4.7. Law of Equilibrium
Consider a system with two one-electron energy levels having the same energy in equilibrium with an electrochemical potential $\mu$: $x = \frac{e}{kT}$.

4.7a If we assume that the interaction energy $U_0 = 0$, the average number of electrons in the system (both levels added together) at equilibrium is given by

(a) $\langle N \rangle = \frac{2}{1 + 2e^x}$
(b) $\langle N \rangle = \frac{2}{1 + e^x}$
(c) $\langle N \rangle = \frac{1}{1 + 2e^x}$
(d) $\langle N \rangle = \frac{1}{2 + e^x}$
(e) None of the above

\[ 1 = p_{00} + p_{01} + p_{10} + p_{11} = \frac{1 + 2e^{-x} + e^{-2x}}{Z} \rightarrow Z = \left(1 + e^{-x}\right)^2 \]
\[ \langle N \rangle = p_{01} + p_{10} + 2p_{11} = \frac{2e^{-x} + 2e^{-2x}}{1 + e^{-x}} = \frac{2}{1 + e^{-x}} = \frac{2}{2 + e^x} \]

4.7b If we assume that the interaction energy $U_0$ is extremely large so that the probability of having two electrons is zero, then the average number of electrons in the system at equilibrium is given by

(a) $\langle N \rangle = \frac{2}{2 + e^x}$
(b) $\langle N \rangle = \frac{2}{1 + e^x}$
(c) $\langle N \rangle = \frac{1}{1 + 2e^x}$
(d) $\langle N \rangle = \frac{1}{2 + e^x}$
(e) None of the above

\[ 1 = p_{00} + p_{01} + p_{10} = \frac{1 + 2e^{-x}}{Z} \rightarrow Z = 1 + 2e^{-x} \]
\[ \langle N \rangle = p_{01} + p_{10} = \frac{2e^{-x}}{Z} = \frac{2e^{-x}}{1 + 2e^{-x}} = \frac{2}{2 + e^x} \]