Nanophotonic Modeling Lecture 1.2: Bloch Theorem

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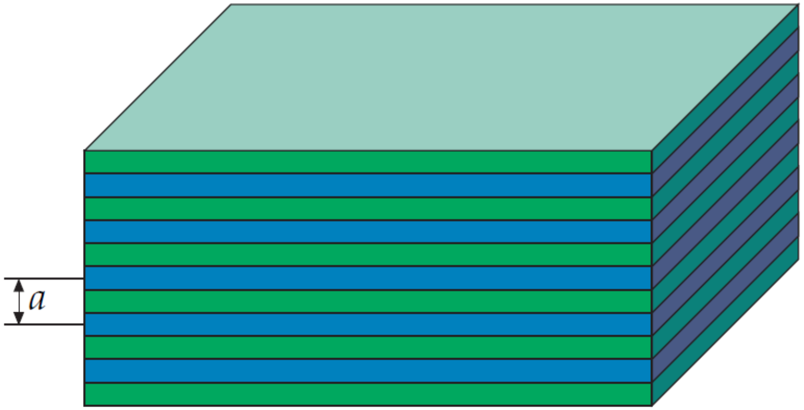
Electron Dilemma Also Found in Optics

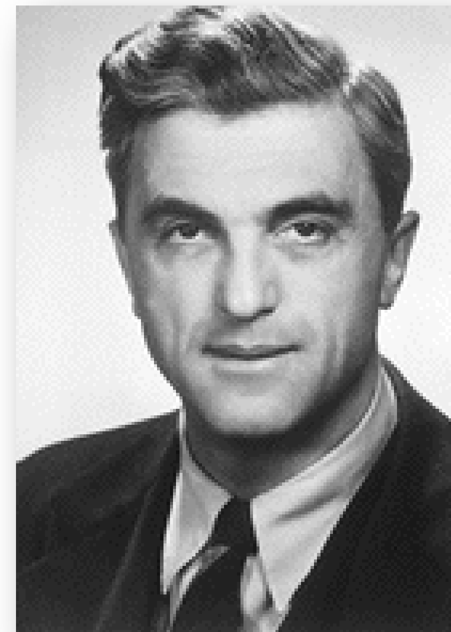
- Example: 1D stack of dielectric materials
- Dielectric Constant:
$$\epsilon(x) = \begin{cases} 1, & 0 < x < a/2 \\ 13, & a/2 < x < a \end{cases}$$
- And, $\epsilon(x+a) = \epsilon(x)$
- Boundary conditions:
$$E(x+a) = E(x)?$$
- Will each photon be stuck in its own little layer?



Bloch Theorem

"When I started to think about it, I felt that the main problem was to explain how the electrons could sneak by all the ions in a metal …. By straight Fourier analysis, I found to my delight that the wave differed from the plane wave of free electron only by a periodic modulation."

--Felix Bloch, *Physics Today* (1976)



Bloch Theorem

- Asserts that solution in periodic potential is always a product of two terms:
  - a periodic function (with the same period)
  - a plane wave
- Mathematically, we can write:

$$\Psi(x) = Ae^{ikx}u(x)$$

where $u(x + a) = u(x)$

Bloch Theorem: Numerical Solution

- Use Bloch's theorem to solve the eigenproblem numerically

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\left[e^{ikx}u(x)\right] = E(k)e^{ikx}u(x)$$

- What basis to use for periodic function?

Bloch Theorem: Real-Space Basis

- Real space is most obvious, with uniform grid
- Pull out plane wave from eigenvector to reduce complexity:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]u(x) = \left[E(k) - \frac{\hbar^2 k^2}{2m}\right] u(x)$$

- Immediate problem: not positive definite