The Materials Science of Rechargeable Batteries

HW #3

1. You came back from a nice relaxing weekend and you find on your workbench three carefully labelled electrochemical cells: cell (I) which is a full cell with cathode, $h_c$, separator, $h_s$, and anode, $h_a$, layers; cell (II) which is a cathode half cell with cathode, $h_c$, and separator, $h_s$, layers only; and cell (III) which is an anode half cell with anode, $h_a$, and separator layers, $h_s$, only. Your manager has determined the tortuosity of each cell. So, cell (I) has a tortuosity $\tau_I = 5/2$, cell (II) has a tortuosity $\tau_{II} = 5/2$, and cell (III) has a tortuosity $\tau_{III} = 3$. She also left a nice sticky note where she reports the thickness of each layer: $h_c = 200\mu m$, $h_a = 100\mu m$, and $h_s = 25\mu m$.

i) Please determine the tortuosity of each of the individual layers.
(a) $\tau_c = 2, \tau_s = 5, \tau_a = 3$
(b) $\tau_c = 5, \tau_s = 2.2, \tau_a = 2$
(c) $\tau_c = 1.7, \tau_s = 1.5, \tau_a = 9$
(d) $\tau_c = 2.2, \tau_s = 5, \tau_a = 2.5$
(e) $\tau_c = 1.75, \tau_s = 1.5, \tau_a = 2.5$

ii) (10/60 points) Estimate the porosity of each layer assuming a Bruggeman-type model for the electrode layers.
(a) $\epsilon_c = 0.3, \epsilon_s = 0.1, \epsilon = 0.5$
(b) $\epsilon_c = 0.32, \epsilon_s = 0.44, \epsilon = 0.11$
(c) $\epsilon_c = 0.21, \epsilon_s = 0.04, \epsilon = 0.16$
(d) $\epsilon_c = 0.44, \epsilon_s = 0.35, \epsilon = 0.012$
(e) $\epsilon_c = 0.04, \epsilon_s = 0.2, \epsilon = 0.25$
2. (40 points) For a Bruggeman-type porous electrode, propose a measure of performance that simultaneously minimizes the tortuosity and maximizes the reactivity area per unit volume of a battery as a function of average porosity of particle size, \( r_p \). Based on this proposed measure, what is the optimal porosity value, \( \epsilon \)?

(a) \( \epsilon = 1/2 \) and \( R = 3/r_p \)
(b) \( \epsilon = 2/3 \) and \( R = 1/r_p \)
(c) \( \epsilon = 1/3 \) and \( R = 3/r_p \)
(d) \( \epsilon = 1/4 \) and \( R = 4/r_p \)
(e) \( \epsilon = 1/3 \) and \( R = 2/r_p \)