Nanophotonic Modeling
Lecture 4.5: Beam Propagation Method

Prof. Peter Bermel
Beam Propagation Method

• Starting from the Helmholtz equation:

\[-\nabla^2 \psi = \left(\frac{n\omega}{c}\right)^2 \psi\]

• One can assume a solution of the form:

\[\psi = \phi e^{-j\beta z}\]

• Where \(\phi\) is slowly varying, which gives rise to:

\[-\nabla^2 \phi + 2j\beta \hat{\mathbf{z}} \cdot \nabla \phi = k^2_{\perp} \phi\]
Beam Propagation

- To simplify problem, drop second derivatives in $z$ – now we can write as:

$$
\frac{\partial \phi}{\partial z} = \frac{j}{2\beta} \nabla^2 \phi + \frac{jk^2}{2\beta} \phi
$$

- Can simplify by defining two operators:

$$
U = \frac{j}{2\beta} \nabla^2 \\
W = \frac{jk^2}{2\beta}
$$

$$
\frac{\partial \phi}{\partial z} = (U + W)\phi
$$
Nonlinear Schrödinger Equation

- Can derive expressions suitable for understanding fibers with dispersion and Kerr nonlinearity:

\[
\begin{align*}
U &= -\frac{j\beta_2}{2} \frac{\partial^2}{\partial t^2} \\
W &= -\alpha + \frac{j\kappa}{2} |\phi|^2 \\
\frac{\partial \phi}{\partial z} &= (U + W)\phi
\end{align*}
\]
Nonlinear Schrödinger Equation

• In the presence of nonlinearity, don’t actually know the value of $W(z+h)$
• Can obtain the result iteratively
  – Use $W(z)$ to evaluate $W(z+h)$
  – Work backwards to refine guess for $\phi(z+h)$
• After a few iterations, generally reach a self-consistent solution
Beam Propagation Method

• For a small z-step of size $h$, we can formally write a solution:

$$\phi(z + h) = e^{h(U+W)}\phi(z)$$

• If we know that $U$ and $W$ operators commute, we can rewrite as:

$$\phi(z + h) = e^{hU}e^{hW}\phi(z)$$

$$\phi(z + h) = e^{hU/2}e^{hW}e^{hU/2}\phi(z)$$