Fundamentals of Nanotransistors
Unit 3 Homework
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In the Unit 3 lectures, I largely used Maxwell-Boltzmann (nondegenerate) statistics for carriers, because it simplified the calculations but still illustrated the key points. In practice, one would need to include Fermi-Dirac statistics, and that complicates things by bringing in Fermi-Dirac integrals, but the fully degenerate \( T = 0 \text{ K} \) case is just as easy to solve as the nondegenerate case, and it is also illustrative of more general principles. In this homework assignment, you will re-do many of the calculations done in the lectures, but this time assuming \( T = 0 \text{ K} \). Hopefully this will deepen your understanding of the concepts discussed in Unit 3.

1) Consider a modern, silicon, N-channel MOSFET with the following parameters:

\[
L = 20 \text{ {nm}} \\
C_{\text{inv}} = 2.8 \mu \text{C/cm}^2 \\
\mu_n = 200 \text{ {cm}^2/V-s} \\
R_S = R_D = 100 \Omega - \mu\text{m} \\
V_{DD} = 0.7 \text{ V} \\
T = 300 \text{ K} \\
m^* = 0.19 m_0
\]

Compute the ballistic mobility and compare its value to the scattering limited, bulk mobility, \( \mu_n = 200 \text{ {cm}^2/V-s} \). You may assume non-degenerate (Maxwell-Boltzmann) statistics for electrons. (In the last problem in this HW assignment, we will re-visit this problem assuming fully degenerate carrier statistics.)

2) To compute the current in an N-channel MOSFET, we would begin with the Landauer expression,

\[
I = \left( \frac{2q}{\hbar} \right) \int_{E_1}^{E_2} T(E) M(E) (f_1 - f_2) dE,
\]

Assume that contact one is grounded and that a positive voltage (not necessarily small) has been applied to contact 2. Assume 2D electrons, \( T = 0 \text{ K} \), that the transmission, \( T_0 \), is independent of energy, and answer the following questions.
Unit 3 HW (continued)

2a) Determine the limits of integration, $E_1$ and $E_2$, for the integral in the Landauer expression.

2b) Evaluate the integral to obtain an expression for the drain current of an N-channel MOSFET at $T = 0$ K.

We will see later in this HW assignment that this equation can be used to compute the current of a ballistic MOSFET.

3) In the Landauer approach, current is proportional to the number of channels in the Fermi window:

$$\langle M \rangle \equiv \int M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE$$

At $T = 0$ K, this becomes

$$\langle M \rangle = M(E_F)$$

Compute the number of channels for a MOSFET under high gate bias with an electron density of $n_S = 10^{13}$ cm$^{-3}$. Assume a silicon MOSFET with $m^* = 0.19m_0$, $g_V = 2$, $T = 0$ K, and that $W = 100$ nm. Assume a small drain bias so that $E_{F1} \approx E_{F2} = E_F$.

4) In Lecture 3.3, we showed that for small drain biases, current is proportional to voltage, $I = GV$, and that at $T = 0$ K:

$$G(T = 0 \text{ K}) = \frac{2e^2}{h} T \langle M \rangle F(E_F)$$

Show that the solution to problem 2) simplifies to this expected result for small voltages.

5) In Lecture 3.3, we showed that for non-degenerate, Maxwell-Boltzmann, carrier statistics,

$$I_{ON}^{\text{ball}} = qn_s v_T$$

where

$$v_T = \sqrt{\frac{2k_B T}{\pi m^*}}$$

is the nondegenerate uni-directional thermal velocity.

Use the results of problem 2) to determine the ballistic on-current for $T = 0$ K.
Unit 3 HW (continued)

6) The ballistic injection velocity is an important quantity for a MOSFET. Compare the value of the ballistic injection velocity for a typical Si MOSFET computed assuming nondegenerate statistics to the value computed assuming fully degenerate ($T = 0$ K) statistics. Assume the following parameters:

\[ T = 300 \text{ K} \]
\[ m^* = 0.19 m_0 \]
\[ g_v = 2 \]
\[ n_S = 10^{13} \text{ cm}^{-2} \]

7) In Lecture 3.4, we derived the following expression for the ballistic channel conductance assuming non-degenerate carrier statistics

\[ G_{CH} = W Q_n(V_{GS}, V_{DS}) \frac{v_T}{2(k_B T / q)} \ (*) \]

Derive the corresponding expression for the fully degenerate ($T = 0$ K) case.

8) The ballistic model we developed in Lecture 3.4,

\[ I_{DS} = W |Q_n(V_{GS}, V_{DS})| v_T \left( \frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}} \right) \]

assumed non-degenerate carrier statistics. In this case, the drain current saturates when $V_{DS} >> k_B T$. Answer the following questions about the $T = 0$ K drain saturation voltage.

8a) Derive an expression for $V_{DSAT}$.

8b) Compare nondegenerate and degenerate drain saturation voltages for a silicon MOSFET biased at a gate voltage so that $n_S = 10^{13}$ cm$^{-3}$.

9) Now let’s re-visit prob. 1) but this time assuming fully degenerate conditions. Answer the following questions.

9a) Derive an expression for the ballistic mobility at $T = 0$ K.

9b) Numerically evaluate the ballistic mobility for the MOSFET of prob. 1) and compare your answer to the result obtained by assuming nondegenerate conditions.