Nanophotonic Modeling
Lecture 3.2: Finite Difference Time Domain Method

Prof. Peter Bermel
Equations for Light Propagation

\[ \frac{\partial B}{\partial t} = -\nabla \times \vec{E} - J_B - \sigma_B B \]

\[ \frac{\partial D}{\partial t} = \nabla \times \vec{H} - \vec{J} - \sigma_D D \]

\[ D = \epsilon E \]

\[ H = \frac{B}{\mu} \]

**Fictitious magnetic current**

**Fictitious magnetic conductivity**

**Material absorption**

**Radiating dipole source**

**dielectric function** \( \epsilon(x) = n^2(x) \)

**magnetic permeability** \( \mu(x) \)
Numerical PDE Solvers

• In context of electromagnetism, key independent variables are time and space
• Can approach problem by using ODE solution methods that ‘talk’ to each other
• Strategy for communication:
  – Discretize on Yee lattice
  – Ensure communication between neighbors
  – Alternate E & H field evaluation with leapfrog method
1D FDTD

- Maxwell’s equations in 1D simplify to:

\[
\mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}, \quad \varepsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x}
\]

- We can discretize them on a 1D chain:

\[
\frac{H_y^{q+\frac{1}{2}}[m + \frac{1}{2}] - H_y^{q-\frac{1}{2}}[m + \frac{1}{2}]}{\Delta_t} = \frac{E_z^q[m + 1] - E_z^q[m]}{\Delta_x}
\]
1D FDTD

• This yields 2 update equations:

\[
H_y^{q+\frac{1}{2}}\left[ m + \frac{1}{2} \right] = H_y^{q-\frac{1}{2}}\left[ m + \frac{1}{2} \right] + \frac{\Delta t}{\mu \Delta x} \left( E_z^q[m+1] - E_z^q[m] \right)
\]

\[
E_z^{q+1}[m] = E_z^q[m] + \frac{\Delta t}{\epsilon \Delta x} \left( H_y^{q+\frac{1}{2}}\left[ m + \frac{1}{2} \right] - H_y^{q+\frac{1}{2}}\left[ m - \frac{1}{2} \right] \right)
\]

• We can then solve these at alternating times to obtain second order accuracy

• But what do we do in higher dimensions?
Yee Lattice

• Proposed by Yee in 1966
• Lattice staggers E and H field locations by $\frac{1}{2}$ grid points for evaluation:
  – E-fields on edges
  – H-fields on faces
• Allows for maximum numerical accuracy and efficiency