ECE606: Solid State Devices
Lecture 34: MOSCAP Frequency Response

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Outline

1. Background

2. Small signal capacitances

3. Large signal capacitance

4. Conclusion

Ref: Sec. 16.4 of SDF
# Topic Map

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Small Signal Equivalent Circuit

For insulated devices, consider only majority carrier junction capacitance $C_J$

Low frequency

High frequency

$C_J/C_0$

$V_G$

G0

C_D

C_J
Junction Capacitance

\[ V_G + \nu_s \sin \omega t \]

\[ C_G \equiv \frac{dQ_G}{dV_G} = \frac{d(-Q_s)}{dV_G} \]

\[ V_G = \psi_s - \frac{Q_s}{C_o} \]

\[ \frac{dV_G}{d(-Q_s)} = \frac{d\psi_s}{d(-Q_s)} + \frac{1}{C_o} \]

\[ \frac{1}{C_G} = \frac{1}{C_S} + \frac{1}{C_O} \]
Junction Capacitance

\[ \frac{1}{C_G} = \frac{1}{C_S} + \frac{1}{C_O} \]

\[ C_S \equiv \frac{d(-Q_S)}{d\psi_S} \]

\[ Q_S(\psi_S) \]

which we already understand!
Definition of $m$ for later use

$m = (1 + C_S / C_O)$

‘body effect coefficient’

$m = (1 + \kappa_S x_O / \kappa_0 W_T)$

in practice:

$1.1 \leq m \leq 1.4$

\[ \psi_s = \frac{C_O}{C_O + C_S} V_G \equiv \frac{V_G}{m} \]
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Junction Capacitance *in accumulation*

\[ C_{j,acc} \approx \frac{K_{ox} \varepsilon_0}{x_0} \equiv C_0 \]

\[ C_{j,acc} = \frac{C_0 C_{s,acc}}{C_0 + C_{s,acc}} \]

\[ C_{s,acc} = \frac{\kappa_s \varepsilon_0}{W_{acc}} \]

Low frequency \( \frac{C}{C_0} \)
Junction Capacitance \textit{in depletion}

\[ C_{j,\text{dep}} = \frac{C_0 C_s}{C_0 + C_s} = \frac{C_0}{1 + C_0/C_s} \]

\[ = \frac{C_0}{1 + \frac{\kappa_o \varepsilon_0}{x_0} / \sqrt{\frac{\kappa_s \varepsilon_0}{W}}} = \frac{C_0}{\sqrt{1 + \frac{V_G}{V_\delta}}} \]

\[ V_G = \frac{q N_A W}{\kappa_o \varepsilon_0} x_0 + \left(\frac{q N_A W^2}{2 \kappa_s \varepsilon_0}\right) \]

\[ \frac{\kappa_o W}{\kappa_s x_0} = \sqrt{1 + \frac{V_G}{V_\delta}} - 1 \]

\[ \frac{C}{C_0} \]

\[ V_G \]
Junction capacitance *in inversion*

\[ C_{j,\text{inv}} \approx \frac{\kappa_s \varepsilon_0}{x_0} \equiv C_0 \]

\[ C_{j,\text{inv}} = \frac{C_o C_{\text{inv}}}{C_o + C_{\text{inv}}} \quad C_{\text{inv}} \equiv \frac{\kappa_s \varepsilon_0}{W_{\text{inv}}} \]

\[ \frac{C}{C_o} \]

Low frequency

\[ V_G' > V_{T'} \]

Exposed Acceptors

Electrons

\[ +Q \]

\[ -Q \]

\[ V_G \]
Equivalent Oxide Thickness

\[ Q_i = -C_G \left( V_G - V_T \right) \]

\[ C_G = C_{j,\text{inv}} = \frac{C_o C_{\text{inv}}}{C_{\text{inv}} + C_o} < C_o \]

\[ C_o = \frac{\kappa_o \varepsilon_o}{x_0} \quad C_{\text{inv}} \equiv \frac{\kappa_s \varepsilon_o}{W_{\text{inv}}} \]

\[ C_G = \frac{\kappa_{ox} \varepsilon_o}{EOT_{\text{elec}}} \]

\[ EOT_{\text{elec}} = x_O + \left( \frac{\kappa_{ox} \varepsilon_o}{\kappa_s \varepsilon_o} \right) W_{\text{inv}} > x_O \]

‘Equivalent oxide thickness - electrical’
High frequency curve at inversion

$$C_{j,\text{inv}} \approx \frac{K_s \epsilon_0}{x_0} \equiv C_0$$

What about high frequency part of the curve?
Response Time

Dielectric Relaxation

\[ \tau = \frac{\sigma}{K_s \varepsilon_0} \]

SRH Recombination-Generation

\[ R = \frac{np - n_i^2}{\tau_n (p + p_1) + \tau_p (n + n_1)} \rightarrow \frac{-n_i}{\tau_n + \tau_p} \]

Ref. Lecture no. 15
High frequency response in MOS-C

Low Frequency

High Frequency

\( \Delta Q \)

\( W_T \)

\(-\Delta Q\)

\( C/C_0 \)

Low frequency

High frequency

\( V_G \)
Ideal vs. Real C-V Characteristics

Flat band voltage ...

Threshold voltage ...

$C/C_0$
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Large Signal Deep Depletion

\[ C_{j,\text{dep}} = \frac{C_0 C_s}{C_0 + C_s} = \frac{C_0}{1 + \frac{\kappa_{ox} W}{\kappa_s x_0}} \]

\[ = \frac{C_0}{\sqrt{1 + \frac{V_G}{V_\delta}}} \]

(even beyond threshold)
Relaxation from Deep Depletion

Depending on the measurement frequency, it will either merge with low-freq. or high-freq. curve.

\[ C/C_{ox} \]

Low frequency

High frequency

Deep depletion

\[ \Delta Q \]

\[ \rho(x) \]

\[ W_{dm} \]

\[ N_A \]

\[ x_0 \]
Ideal vs. Real C-V Characteristics

Flat band voltage ...

Threshold voltage ...

\[
\frac{C}{C_0} \quad V_G
\]
Low or High frequency?

Typically observe high-frequency CV

\[ G = \frac{n_i}{2\tau} \]

Typically observe low-frequency CV
No deep-depletion as well

What happens if I shine light on a MOS capacitor?
Summary

1) Since current flow through the oxide is small, we are primarily interested in the junction capacitance of the MOS-capacitor.

2) High frequency of MOS-C is very different than low-frequency C-V. In MOSFET, we only see low frequency response.

3) Deep depletion is an important consideration for MOS-capacitor that does not happen in MOSFETs.