ECE606: Solid State Devices
Lecture 35: MOSFET I-V Characteristics (I)

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Outline

1. Introduction

2. Sub-threshold (depletion) current

3. Super-threshold, inversion current

4. Conclusion
Subthreshold Region ($V_G < V_{th}$)
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\[ \Delta n = \psi \beta \left( \frac{W}{Q_n} \right) \]

\[ I_D (A) = N_A \frac{W}{Q_n} \left( \frac{V_G - V_{th}}{m} \right)^2 \]

\[ V_{G, subthreshold} \approx \frac{V_{th}}{2} \]
Recall the definition of body coefficient ($m$)

$$m = \left(1 + \frac{C_S}{C_O}\right)$$

‘Body Effect Coefficient’

$$m = \left(1 + \frac{\kappa_S x_O}{\kappa_0 W_T}\right)$$

in practice:

$$1.1 \leq m \leq 1.4$$
Outline

1. Introduction
2. Sub-threshold (depletion) current
3. Super-threshold, inversion current
4. Conclusion
Post-Threshold MOS Current \((V_G > V_{th})\)

\[
I_D = -\frac{W}{L_{ch}} \mu_{eff} \int_{0}^{V_{DS}} Q_i(V) dV
\]

1) Square Law

\[
Q_i(V) = -C_G \left[ V_G - V_T - V \right]
\]

2) Bulk Charge

\[
Q_i(V) = -C_G \left( V_G - V_{FB} - 2\psi_B - V - \frac{\sqrt{2q\varepsilon_{Si}N_A(2\phi_B + V)}}{C_O} \right)
\]

3) Simplified Bulk Charge

\[
Q_i(V) = -C_G \left[ V_G - V_T - mV \right]
\]

4) “Exact” (Pao-Sah or Pierret-Shields)
Gated doped or p-MOS with adjacent $n^+$ region
a) gate biased at flat-band
b) gate biased in inversion
The Effect of Drain Bias

2D band diagram for an n-MOSFET

a) device

b) equilibrium (flat band)

c) equilibrium ($\psi_S > 0$)

d) non-equilibrium with $V_G$ and $V_D > 0$ applied

Effect of a Reverse Bias at Drain

\[ \psi_S = 2\psi_B + V_R \]

Gated doped or p-MOS with adjacent, reverse-biased \( n^+ \) region

a) gate biased at flat-band
b) gate biased in depletion
b) gate biased in inversion

Inversion Charge in the Channel

\[ Q_i = -C_{ox} (V_G - V_{th} - V) + qN_A (W_T(V) - W_T(V = 0)) \]
Inversion Charge at one point in Channel

\[ V_{th} = 2\phi_F - \frac{qN_A W_T(V = 0)}{C_{ox}} \]

\[ V^*_{th} = (2\phi_F + V) - \frac{qN_A W_T(V)}{C_{ox}} \]

\[ V^*_{th} = V_{th} + V - \frac{qN_A (W_T(V) - W_T(V = 0))}{C_{ox}} \]

\[ Q_i = -C_{ox} (V_G - V^*_{th}) \]
Approximations for Inversion Charge

\[ Q_i = -C_o(V_G - V_{th} - V) + q_A \left( W_T(V) - W_T(V = 0) \right) \]

\[ = -C_o(V_G - V_{th} - V) + \left[ \sqrt{2q\kappa_S\varepsilon_o N_A(2\phi_B + V)} - \sqrt{2q\kappa_S\varepsilon_o N_A(2\phi_B)} \right] \]

Approximations:

\[ Q_i \approx -C_{ox}(V_G - V_{th} - V) \quad \text{Square law approximation} \ldots \]

\[ Q_i \approx -C_{ox}(V_G - V_{th} - mV) \quad \text{Simplified bulk charge approximation} \ldots \]
The MOSFET

\[ F_n = F_p = E_F \]

\[ F_n = F_p - qV_D \]

\[ F_n \] increasingly negative from source to drain
(reverse bias increases from source to drain)
$Q_i(y) = -C_{ox} \left[ V_G - V_{th} - mV(y) \right]$
Another view of Channel Potential

Source

N+

P-doped

Drain

N+

FP

FN

EC

EF

EV

N+ N+

Source Drain

x

x

E_C E_F E_V

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Square Law Theory

\[ J_1 = \frac{Q_1 \mu \mathcal{E}_1}{dx} \bigg|_{x_1} \]

\[ J_2 = \frac{Q_2 \mu \mathcal{E}_2}{dx} \bigg|_{x_2} \]

\[ J_3 = \frac{Q_3 \mu \mathcal{E}_3}{dx} \bigg|_{x_3} \]

\[ J_4 = \frac{Q_4 \mu \mathcal{E}_4}{dx} \bigg|_{x_4} \]

\[ \sum_{i=1,N} \frac{J_i \, dy}{\mu} = \sum_{i=1,N} Q_i \, dV \]

\[ \frac{J_D}{\mu} \sum_{i=1,N} dy = \int_{0}^{V_D} C_{ox} (V_G - V_{th} - mV) \, dV \]

\[ J_D = \frac{\mu C_{ox}}{L_{ch}} \left[ (V_G - V_{th}) V_D - m \frac{V_D^2}{2} \right] \]
Square Law or Simplified Bulk Charge Theory

\[ I_D = W \frac{\mu C_{ox}}{L_{ch}} \left[ (V_G - V_{th})V_D - m \frac{V_D^2}{2} \right] \]

\[ \frac{dI_D}{dV} = 0 = (V_G - V_{th}) - mV_D \Rightarrow V_{D,sat} = \frac{(V_G^* - V_{th})}{m} \]

\[ V_{DSAT} = (V_{GS} - V_T)/m \]

\[ I_D = \frac{W \mu C_o}{2mL_{ch}} (V_G - V_T)^2 \]

\[ J_D = \frac{\mu C_{ox}}{L_{ch}} \left[ (V_G - V_{th})V_D - m \frac{V_D^2}{2} \right] \]

\[ I_D = \mu C_o \frac{W}{L} (V_G - V_T)V_D \]
Why does the curve roll over?

\[ I_D = \frac{W \mu C_o}{2mL_{ch}} (V_G - V_T)^2 \]

\[ V_{DSAT} = \frac{(V_{GS} - V_T)}{m} \]

\[ Q_i \approx -C_{ox} (V_G - V_{th} - mV) \]

loss of inversion
Linear Region (Low $V_{DS}$)

$$I_D = \mu C_o \frac{W}{L_{ch}} (V_G - V_T) V_D$$

$$= \frac{V_{DS}}{R_{CH}}$$

Slope gives mobility

$V_{DS}$ small

Actual

Mobility degradation at high $V_{GS}$

Subthreshold Conduction

Intercept gives $V_T$
Summary

1) MOSFET differs from MOSCAP in that the field from the S/D contacts now causes a current to flow.

2) Two regimes, diffusion-dominated Subthreshold and drift-dominated super-threshold characteristics, define the $I_D$-$V_D$-$V_G$ characteristics of a MOSFET.

3) The simple bulk charge theory allows calculation of drain currents and provide many insights, but there are important limitations of the theory as well.