frac{\partial p}{\partial t} = \frac{-1}{q} \nabla \cdot \mathbf{J}_p - r_P + g_P \]

\[ \mathbf{J}_p = q p \mu_p E - q D_P \nabla p \]
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Meaning of Effective Mass ...

\[
\left(-\frac{\hbar^2}{2m_0} \frac{d^2}{dx^2} + U_{\text{crys}}(x) + U_{\text{ext}}(x)\right)\psi = E\psi
\]

\[
\left(-\frac{\hbar^2}{2m^*} \frac{d^2}{dx^2} + U_{\text{ext}}(x)\right)\phi = E\phi
\]
Drift by Electric field ....

\[ J_n = qn\mu_n\mathcal{E} \]

\[ \frac{d(m_n^*\nu)}{dt} = -q\mathcal{E} - \frac{m_n^*\nu}{\tau_n} \]

\[ \nu(t) = -\frac{q\tau_n}{m_n^*}\mathcal{E}\left[1 - e^{-\frac{t}{\tau_n}}\right] \]
Drift by Electric field ....

\[ \nu(t) = -\frac{q\tau_n}{m_n^*} \mathcal{E} \left[ 1 - e^{-\frac{t}{\tau_n}} \right] \]

\[ = -\frac{q\tau_n}{m_n^*} \mathcal{E} \quad (t \to \infty, \ 1-2 \ \text{ps}) \]

\[ \equiv \mu_n \mathcal{E} \]

\[ J_n = qn \mu_n \mathcal{E} \]

(Theory valid once \( t > 1-2 \ \text{ps} \))
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Mobility and Physics of Scattering Time

\[ \mu_n = \frac{q \tau_n}{m_n^*} \]

Fermi’s Golden rule ...

\[ \tau_n^{-1} \sim \left| \frac{2\pi}{\hbar} \int_{-\infty}^{\infty} \psi^*(x)U(x)\psi(x)dx \right|^2 \]
Phonon and Ionized Impurity Scattering

Ionized impurity

\[ \tau_n \sim \frac{T^{3/2}}{N_D} \]

Higher temperature, more phonon scattering

\[ \tau_n \sim T^{-3/2} \]
Multiple Scattering Events

- Ionized impurity
- Phonon scattering
- others ....

\[
\frac{1}{\mu_n} = \frac{1}{\mu_{ph}} + \frac{1}{\mu_{II}}
\]

\[
\Rightarrow \mu_n = \frac{\mu_{ph}\mu_{II}}{\mu_{ph} + \mu_{II}}
\]

\[
= \mu_{\text{min}} + \left( \frac{\mu_{ph}\mu_{II}}{\mu_{ph} + \mu_{II}} - \mu_{\text{min}} \right)
\]

\[
= \mu_{\text{min}} + \left( \frac{\mu_0}{1 + (N_I/N_0)^\alpha} \right)
\]

\[

t_n = \frac{1}{\tau_n} + \frac{1}{\tau_{II}} + \frac{1}{\tau_{ph}} + \frac{1}{\tau_s} + \cdots
\]

\[
\frac{1}{\mu_n} = \frac{m_n^*}{q\tau_n}
\]

Matthession Rule ....
Model for Ionized impurity Scattering

\[ \mu_n = \mu_{n,\text{min}} + \left( \frac{\mu_{0,n}}{1 + \left( \frac{N_I}{N_{0,n}} \right)^\alpha_n} \right) \]

\[ m_n^* \]

[Graph showing electron and hole mobilities as functions of \( N_A \) or \( N_D \) with values at different concentrations and temperatures.]

\[ \mu_{n,\text{min}} \]
Temperature-dependent Mobility

\[ \mu_n \sim \tau_n \sim T^{-3/2} \]
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Mobility at High Fields?

What causes velocity saturation at high fields?

Where does all the mobility formula in device simulator come from?
Velocity Saturation in Si/Ge

\[ E = 0 \quad J_1 = J^+ - J^- = 0 \]

\[ E \ll E_c \quad J_2 = J^+ - J^- > J_1 \]

\[ E \approx E_c \quad J_3 = J^+ - J^- > J_2 \]

\[ E \gg E_c \quad J_4 = J^+ - J^- \approx J_3 \]
What type of scattering would you need for inter-valley transfer?
Doping dependent Resistivity

\[ \mathcal{E} = \rho J \]

\[ J = q(\mu_n n + \mu_p p)\mathcal{E} \]

\[ \rho = \frac{1}{q(\mu_n n + \mu_p p)} \]

\[ = \frac{1}{q\mu_n N_D} \text{ ... for n-type} \]

\[ = \frac{1}{q\mu_p N_A} \text{ ... for p-type} \]
Conclusion

1) Poisson and drift-diffusion equations form a complete semi-classical transport model that can explain wide variety of device phenomena.

2) Drift current results from response of electrons/holes to electric field. The physics of mobility is complex and material dependent.

3) Constancy of low-field mobility can be checked by experiments.