# Nanophotonic Modeling Lecture 2.7: S-Matrices with Periodicity II

Prof. Peter Bermel

# S-Matrices: Periodic Solution Strategy

- Can rephrase:

$$[(\omega^2 - \mathcal{K})\mathcal{E}(\omega^2 - \mathcal{K}) - \omega^2 K]\phi = q^2(\omega^2 - \mathcal{K})\phi$$

- Where *H*-fields are written as:

$$\begin{pmatrix} h_x(z) \\ h_y(z) \end{pmatrix} = \sum_n \begin{pmatrix} \phi_{x_n} \\ \phi_{y_n} \end{pmatrix} (e^{iq_n z} a_n + e^{iq_n(d-z)} b_n)$$

$$h_{\parallel}(z) = \Phi[\hat{\mathrm{f}}(z)a + \hat{\mathrm{f}}(d-z)b]$$

- And where *E*-fields are given by:

$$\begin{pmatrix} -e_y(z) \\ e_x(z) \end{pmatrix} = \sum_n \begin{pmatrix} \hat{\eta} & 0 \\ 0 & \hat{\eta} \end{pmatrix} \left[ q_n^2 + \begin{pmatrix} \hat{k}_x \hat{k}_x & \hat{k}_x \hat{k}_y \\ \hat{k}_y \hat{k}_x & \hat{k}_y \hat{k}_y \end{pmatrix} \right] \times \begin{pmatrix} \phi_{x_n} \\ \phi_{y_n} \end{pmatrix} \frac{1}{q_n} (e^{iq_n z} a_n - e^{iq_n(d-z)} b_n)$$

$$e_{\parallel}(z) = (\omega^2 - \mathcal{K})\Phi \hat{q}^{-1}[\hat{\mathrm{f}}(z)a - \hat{\mathrm{f}}(d-z)b]$$

# S-Matrices: Periodic Solution Strategy

- Interface matrix in WC notation:

$$\begin{pmatrix} \hat{f}_l a_l \\ b_l \end{pmatrix} = I(l, l+1) \begin{pmatrix} a_{l+1} \\ \hat{f}_{l+1} b_{l+1} \end{pmatrix} = \begin{pmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{pmatrix} \begin{pmatrix} a_{l+1} \\ \hat{f}_{l+1} b_{l+1} \end{pmatrix}$$

- Where:

$$\begin{aligned} I(l, l+1) &= M_l^{-1} M_{l+1} \\ &= \tfrac{1}{2} \hat{q}_l \Phi_l^T (\omega^2 - \mathcal{K}_{l+1}) \Phi_{l+1} \hat{q}_{l+1}^{-1} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \\ &\quad + \tfrac{1}{2} \Phi_l^T (\omega^2 - \mathcal{K}_l) \Phi_{l+1} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \end{aligned}$$

# S-Matrix Construction: Recap

- We can relate the *s*-interface matrix to the *t*-interface matrix from before:

$$s^{(p)} = \begin{bmatrix} t^{(p)}_{11} - t^{(p)}_{12}\, t^{(p)\,-1}_{22}\, t^{(p)}_{21} & t^{(p)}_{12}\, t^{(p)\,-1}_{22} \\ -t^{(p)\,-1}_{22}\, t^{(p)}_{21} & t^{(p)\,-1}_{22} \end{bmatrix}$$

- Then iteratively construct next S-matrix via:

$$\begin{bmatrix} T^{(p)}_{uu} & R^{(p)}_{ud} \\ R^{(p)}_{du} & T^{(p)}_{dd} \end{bmatrix} = \begin{bmatrix} \tilde{t}^{(p)}_{uu} \left[1 - R^{(p-1)}_{ud}\, \tilde{r}^{(p)}_{du}\right]^{-1} T^{(p-1)}_{uu} \\ R^{(p-1)}_{ud} + T^{(p-1)}_{dd}\, \tilde{r}^{(p)}_{du} \left[1 - R^{(p-1)}_{ud}\, \tilde{r}^{(p)}_{du}\right]^{-1} T^{(p-1)}_{uu} \end{bmatrix}$$

# S-Matrices: Periodic Solution Strategy

- In WC's notation:

$$S_{11}(l',l+1) = (I_{11} - \hat{f}_l S_{12}(l',l) I_{21})^{-1} \hat{f}_l S_{11}(l',l)$$

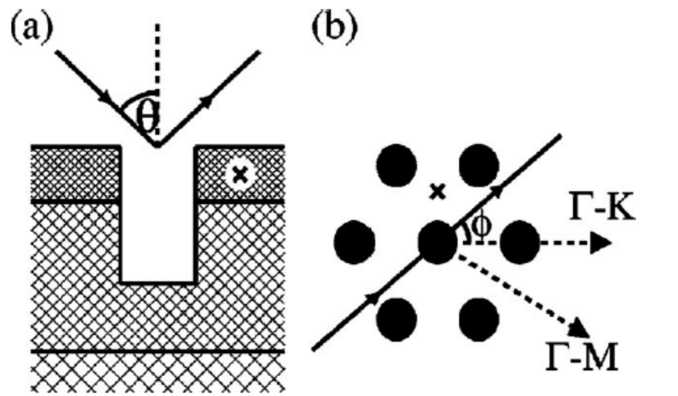
$$S_{12}(l',l+1) = (I_{11} - \hat{f}_l S_{12}(l',l) I_{21})^{-1} \times (\hat{f}_l S_{12}(l',l) I_{22} - I_{12}) \hat{f}_{l+1}$$

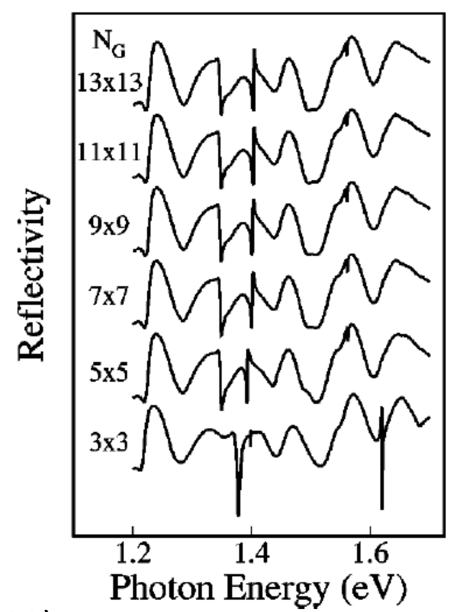
$$S_{21}(l',l+1) = S_{22}(l',l) I_{21} S_{11}(l',l+1) + S_{21}(l',l)$$

$$S_{22}(l',l+1) = S_{22}(l',l) I_{21} S_{12}(l',l+1) + S_{22}(l',l) I_{22} \hat{f}_{l+1}$$

# S-Matrix Simulations

Transmission through triangular lattice converges as number of plane waves $N_G$ increases




$$\varepsilon(\mathbf{G}) = \begin{cases} 2(\varepsilon_a - \varepsilon_b)\beta J_1(G\rho)/(G\rho) & \mathbf{G} \neq 0 \\ \varepsilon_a \beta + \varepsilon_b(1-\beta) & \mathbf{G} = 0, \end{cases}$$

Whittaker & Culshaw, *Phys. Rev. B* **60**, 2610 (1999)