ECE606: Solid State Devices
Lecture 33: MOSCAP Electrostatics (II)

Muhammad Ashraful Alam
alam@purdue.edu
Outline

1. Review
2. Induced charges in depletion and inversion
3. Exact solution of electrostatic problem
4. Conclusion

REF: Chapters 15-18 from SDF
## Topic Map

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Threshold for Inversion

\[ V_G = \frac{qN_A x_0}{\kappa_{ox} \varepsilon_0} \sqrt{\frac{2 \kappa_{ox} \varepsilon_0}{qN_A}} \sqrt{\psi_s} + \psi_s \]

\[ V_{th} = \frac{qN_A x_0}{\kappa_{ox} \varepsilon_0} \sqrt{\frac{2 \kappa_{ox} \varepsilon_0}{qN_A}} \sqrt{2\phi_F + 2\phi_F} \]
What happens when surface potential is $2\phi_F$?

\[ V_{th} = \frac{qN_A x_0}{\kappa_{ox} \varepsilon_0} \sqrt{\frac{2\kappa_{ox} \varepsilon_0}{qN_A}} \sqrt{2\phi_F + 2\phi_F} \]

\[ n_{1s} = n_i e^{(E_F - E_{i,s}) \beta} \]

\[ = n_i e^{(E_F - E_{i(bulk)}) \beta} \times e^{(E_{i(bulk)} - E_{is}) \beta} \]

\[ = n_i e^{-\phi_F \beta} e^{(E_{i(bulk)} - E_{is}) \beta} \]

\[ n_{1s} = n_i e^{-\phi_F \beta} e^{2\phi_F \beta} \]

\[ = n_i e^{\phi_F \beta} = N_A \]

Electron concentration equals background acceptor concentration.
A little bit about scaling ....

\[ V_{th} = \frac{q N_A x_0}{\kappa_{ox} \varepsilon_0} \sqrt{\frac{2 \kappa_{ox} \varepsilon_0}{q N_A}} \sqrt{2 \phi_F} + 2 \phi_F \]

Reduce \( V_{th} \) by ...

Reducing oxide thickness
(from 1000 A in 1970s
to 10 A now)

Increase dielectric constant
(SiO\(_2\) historically, HfO\(_2\) now in Intel Penryn)
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Induced charges below Threshold

\[ n_{1s} = n_i e^{-\phi_F} e^{(E_{i(bulk)} - E_{is})\beta} \]

\[ \equiv B e^{q\psi_s \beta} \]

\[ \log_{10} Q_s(\psi_s) \]

\[ C / c^2 m \]

\[ \sim e^{-q\psi_s / 2k_B T} \]

\[ \sim \sqrt{\psi_s} \]

\[ V_G = \frac{qN_A x_0}{\kappa_{ox} \varepsilon_0} \sqrt{\frac{2\kappa_{ox} \varepsilon_0}{qN_A}} \sqrt{\psi_s} + \psi_s \]
Integrated charges below Threshold

\[ \frac{Q_i}{q} = \int_0^\infty n(x) \, dx \approx \int_0^{\infty} \frac{n_i^2}{N_B} e^{q\psi(x)\beta} \, dx \]

\[ = \frac{n_i^2}{N_B} \int_0^\infty e^{q\psi(x)\beta} \frac{1}{\mathcal{E}(x)} \, d\psi \]

\[ \approx \frac{1}{\langle \mathcal{E}(x) \rangle} \frac{n_i^2}{N_B} \int_0^\infty e^{q\psi(x)\beta} \, d\psi \]

\[ = \left( \frac{k_B T}{q} \right) \frac{n_i^2}{\langle \mathcal{E}(x) \rangle} \mathcal{E}(x) \equiv W_{\text{inv}} \times n_s \]
Charges above Threshold

\[ V_G = \psi_s + \mathcal{E}_{ox} x_o = \psi_s - \left[ \frac{Q_i(\psi_s) + Q_F}{K_{ox} \mathcal{E}_0} \right] x_o \]

\[ V_{th} = 2\phi_F + \mathcal{E}_{ox} x_o = 2\phi_F - \left( \frac{Q_i(2\phi_F) + Q_F}{K_{ox} \mathcal{E}_0} \right) x_o \]

\[ V_G - V_{th} = (\psi_s - 2\phi_F) + \frac{Q_i(\psi_s) - Q_i(2\phi_F)}{K_{ox} \mathcal{E}_0} x_o \]

\[ Q_i = C_{ox} (V_G - V_{th}) \]
Linear Charge Build-up Above Threshold?

\[ \log_{10} \left| Q_S (\psi_S) \right| \]

- Small changes \( \psi_s \) in changes \( Q_i \) a lot.
- Change in \( Q_i \) changes \( E_{ox} \), because \( E_{ox} = Q_i / \kappa_0 e_0 \).
- \( V_{ox} \) is large because \( V_{ox} = E_{ox} x_0 \), i.e. most of the drop above \( 2\psi_F \) goes to \( V_{ox} \).
- Acts like a parallel plate capacitor, hence the inversion equation.
Tunneling Current

\[ J_T = J_{s \rightarrow g} - J_{g \rightarrow s} \]
\[ = \left[ Q_i(V_G) e^{-\Delta E_C \beta} - q n_m e^{-\Delta E_C \beta} e^{-q V_{ox} \beta} \right] \nu_{th} \]
\[ = \left[ Q_i(V_G) - q n_m e^{-q V_{ox} \beta} \right] \nu_{th} T \quad T \equiv e^{-\Delta E_C \beta} \]

\[ J_T = \left[ Q_i(V_G) - q n_m e^{-q V_G \beta} \right] \nu_{th} \langle T(E) \rangle \]
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A step back: ‘Exact’ Solution of $Q_S(\psi_S)$

\[ \nabla \cdot \vec{D} = \rho \]

\[ \nabla \cdot \left( \frac{\vec{J}_n}{-q} \right) = (G - R) \]

\[ \nabla \cdot \left( \frac{\vec{J}_p}{q} \right) = (G - R) \]

\[
\frac{d^2 \psi}{dx^2} = \frac{-q}{\kappa_{si} \varepsilon_0} \left[ p_0(x) - n_0(x) + N_D^+ - N_A^- \right]
\]

Approximate ...

\[
V_G = \frac{qN_A x_0}{\kappa_{ox} \varepsilon_0} \sqrt{\frac{2 \kappa_{ox} \varepsilon_0}{qN_A}} \sqrt{\psi_S} + \psi_S
\]
Normalized Variable (to save some writing)...

\[ E_C(x) = \text{constant} - q\psi(x) \]

\[
\psi(x) = \frac{E_{C,\text{bulk}} - E_C(x)}{q}
\]

\[
u = \psi(x) = \frac{E_{i(\text{bulk})} - E_{i(x)}}{k_B T / q}
\]

\[
u_S = \frac{\psi_S}{k_B T / q} = \frac{E_{i(\text{bulk})} - E_{i(\text{surface})}}{k_B T}
\]

\[
u_F = \frac{\phi_F}{k_B T / q} = \frac{E_{i(\text{bulk})} - E_F}{k_B T}
\]
Normalized Variable (to save some writing!)

\[ p(x) = n_i e^{[E_i(x) - E_F] \beta} = n_i e^{(U_F - U)} \]

\[ n(x) = n_i e^{-[E_i(x) - E_F] \beta} = n_i e^{-(U_F - U)} \]

\[ N_D^+ = n_i e^{[E_F - E_{i,\text{bulk}}] \beta} = n_i e^{-(U_F)} \]

\[ N_A^- = n_i e^{-[E_F - E_{i,\text{bulk}}] \beta} = n_i e^{(U_F)} \]
Poisson-Boltzmann Equation

\[
\frac{d^2 \psi}{dx^2} = \frac{-q}{\kappa_s \varepsilon_0} \left[ p(x) - n(x) + N_D^+ - N_A^- \right]
\]

\[
\frac{q}{k_B T} \frac{d^2 U}{dx^2} = \frac{-qn_i}{\kappa_s \varepsilon_0} \left[ e^{+(U_F-U)} - e^{-(U_F-U)} + n_e^{-U_F} - n_i e^{U_F} \right] \equiv g(U, U_F)
\]

Can be evaluated at any \( U \)

\[
\left(2 \frac{dU}{dx}\right) \times \frac{d^2 U}{dx^2} = -\left(\frac{n_i k_B T}{\kappa_s \varepsilon_0}\right) g(U, U_F) \times \left(2 \frac{dU}{dx}\right)
\]

\[
\frac{d}{dx} \left(\frac{dU}{dx}\right)^2 \ dx = -\left. \frac{1}{2L_D^2} \ g(U, U_F) \left(2 \frac{dU}{dx}\right) \right|_0^U \ dx
\]

\[
-\frac{q \varepsilon(x)/kT}{0} \ d \left(\frac{dU}{dx}\right)^2 = -\frac{1}{L_D^2} \ U(x) \ g(U, U_F) \ dU
\]
Exact Solution (continued)

\[-q \mathcal{E}(x)/kT \int_0^U d \left( \frac{dU}{dx} \right)^2 = -\frac{1}{L_D^2} \int_0^U g(U, U_F) dU\]

\[\left[ \frac{q \mathcal{E}(x)}{kT} \right]^2 = \frac{1}{L_D^2} \int_0^U g(U, U_F) dU \equiv \frac{F^2(U, U_F)}{L_D^2}\]

\[\mathcal{E}_s = \frac{k_B T}{qL_D} F(U_s, U_F)\]

\[V_G = \psi_s + \left[ \frac{\kappa_s}{\kappa_{ox}} \mathcal{E}_s \right] x_0 = \psi_s + \frac{\kappa_s}{\kappa_{ox}} \frac{k_B T}{qL_D} F(U_s, U_F) x_0\]

Compare ...

\[V_G = \frac{qN_A x_0}{\kappa_{ox} \epsilon_0} \sqrt{\frac{2\kappa_{ox} \epsilon_0}{qN_A}} \sqrt{\psi_s + \psi_s}\]
How does the calculation go ...

\[
\left( \frac{q\mathcal{E}(x)}{kT} \right)^2 = \frac{1}{L_D^2} \int_0^{U(x)} g(U, U_F) dU \equiv \frac{F^2(U, U_F)}{L_D^2}
\]

\[V_G = \psi_s + \frac{\kappa_s}{\kappa_{ox}} \mathcal{E}_s x_0 = \psi_s + \frac{\kappa_s}{\kappa_{ox}} \frac{k_B T}{q L_D} F(U_s, U_F) x_0
\]

Begin with a surface potential

Calculate \( U_s \) and then divide \( U_s \) by N points.

Calculate \( g(U, U_F) \) at those points and integrate to find \( F(U_s, U_F) \)

Find \( V_G \).
Exact Solution...

Accumulation

Depletion

Inversion (weak)

Inversion (strong)
“Exact” solution is not really exact ...

\[ \frac{d^2 \psi}{dx^2} = \frac{-q}{\varepsilon} \left[ p(x) - n(x) |\psi(x)|^2 + N_D^+ - N_A^- \right] \]

Wave function should be accounted for

Bandgap widening near the interface must also should be accounted for.

Assumption of nondegeneracy may not always be valid

Wave function, not potential!
Our discussion today was focused on calculating the induced charge in the depletion and inversion region as a function of gate bias.

We found that we could calculate the tunneling current from the inversion changes by using the thermionic emission theory.

We also discussed the “exact” solution of the MOS-capacitor electrostatics. The “exact” solution is mathematically exact, but not necessarily physically exact solution of the electrostatic problem.