ECE606: Solid State Devices
Lecture 22: Non-ideal effects

Muhammad Ashraful Alam
alam@purdue.edu
Outline

1) **Non-ideal effects: Junction recombination**

2) Non-ideal effects: Impact ionization

3) Conclusion

Ref. Semiconductor Devices Fundamentals, Chapter 6
## Topic Map

<table>
<thead>
<tr>
<th></th>
<th>Equilibrium</th>
<th>DC</th>
<th>Small Signal</th>
<th>Large Signal</th>
<th>Circuits</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Diode</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Schottky</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>BJT/HBT</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>MOSFET</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Various Regions of I-V Characteristics

1. Diffusion limited
2. Ambipolar transport
3. High injection
4. R-G in depletion
5. Breakdown
6. Trap-assisted R-G
7. Esaki Tunneling
Asad: We should redraw the figure ...
Applying Bias

\[ qV_{bi} \]

\[ E_{C} - E_{F} \]

\[ E_{F} - E_{V} \]

\[ q(V_{bi - V}) \]

\[ E_{C} - F_{n} \]

\[ F_{p} - E_{V} \]
(4,6) Junction Recombination

\[ I_R = -qA \int_0^W \frac{\partial n}{\partial t} \, dx \]

\[ \frac{\partial n}{\partial t} = -\frac{[n(x)p(x) - n_i^2]}{\tau_p[n(x) + n_1] + \tau_n[p(x) + p_1]} \]

Assume \( \tau_n = \tau_p \), \( E_i = E_T \), \( n_1 = p_1 = n_i \)

\[ \frac{\partial n}{\partial t} = -\frac{n_i^2(e^{qV_A/kT} - 1)}{\tau[n(x) + p(x) + 2n_i]} \]

Note: Do you remember this HW?
(np) Product within the Junction

Mass action in non-equilibrium

\[ n(x)p(x) = n_i^2 e^{(F_N - F_P)/kT} \]

\[ = n_i^2 e^{qV_A/kT} \]
Electron/Hole Concentrations at Junction

\[ E_i(x) = E_{iL} - qV(x) \]

\[ n(x) = n_i e^{(F_N - E_i(x))/kT} = n_i e^{[F_N - E_{iL} + qV(x)]/kT} \]

\[ p(x) = \frac{n_i^2 e^{qV_A/kT}}{n_i e^{[F_N - E_{iL} + qV(x)]/kT}} = n_i e^{-[F_N - E_{iL} + qV(x)]/kT + qV_A/kT} \]
Junction Recombination

\[ U_{FN} = \frac{F_N - E_{iL}}{kT} \quad U_A = \frac{V_A}{kT / q} \]

\[ \frac{\partial n}{\partial t} = -\frac{n_i (e^{U_A} - 1)}{\tau [e^{U_{FN}+U} + e^{-U_{FN}-U+U_A}]} \]

\[ I_R = -qA \left( \frac{n_i}{\tau} \right) \times sinh \left( \frac{U_A}{2} \right) \times \int_0^W \frac{dx}{cosh[U_{FN} + U - U_A/2]} \]

\[ \Rightarrow \frac{\partial n}{\partial t} = -\frac{n_i}{\tau} \frac{e^{U_A/2} (e^{U_A/2} - e^{-U_A/2})}{e^{U_{FN}+U-U_A/2} + e^{-U_{FN}-U+U_A/2}} \]

\[ \Rightarrow \frac{\partial n}{\partial t} = -\frac{n_i}{\tau} \frac{sinh(U_A/2)}{cosh[U_{FN} + U - U_A/2]} \]
Junction Recombination in Forward Bias

\[ \frac{\partial n}{\partial t} = -\frac{n_i}{\tau} \frac{\sinh(U_A/2)}{\cosh(U_{FN} + U - U_A/2)} \]

\[ I_R \approx -qA\left(\frac{n_i}{\tau}\right)\sinh\left(\frac{U_A}{2}\right) \int_0^W dx \frac{dx}{e^{(U_{FN} + U - U_A/2)}} \]

\[ I_R \approx -qA\left(\frac{n_i}{\tau}\right) \times \sinh\left(\frac{U_A}{2}\right) \int_0^W dx \frac{dx}{e^{-(\varepsilon_{\text{max}}x)/(kT/2q)}} \]

\[ I_{\text{Dep}} = -qA \left[ \frac{kT}{2q\varepsilon_{\text{max}}} \right] \left[ \frac{n_i}{\tau} e^{qV_A/2kT} \right] \]

Effective width  
Excess Carrier at mid-junction
Junction Leakage in Practice

Insulating Layer

r_j  d  r_j
p  n
Junction Recombination in Reverse Bias

\[ \frac{\partial n}{\partial t} = -\frac{n_i}{2\tau} \]  
(Remember: Recombination in depletion region)

\[ I_R \approx -qA \int_0^W \left( \frac{n_i}{2\tau} \right) dx \]

\[ = -qA \frac{n_i W}{2\tau} \propto \sqrt{V_{bi} - V_A} \]
Outline

1) Non-ideal effects: Junction recombination
2) Non-ideal effects: Impact ionization
3) Conclusion
Avalanche Breakdown

1. Diffusion limited
2. Ambipolar transport
3. High injection
4. R-G in depletion
5. **Breakdown**
6. Trap-assisted R-G
7. Esaki Tunneling
Asad: We should redraw the figure...
Nonlinearity due to Impact-Ionization
\[ I_n(x + dx) = I_n(x) + \alpha_n I_n(x)dx + \alpha_p I_p(x)dx \]

\[ \frac{I_n(x + dx) - I_n(x)}{dx} = \alpha_n I_n(x) + \alpha_p I_p(x) \]

\[ \Rightarrow \frac{dI_n(x)}{dx} = \alpha_n I_n(x) + \alpha_p I_p(x) \]

\[ \frac{dI_n(x)}{dx} = \alpha_p [I_T - I_n(x)] + \alpha_n I_n(x) \]

\[ \frac{dI_n(x)}{dx} - (\alpha_n - \alpha_p) I_n(x) = \alpha_p I_T \]
Impact-ionization

\[ I_n(0) \approx 0 \]

\[ I_n(W) = \int_0^W \alpha_p e^{\int_0^x (\alpha_n - \alpha_p) dx'} dx + \frac{I_n(0)}{I_T} \]

\[ I_T \]

\[ 1 + \int_0^W (\alpha_p - \alpha_n) e^{\int_0^x (\alpha_n - \alpha_p) dx'} dx \]

\[ I_p(W) + I_n(W) = I_T \Rightarrow I_n(W) \approx I_T \]

\[ I_n(0) \equiv \frac{1}{I_T M_p} \]

\[ 1 - \frac{1}{M_p} \approx 1 = \int_0^W \alpha_p e^{\int_0^x (\alpha_p - \alpha_n) dx'} dx \]
Impact-ionization

$$W = \int_{0}^{x} (\alpha_p - \alpha_n) \, dx'$$

$$\int_{0}^{x} \alpha_p e^{-\alpha \int_{0}^{x}} \, dx \approx 1$$

$$\alpha_p = \alpha_n \Rightarrow \alpha_p W = 1$$

$$\alpha_p = A_0 e^{-B/\epsilon}$$

$$\epsilon(0^-) = \frac{qN_D x_n}{k_s \epsilon_0} = \left[ \frac{2q}{k_s \epsilon_0} \frac{N_D N_A}{N_D + N_A} (V_{bi} - V_A) \right]^{1/2}$$
Impact-ionization: In Practice

Good ....

Bad....

Insulating Layer

Alam ECE-606 S09
Reduced field for p-i-n junction, because $V_{bi}$ (area under the curve) must be the same.
Modern Considerations: Dead Space

What happens if $W$ is less than the mean-free path?
Zener Breakdown vs. Impact Ionization

How do you differentiate between Zener tunneling and impact-ionization?
Conclusion

1) Junction recombination is often used as a diagnostic tool for process maturity. Defects in junction arises from misplaced donor impurities, not necessary from deep-trap impurities.

2) Impact ionization plays an important role in wide variety of devices (e.g. avalanche photo-diodes).

3) In the next class, we will discuss AC response of p-n junction diodes.