Unit 2: Essential Physics of the MOSFET

Lecture 2.1: Energy Band Diagram Review

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An energy band diagram is a plot of the bottom of the conduction band and the top of the valence band vs. position.

Energy band diagrams are a powerful tool for understanding semiconductor devices because they provide qualitative solutions to the semiconductor equations.
The Fermi level in equilibrium

The Fermi level is constant in equilibrium.

\[ J_n = n\mu_n \frac{dF_n}{dx} = 0 = n\mu_n \frac{dE_F}{dx} \rightarrow E_F \text{ is constant} \]
Band bending

What happens when we apply a voltage to the gate?

$+V_G$

$E_C$

$E_i$

$V = 0$

$E_F$

$E_V$

$x$

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Voltage and electron potential energy

\[ E = -qV \]

A positive potential **lowers** the energy of an electron.
Electrostatic potential causes band bending

\[ PE = E_C(x) = E_C(\infty) - qV(x) \]

\[ \frac{dE_C(x)}{dx} = -q \frac{dV(x)}{dx} = qE_x \]

The Fermi level is constant – even with a gate voltage applied because the current is zero.

\[ n(x) = n_i e^{(E_p - E_i(x))/k_BT} \]

\[ p(x) = n_i e^{(E_i(x) - E_F)/k_BT} \]
Band diagrams

1) Draw the band diagram

2) Read the band diagram

\[ V(x) \propto -E_C(x) \]

\[ \mathcal{E} \propto \frac{dE_C(x)}{dx} \]

\[ \log n(x) \propto E_F - E_i(x) \]

\[ \log \rho(x) \propto E_i(x) - E_F \]

\[ \rho(x) \propto \frac{d^2 E_C(x)}{dx^2} \]

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Practice

Sketch vs. position:

- Electrostatic potential
- Electric field
- Electron density
- Hole density
- Space charge density
Another example: NP junction in equilibrium

far from the junction, the bands will be flat

N

\[ n_0 \approx N_D \]

\[ \rho \approx 0 \]

P

\[ p_0 \approx N_A \]

\[ \rho \approx 0 \]

the bands will bend near the junction

far from the junction, the bands will be flat

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Procedure: Equilibrium energy band diagram

1) Begin with $E_F$

2) Draw the E-bands where you know the carrier density then connect the two regions.

3) Then “read” the energy band diagram to obtain the electrostatic potential, electric field, carrier densities, and space charge density vs. position.
Energy band diagram

N-type bands are flat

transition region

P-type bands are flat

$L = -x_n$  $x = 0$  $x = x_p$

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Now, “read” the e-band diagram

1) Electrostatic potential vs. position

2) Electric field vs. position

3) Electron and hole densities vs. position

4) Space-charge density vs. position
1) Electrostatic potential?

\[ V(x) \propto -E_C(x) \]

Diagram showing energy levels \( E_C \), \( E_F \), \( E_i \), and \( E_V \) as functions of \( x \).
Electrostatics: $V(x)$

$V(x) \propto -E_C(x)$

$V_{bi}$ is the “built-in voltage”

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2) Electric field?

\[ \mathcal{E} \propto \frac{dE_C(x)}{dx} \]

Diagram showing the electric field \( E \) as a function of position \( x \) with curves for different fields labeled as follows:

- \( E_C \)
- \( E_F \) (dashed red)
- \( E_i \) (dashed black)
- \( E_V \) (solid black)

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Electric field: $\mathcal{E}(x)$
3) Carrier densities?

\[ \log n(x) \propto E_F - E_i(x) \]

\[ \log p(x) \propto E_i(x) - E_F \]
Carrier densities vs. $x$

$n_{0N} = N_D$

$p_{0N} = n_i^2 / N_D$

$p_{0P} = N_A$

$n_{0P} = n_i^2 / N_A$

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4) Space charge density?

\[ \rho(x) \propto d^2 E_C(x)/dx^2 \]
\[ \rho(x) \propto dE/dx \]

\[ E_C, E_F, E_i, E_V \]

\[ \rho(x) \propto p(x) - n(x) + N_D(x) - N_A(x) \]
Electrostatics: $\rho(x)$

\[ \rho(x) = q(N_D - n(x)) \]

\[ \rho(x) = q(p(x) - N_A) \]
Summary

Three coupled, nonlinear
PDE's in three unknowns:

\[
\frac{\partial p}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_p}{q} \right) + G_p - R_p
\]

\[
\frac{\partial n}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_n}{-q} \right) + G_n - R_n
\]

\[
\nabla \cdot \left( K_S \varepsilon_0 \vec{E} \right) = \rho
\]

Drawing and then reading an E-band diagram
gives us a qualitative solution to these equations.
In the next lecture, we will use energy band diagrams to develop a qualitative understanding of MOSFET IV characteristics.