2.6. Scattering Theory

2.6a. A uniform 1D wire is modeled using just three points 1, 2 and 3 as shown with

\[
H = \begin{bmatrix}
    \varepsilon & t & 0 \\
    t & \varepsilon & t \\
    0 & t & \varepsilon
\end{bmatrix}
\]

For \( t < 0 \) and \( ka > 0 \), the matrices

\[
\begin{bmatrix}
    te^{i\alpha} & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & te^{i\alpha}
\end{bmatrix},
\begin{bmatrix}
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & 0
\end{bmatrix},
\begin{bmatrix}
    -2t \sin ka & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & -2t \sin ka
\end{bmatrix}
\]

represent the following in the order shown:

(a) \( \Sigma_1, \Sigma_2, \Gamma_1, \Gamma_1 + \Gamma_2 \)
(b) \( \Sigma_1, \Sigma_2, \Gamma_1, \Gamma_2 \)
(c) \( \Gamma_1, \Gamma_2, \Sigma_1, \Sigma_2 \)
(d) \( \Sigma_1, \Sigma_2, \Gamma_2, \Gamma_1 \)
(e) \( \Gamma_2, \Gamma_1, \Sigma_2, \Sigma_1 \)

2.6b. If a conductor has a dispersion relation \( E = \varepsilon + 2t \cos ka \) with \( t > 0 \), then

a) group velocity is positive for \( k > 0 \)

b) group velocity is positive for \( k < 0 \)

c) group velocity is positive for \( k = 0 \)

d) group velocity is never positive

e) group velocity is always positive