Nanophotonic Modeling
Lecture 4.14: Thermal Transport Modeling

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Thermal Transport: Conduction

• Can formulate residual as:

\[ r_B = c_V \frac{\partial T}{\partial t} - \nabla \cdot k \nabla T - Q \]

• Writing down Galerkin method:

\[ \int_V \eta r_B \, dV = 0 \]

Integrating by parts and dropping boundary yields:

\[ \int_V \left[ \eta c_V \frac{\partial T}{\partial t} + \nabla \eta \cdot k \nabla T - \eta Q \right] \, dV = 0 \]
Thermal Transport: Conduction

• Now assume $\eta = N_i$ and $T = \sum_i N_i T_i$:

$$\sum_i \left[ \left( \oint N_j c_v N_i \, dS \right) \frac{\partial T_i}{\partial t} + \left( \oint \nabla N_j \cdot k \nabla N_i \, dS \right) T_i - \oint N_j Q \, dS \right] = 0$$

• This can be simplified as:

$$\sum_i \left[ C_{ji} \frac{\partial T_i}{\partial t} + K_{ji} T_i - L_j \right] = 0$$
Thermal Transport: Radiative Transfer

• Heat transfer via photon emission
• For a blackbody, total emission follows Stefan-Boltmann law:

\[ P = \sigma T^4 \]

• Net thermal transfer between two infinite surfaces becomes:

\[ Q = \sigma (T_1^4 - T_2^4) \]
Thermal Transport: Radiative Transfer

• Emission for real materials depends on emissivity
• In thermal equilibrium, Kirchoff’s law states emissivity=absorptivity at each wavelength
• Emission spectrum is given by:
  \[
  \frac{dQ}{d\lambda} = \frac{2\pi hc^2 \varepsilon(\lambda)}{\lambda^5 \left[e^{hc/\lambda kT} - 1\right]}
  \]
• Blackbody result recovered by setting \(\varepsilon(\lambda) = 1\) and integrating
Thermal Transport: Modeling

• Convection amounts to a boundary condition in most problems
• Will thus be first combined with conduction
• Strategy:
  – Create FEM grid for thermal conduction
  – Impose BC’s from convection
  – (Optionally) include radiative transfer from disconnected bodies