Nanophotonic Modeling
Lecture 1.22: Summary of Unit 1

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Maxwell’s Eigenproblem is Solved by the Bloch Theorem

• Maxwell’s eigenproblem is given by:
  \[ \nabla \times \left[ \epsilon^{-1} \nabla \times H \right] = \left( \frac{\omega}{c} \right)^2 H \]

• The Bloch solution for $H$ in a periodic medium is always a product of two terms:
  – a periodic function (with the same period)
  – a plane wave

• This gives us the following matrix problem:
  \[ - (k + G) \times \left[ \epsilon^{-1}_{GG'} (k + G') \times h_{G-G'} \right] = \left( \frac{\omega}{c} \right)^2 h_G \]

• Implemented numerically in MIT Photonic Bands (MPB): http://jdj.mit.edu/mpb/
2D Photonic Crystals

- 2D triangular lattice can give rise to band gap for all polarizations for certain radii
Introducing a pointlike defect creates 3D confinement at one or more bandgap frequencies.
Photonic Crystal Slabs

Photonic bandstructure of square lattice of rods

Line defects create a low-loss waveguide
Rod-Hole 3D PhC

3D bandgap is fairly large

Removing a single rod creates 3D confinement in a very small volume
MPB Reformulates Eigenproblem for Rapid and Efficient Solutions

\[ |H_k^\pm\rangle \cong \sum_{\{m_j\}} \tilde{h}_{\{m_j\}} e^{i \sum_{j,k} m_j \tilde{G}_j \cdot n_k \tilde{R}_k / N_k} = \sum_{\{m_j\}} \tilde{h}_{\{m_j\}} e^{2\pi i \sum_{j} m_j n_j / N_j} \]

\[ A_{\ell m} = - \left( \tilde{k} + \tilde{G}_\ell \right) \times \text{IFFT} \cdots \tilde{\varepsilon}^{-1} \cdots \text{FFT} \cdots \left( \tilde{k} + \tilde{G}_m \right) \times \]

\[ \tilde{\varepsilon}^{-1} = \bar{\varepsilon}^{-1} P + \tilde{\varepsilon}^{-1} (1 - P) \quad P_{ij} = n_i n_j \]