Nanophotonic Modeling
Lecture 4.10: An FEM Waveguide Mode Solver

Prof. Peter Bermel
BPM Mode Solver

• Can extend BPM method to solve for modes, by propagating in the imaginary direction

• First, drop all derivatives in BPM equation:

\[
[K]\{h_{t,l}\} = -\gamma^2 [M]\{h_{t,l}\}
\]

• Second, write down next step in \( z \):

\[
\{h_{t,l}\}_{k+1} = \frac{-2\gamma - 0.5 \Delta z k_o^2 (n_{\text{eff,}l}^2 - n_o^2)}{-2\gamma + 0.5 \Delta z k_o^2 (n_{\text{eff,}l}^2 - n_o^2)} \{h_{t,l}\}_k
\]

• Third, substitute special value of \( \Delta z \):

\[
\Delta z \approx j \frac{4n_o}{(n_{\text{eff,}l}^2 - n_o^2)k_o}
\]
BPM Mode Solver

• Since $\Delta z$ initially unknown, assume largest index possible, and decrease it as needed
• Will eventually converge to correct answer and effective refractive index:
  \[
  n_{\text{eff}, \ell, k}^2 = \frac{\{h_t\}^*_k[K]_k\{h_t\}_k}{k_o^2\{h_t\}^*_k[M]_k\{h_t\}_k}
  \]
• Can use Gram-Schmidt normalization procedure to find higher-order modes:
  \[
  \{h_t\}_{1, \text{new}} = \{h_t\}_1 - \sum_{\ell=1}^{i-1} \frac{\{h_{t,\ell}\}^*[M]\{h_t\}_1}{\{h_{t,\ell}\}^*[M]\{h_{t,\ell}\}}\{h_{t,\ell}\}
  \]
VBPM on a Waveguide: Problem Description

- Cross section defined above; $\lambda = 1.3 \, \mu m$
- Propagation along $z$ is semi-infinite
- Must grid space with first-order triangular elements in cross-sectional plane; choose PML to reduce reflections to $10^{-100}$
- Will vary $\Delta z$ for maximum effectiveness
VBPM on a Waveguide

- Fundamental mode is calculated accurately with 12,800 first-order triangular elements
• Propagation step size in $Z$, known as $\Delta Z$, should equal transverse dimensions for best accuracy.